



University of Pisa

# Neural Modelling and Computational Neuroscience

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# How to send to us exam material?

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- ▶ Email to us (Bacciu, Micheli, Valenza)

[micheli@di.unipi.it, bacciu@di.unipi.it, g.valenza@iet.unipi.it,]

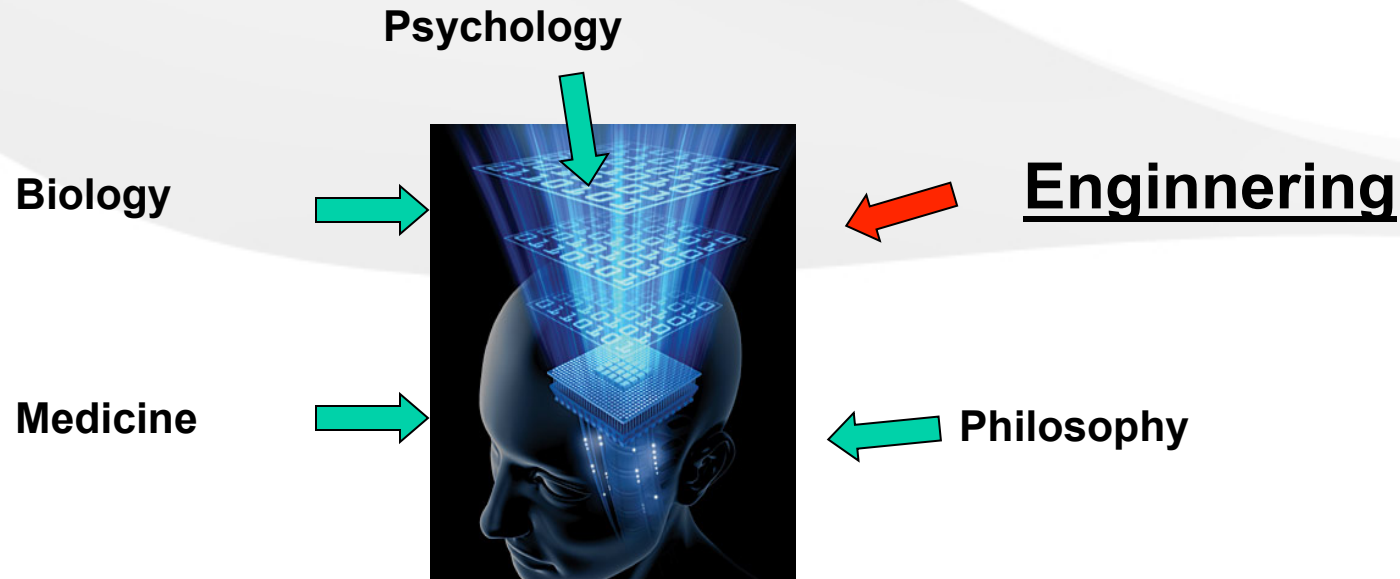
- ▶ **Subject:** [CNS-2016] student Rossi exam material
- ▶ **Body (email text):**
  - ▶ Name Surname, email contact
  - ▶ Master degree programme (Bionics eng. or Computer Science?)
  - ▶ Material attachments (lab source code files, report for the project or slides for the presentation).
  - ▶ Any note you find useful to us

# Neuroscience modeling

- ▶ Introduction to neurophysiology
- ▶ Neural organization and mapping in the brain
- ▶ Introduction to bio-inspired neural modeling
- ▶ Neural modeling:
  - ▶ From perceptron to hodgkin-huxley through Izhikevich,
  - ▶ Spiking neural networks,
  - ▶ The theory of neural group selection,
  - ▶ The role of synaptic delays in a computational brain,
  - ▶ Spike-timing dependent plasticity rule,
  - ▶ Neural memory,
  - ▶ Neural decoding and perception mirror neurons,
  - ▶ Modeling neural cell culture dynamics
- ▶ Introduction to glia and astrocyte cells, the role of astrocytes in a computational brain, modeling neuron-astrocyte interaction, neuron-astrocyte networks,
- ▶ The role of computational neuroscience in neuro-biology and robotics applications.

# Neuroscience

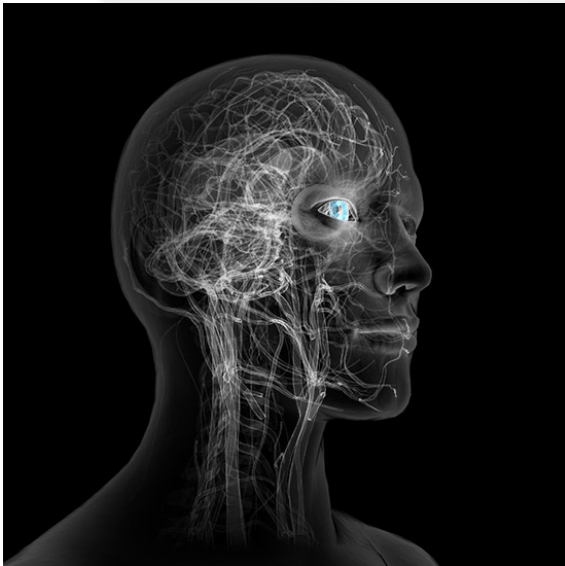
**Neuroscience is the scientific study of the nervous system.**



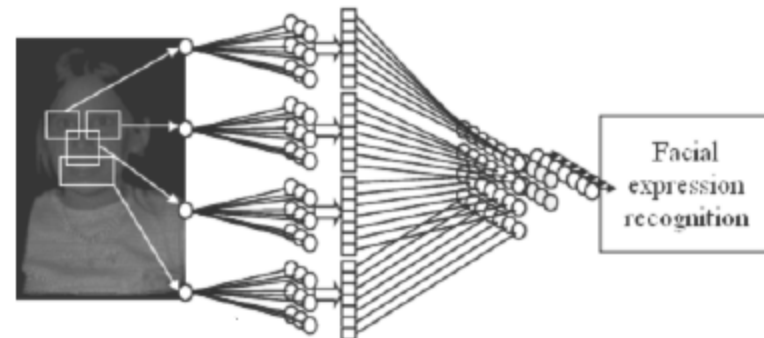
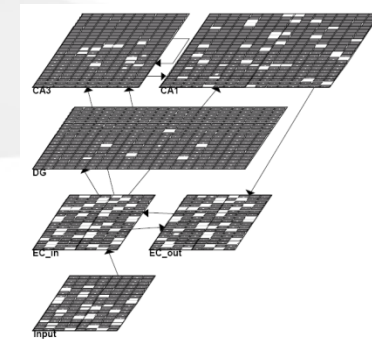
## Computational Neuroscience



## Neural Modeling and Dynamics



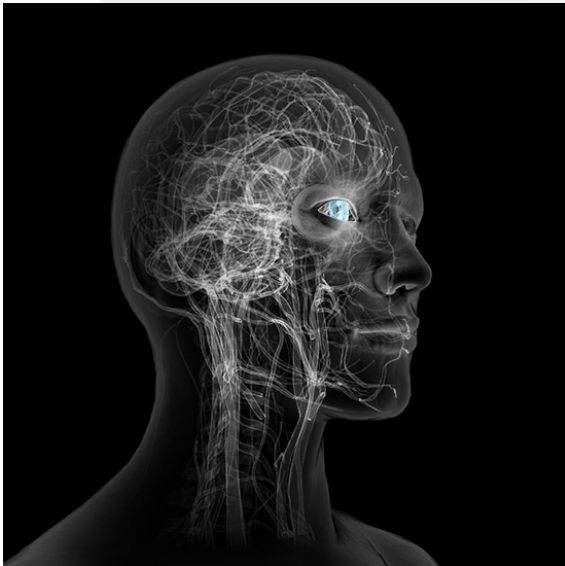
## Artificial Intelligence



## Computational Neuroscience

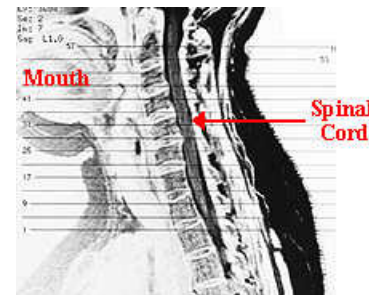
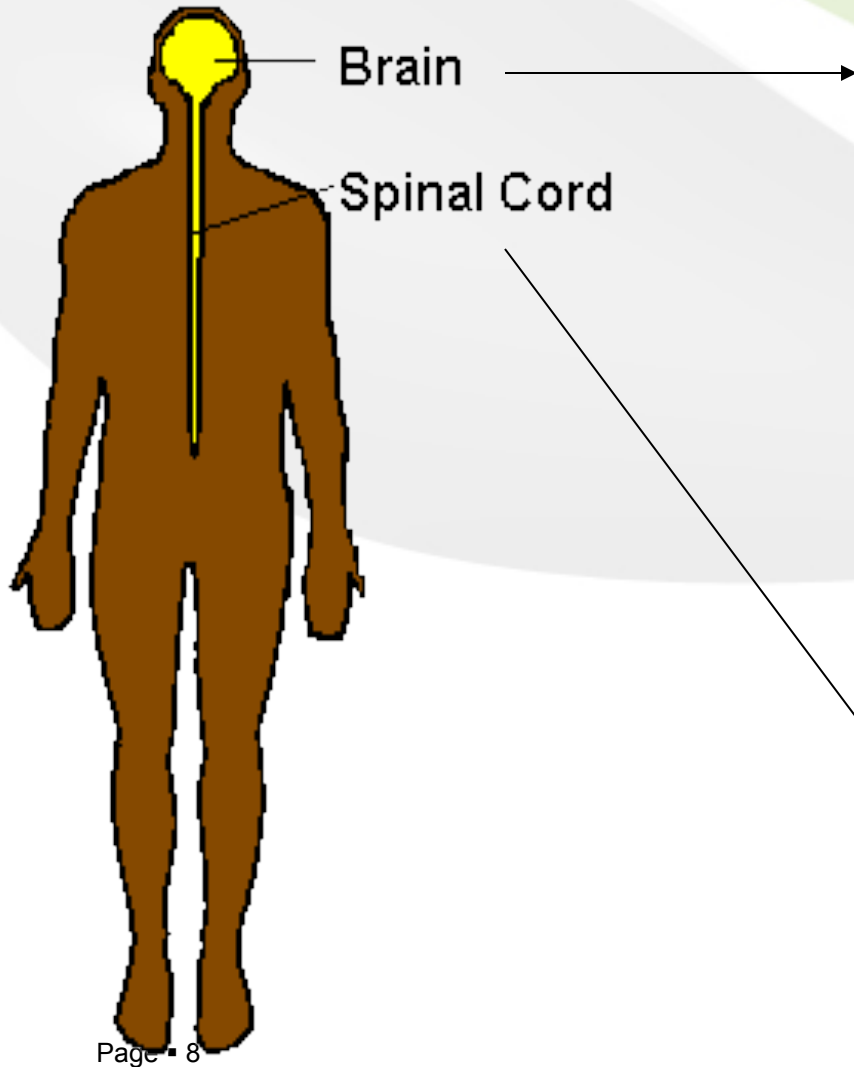


### Neural Modeling and Dynamics



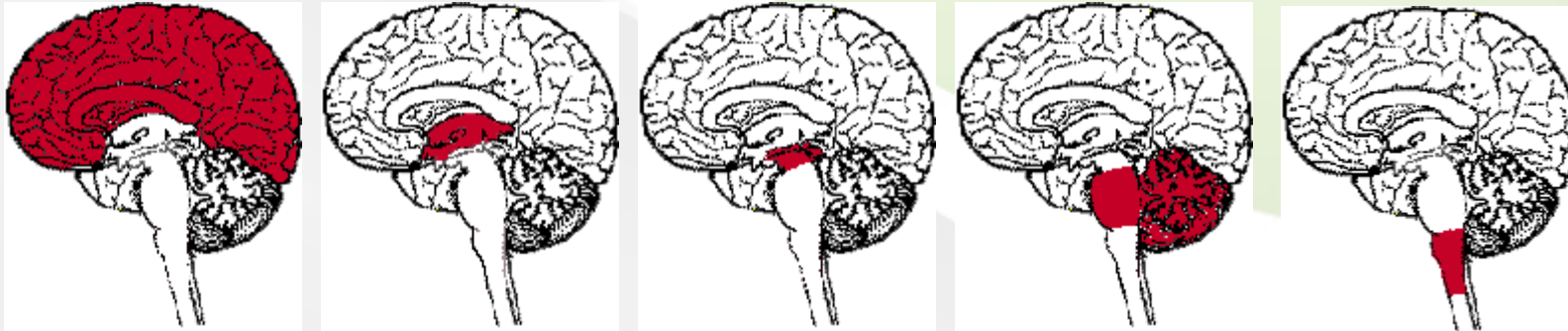
**Aim:** study and definition of physiologically plausible mathematical models to simulate actual neural dynamics

# Central Nervous System (CNS)

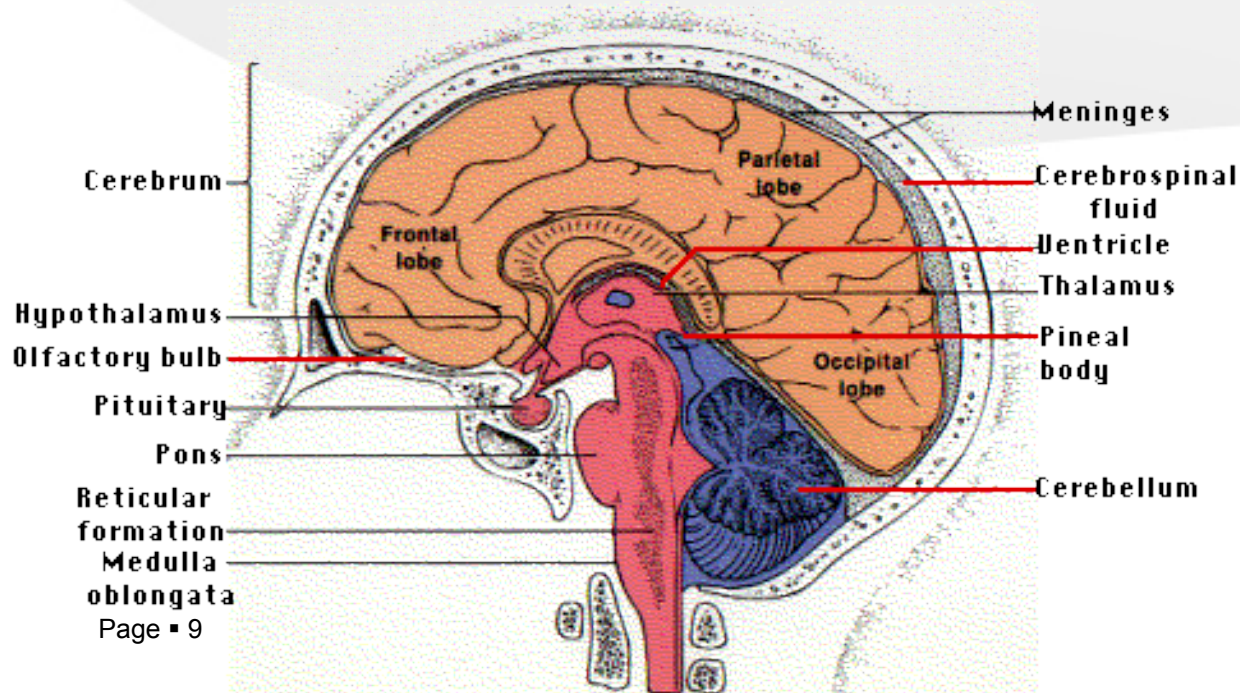




# Encephalon: Anatomical division

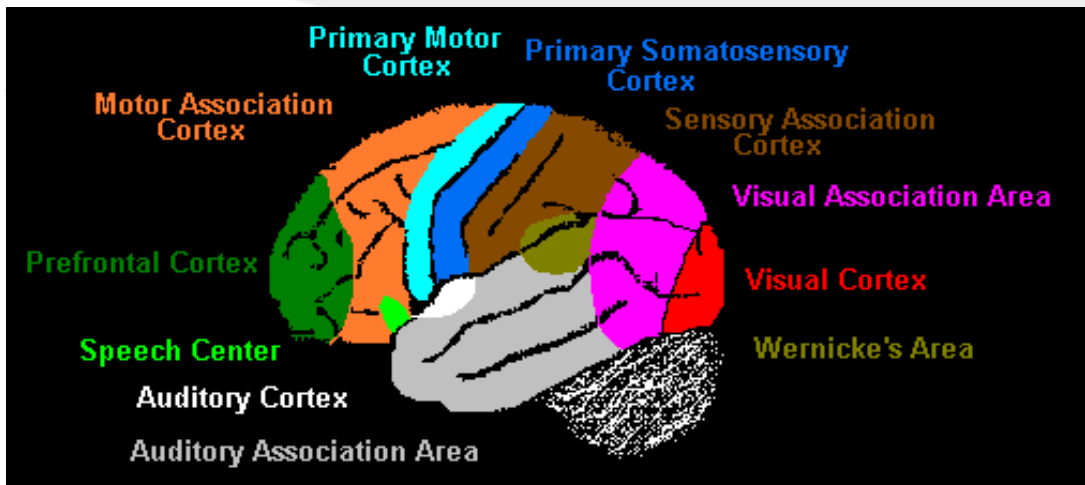
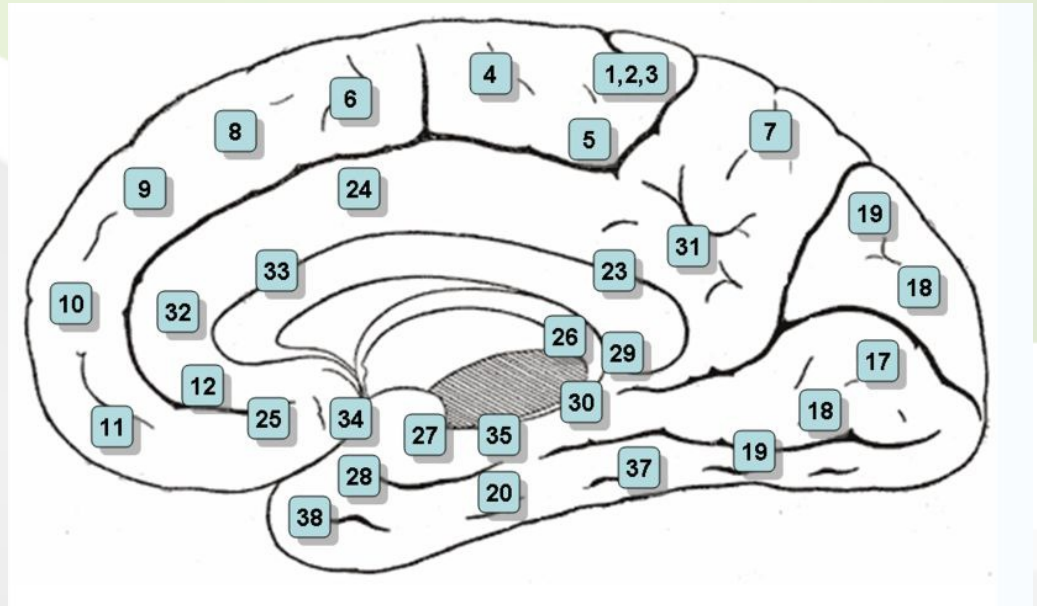


Telencephalon    Diencephalon    Mesencephalon    Metencephalon    Myelencephalon

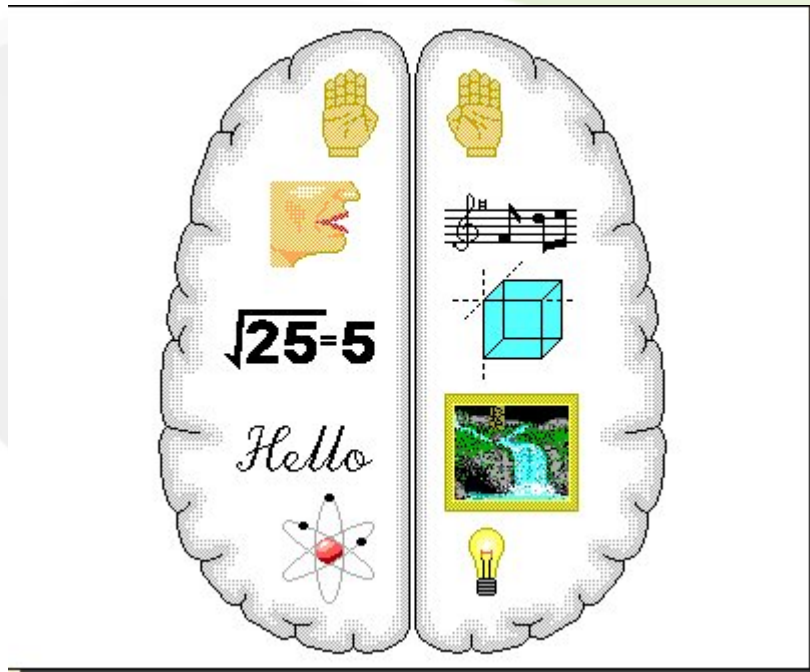


# The Cerebral Cortex: Functional Division

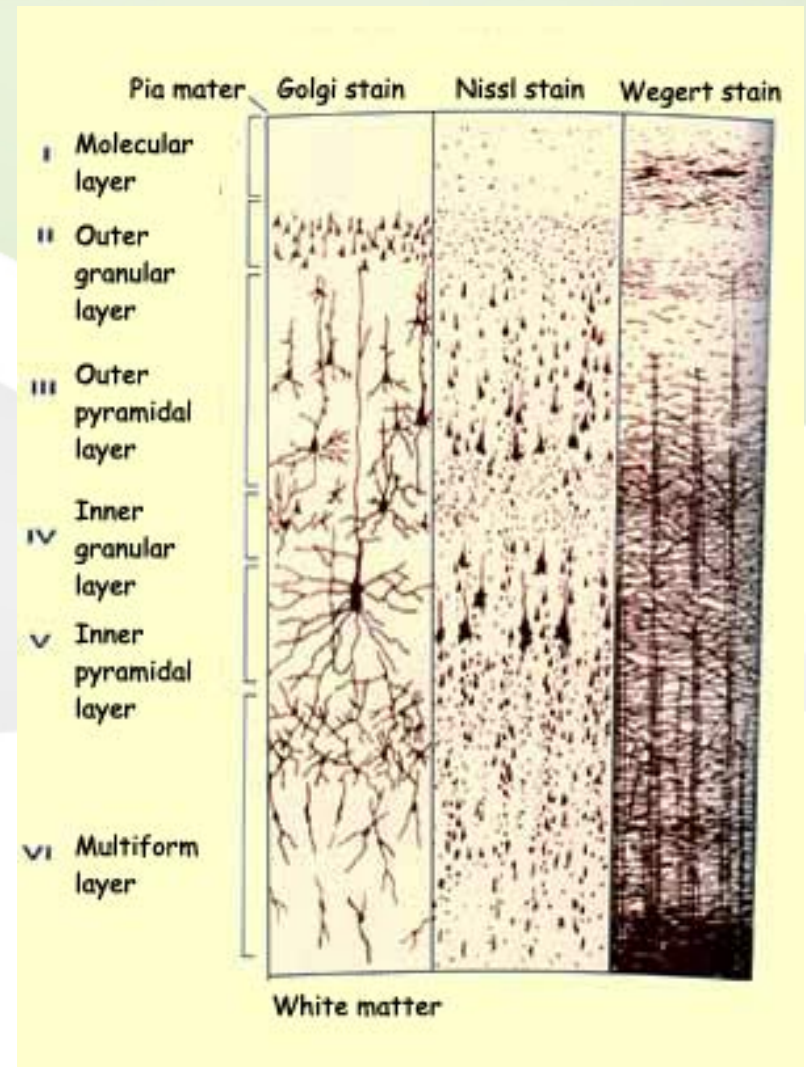
## Brodmann Areas



# The Cerebral Cortex (1)

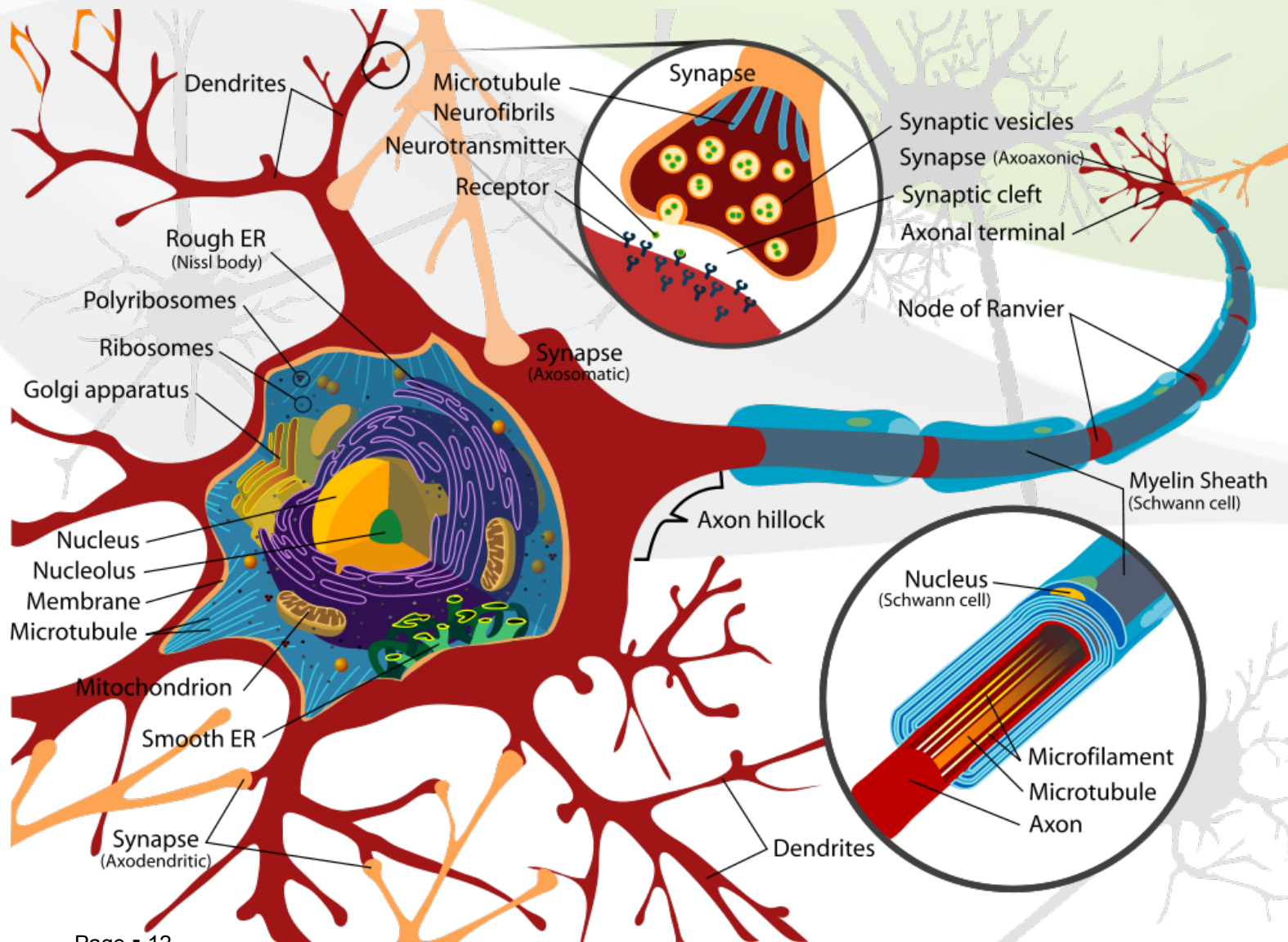


The cerebral cortex is responsible of many cognitive functions such as language, memory, emotional processing, etc.



Six layers of neurons

# Modelling Neural Dynamics



# Levels of Modeling

Brain as a whole

Specific brain systems (visual system,...)

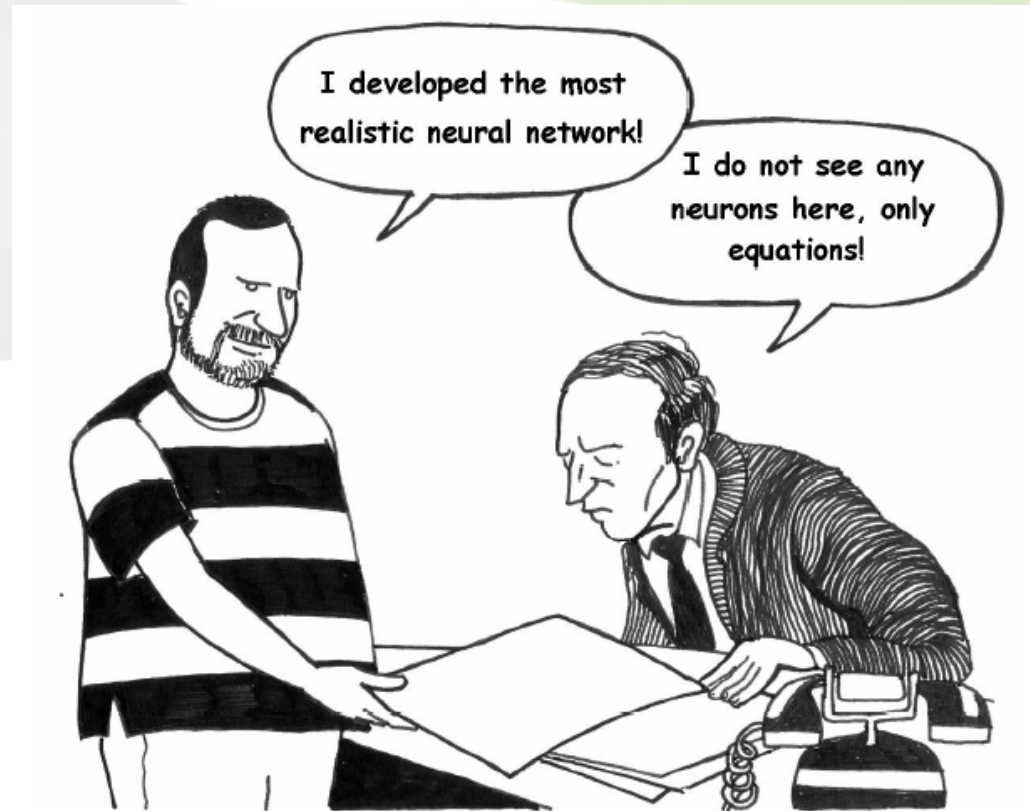
Large scale neural networks

Small neural networks

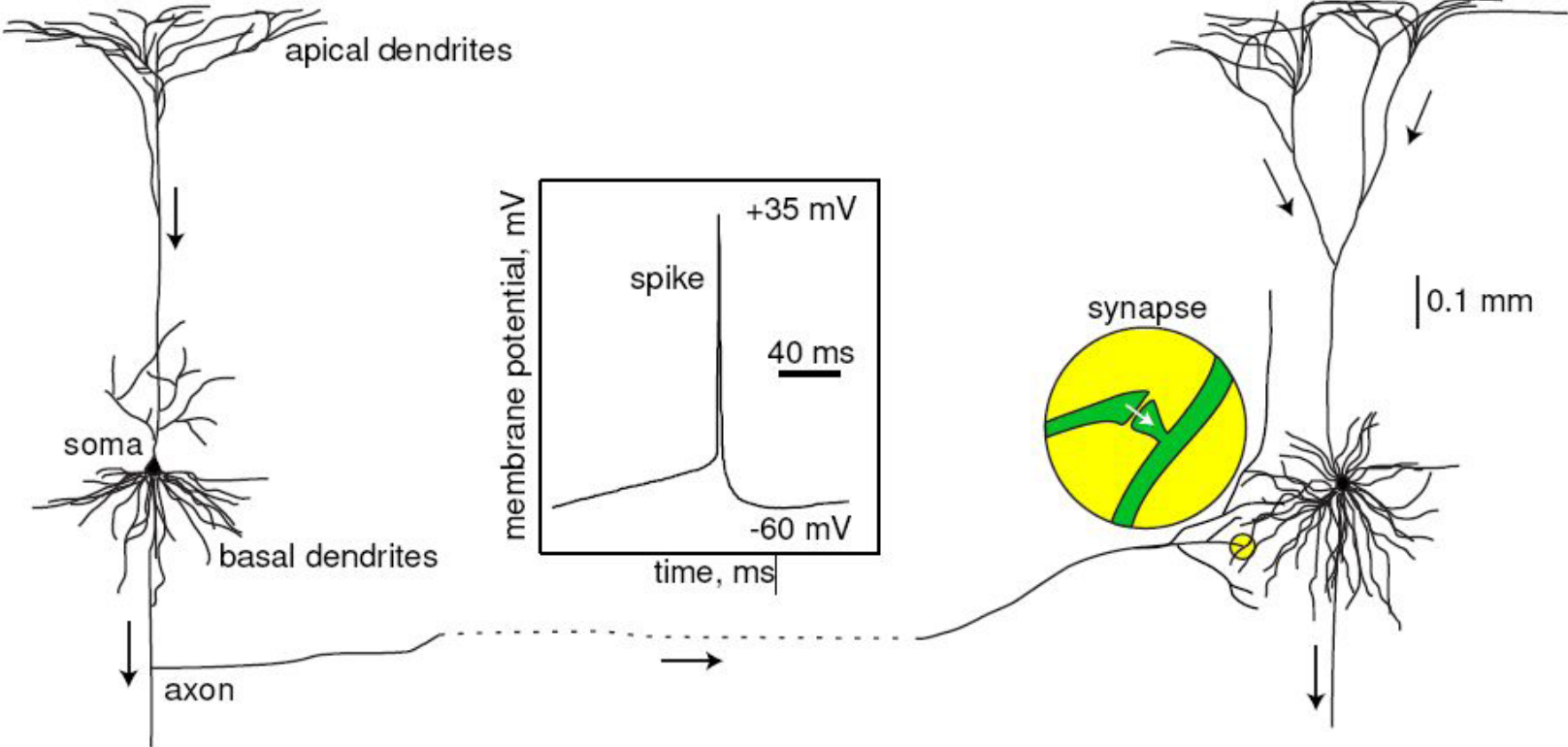
Neurons

Ion channels and synapses

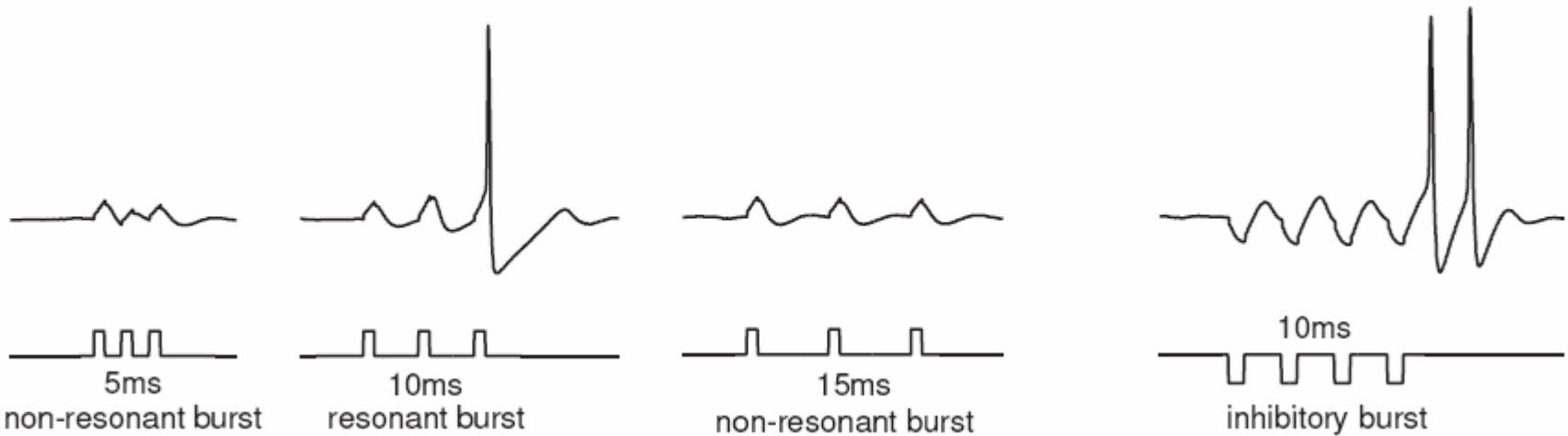
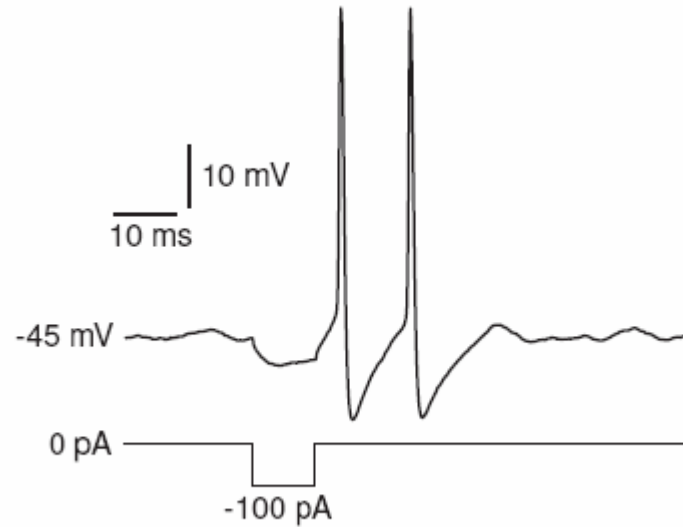
Molecular processes



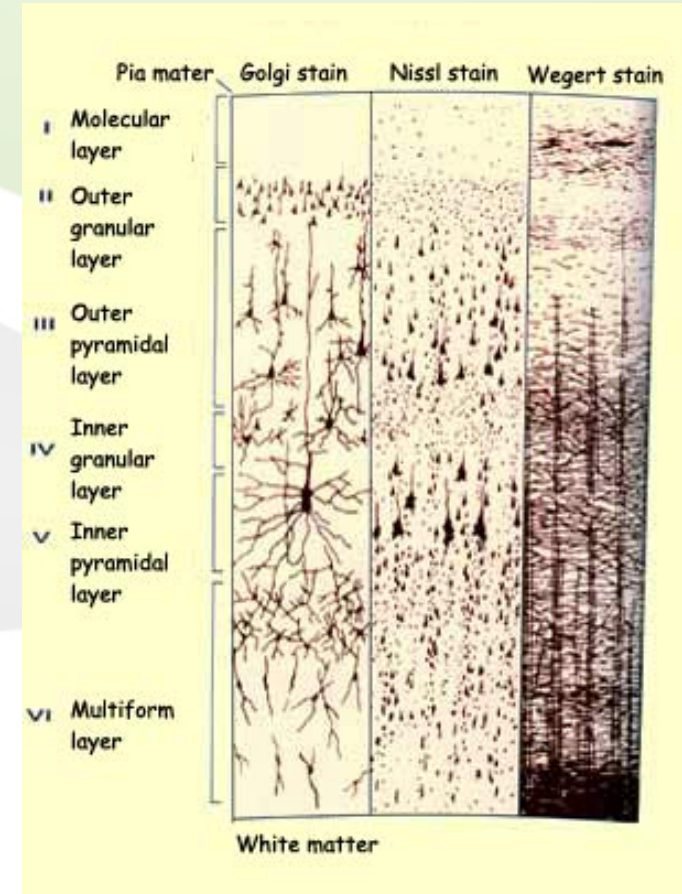
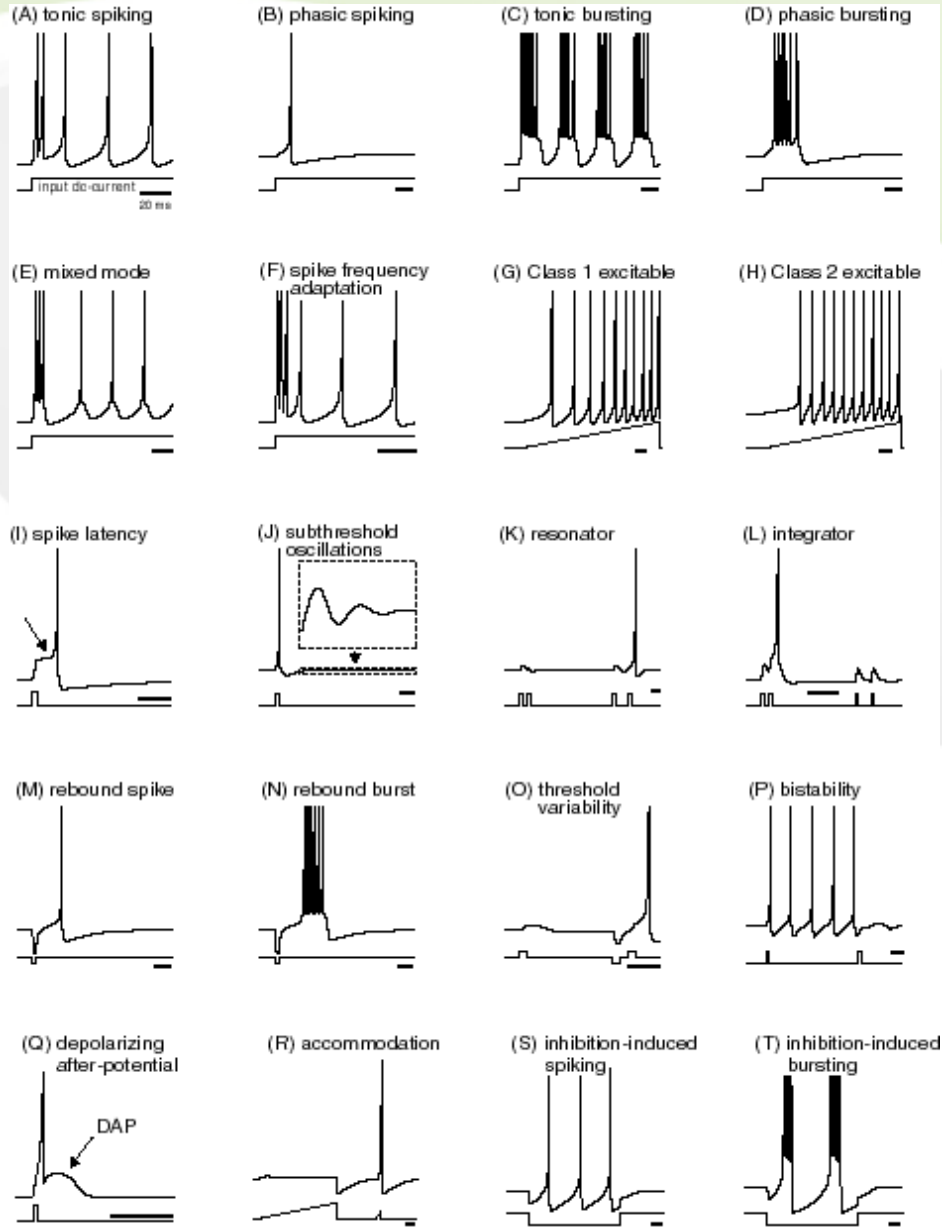
# Neural Spiking



# Neural Spiking



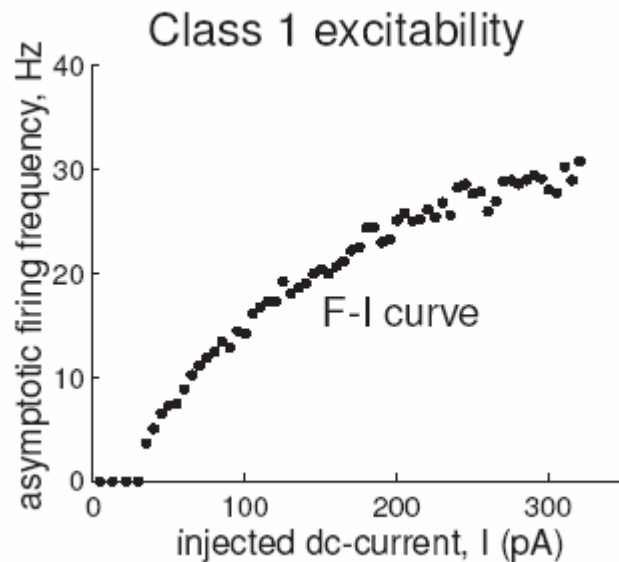
# Particular Neural Dynamics



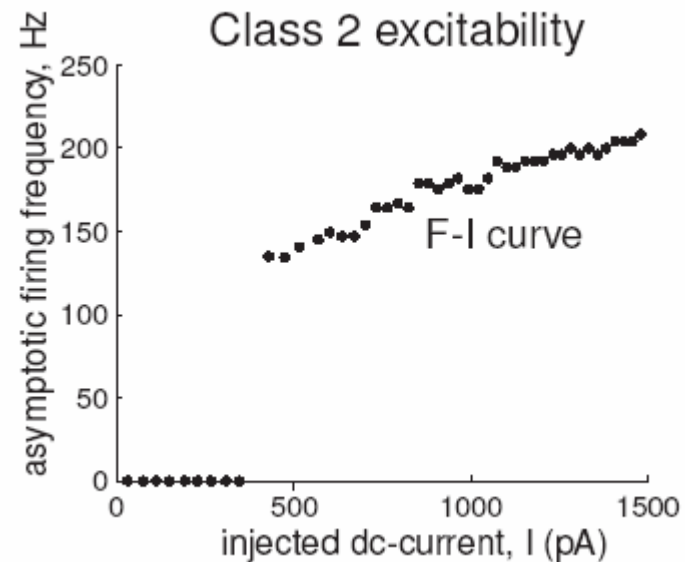


# Hodgking classification of neural excitability

- CLASS 1 NEURAL EXCITABILITY. Action potentials can be generated with arbitrarily low frequency, depending on the strength of the applied current.
- CLASS 2 NEURAL EXCITABILITY. Action potentials are generated in a certain frequency band that is relatively insensitive to changes in the strength of the applied current.
- CLASS 3 NEURAL EXCITABILITY. A single action potential is generated in response to a pulse of current. Repetitive (tonic) spiking can be generated only for extremely strong injected currents or not at all.



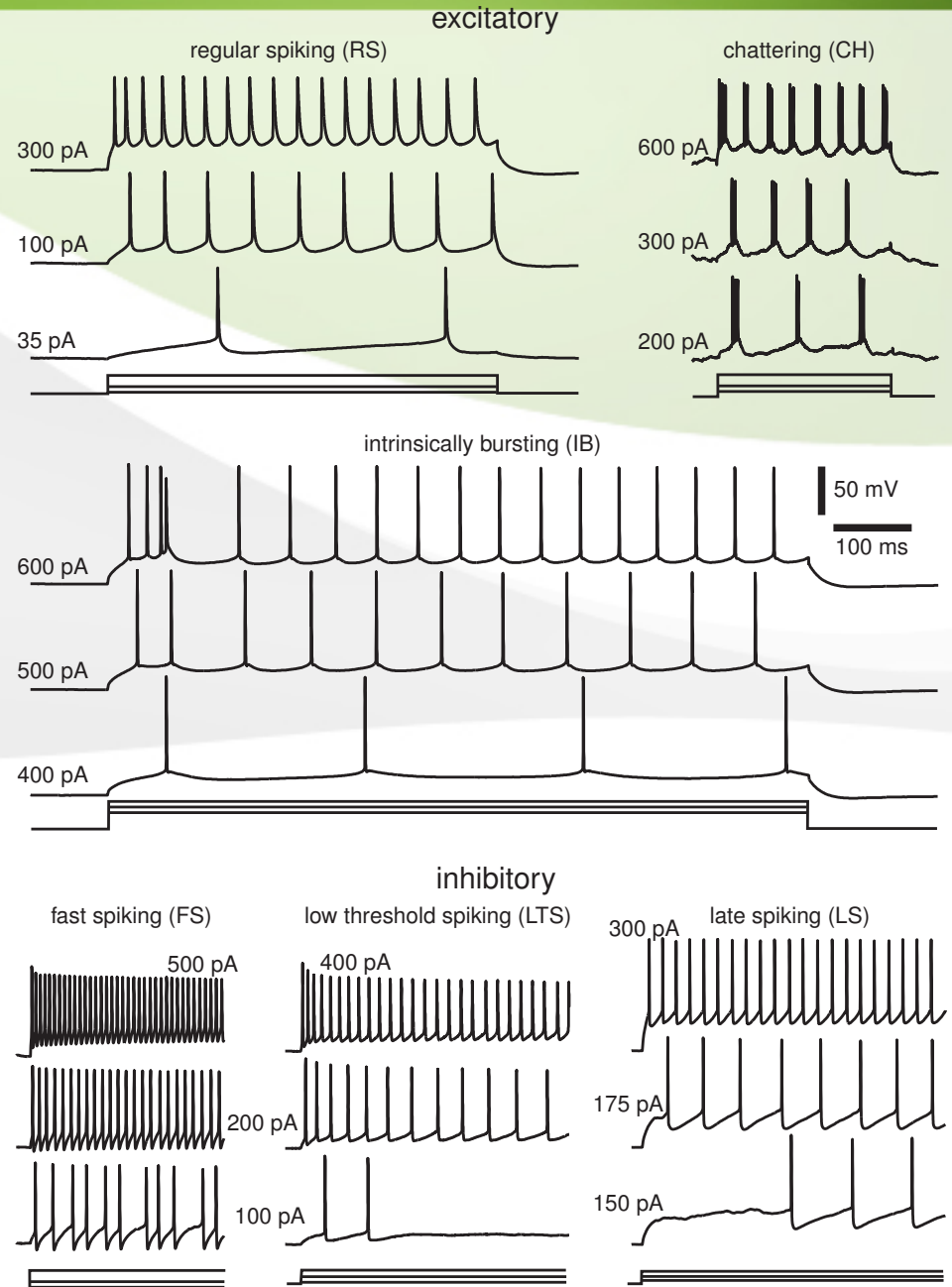
e.g. in cortical pyramidal neurons



e.g. brainstem mesV

# Particular Neural Dynamics in the Neocortex

**Six most fundamental classes** of firing patterns of neocortical neurons in response to pulses of depolarizing dc-current. RS and IB are in vitro recordings of pyramidal neurons of layer 5 of primary visual cortex of a rat, CH was recorded in vivo in cat's visual cortex. FS was recorded in vitro in rat's primary visual cortex, LTS was recorded in vitro in layer 4 or 6 of rat's barrel cortex. LS was recorded in layer 1 of rat's visual cortex.



## Benefits

- Can reproduce activity of single neurons
- Can be used to model detailed changes (external currents or the effect of drugs)

## Disadvantages

- Needs neuron morphology (dendritic layout)
- Needs information about ion channels, synapse position, neurotransmitter type
- Is slow to calculate for large numbers of neurons

=> Need for simplified neuron models

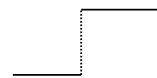
# Modelling Neural Dynamics

## The McCulloch-Pitts neuron (1943)

$$h = \sum_i x_i^{\text{in}}$$

Summation of input (no synaptic weights!)

$$x^{\text{out}} = \begin{cases} 1 & \text{if } h > \Theta \\ 0 & \text{otherwise} \end{cases}$$



Step-wise activation function

-> Birth of artificial neural network (ANN) research

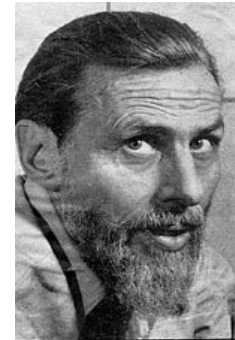
BULLETIN OF  
MATHEMATICAL BIOPHYSICS  
VOLUME 5, 1943

### A LOGICAL CALCULUS OF THE IDEAS IMMANENT IN NERVOUS ACTIVITY

WARREN S. MCCULLOCH AND WALTER PITTS

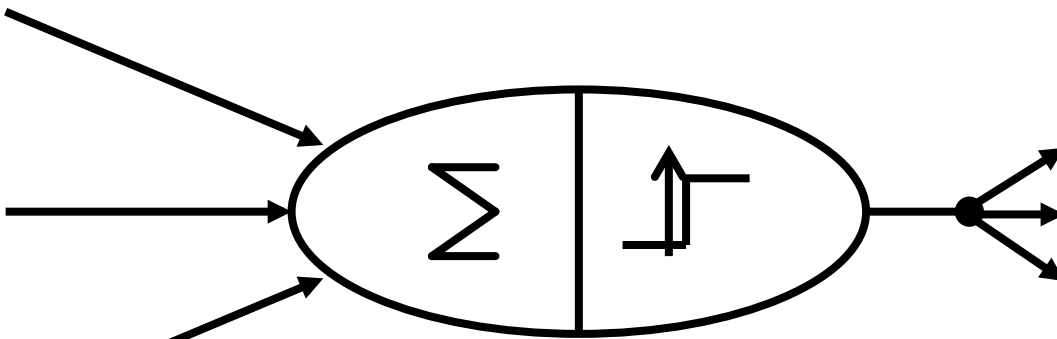
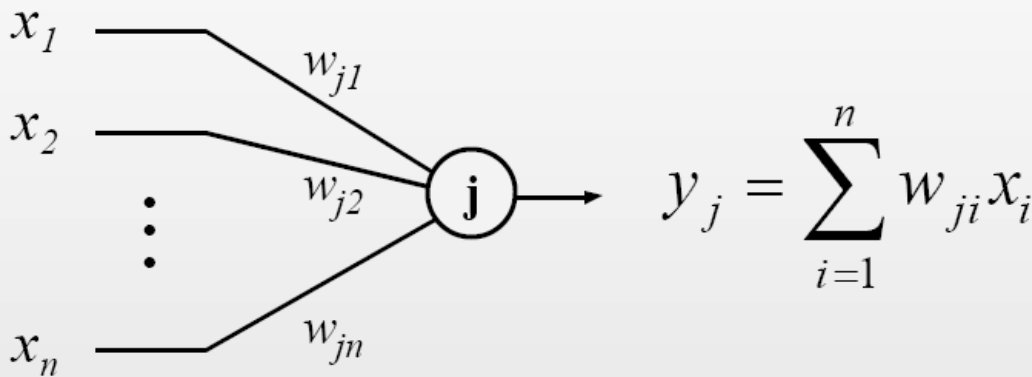
FROM THE UNIVERSITY OF ILLINOIS, COLLEGE OF MEDICINE,  
DEPARTMENT OF PSYCHIATRY AT THE ILLINOIS NEUROPSYCHIATRIC INS  
AND THE UNIVERSITY OF CHICAGO

Because of the "all-or-none" character of nervous activity, r  
events and the relations among them can be treated by means of p  
sitional logic. It is found that the behavior of every net can be des  
in these terms, with the addition of more complicated logical mean  
nets containing circles; and that for any logical expression satis



# Modelling Neural Dynamics

## The first artificial neuron

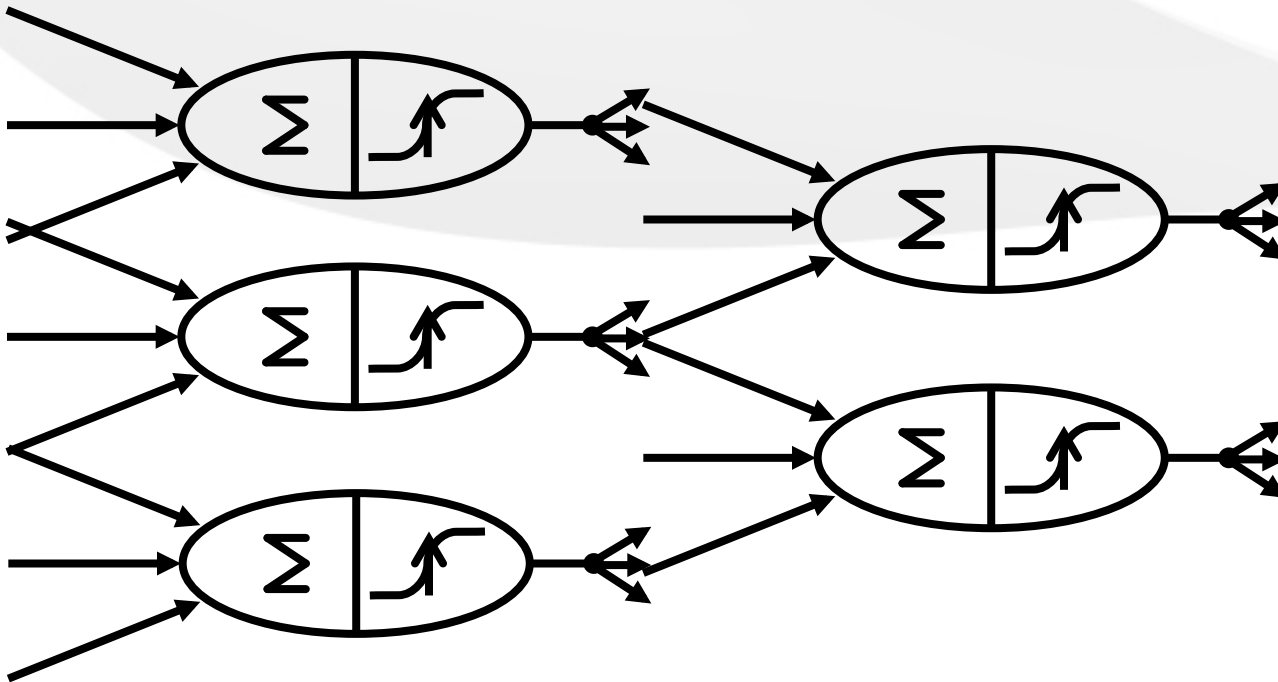


$$y_j = \begin{cases} 1, & \text{if } \sum_{i=1}^n w_{ji} x_i \geq \theta \\ 0, & \text{if } \sum_{i=1}^n w_{ji} x_i < \theta \end{cases}$$

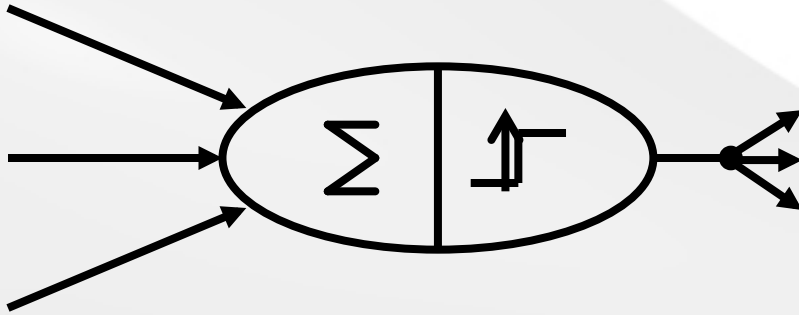
# Modelling Neural Dynamics

$$y_j = f\left(\sum_{i=1}^n w_{ji} x_i\right)$$

Multilayered  
Perception is a  
**universal  
approximator**

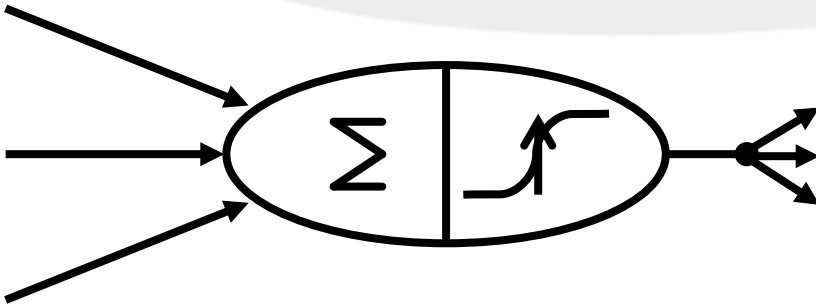


# Modelling Neural Dynamics



$$y = \begin{cases} 1 & \text{spike occurrence} \\ 0 & \text{spike absence} \end{cases}$$

From neurophysiology point of view,  $y$  is **existence of an output spike**


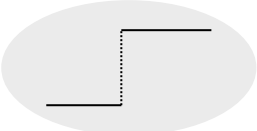


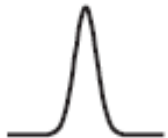


$$y = \frac{\text{Number of spikes}}{\text{Time frame}}$$

From neurophysiology point of view,  $y$  is **firing rate**

**Spike timing is not considered at all!**

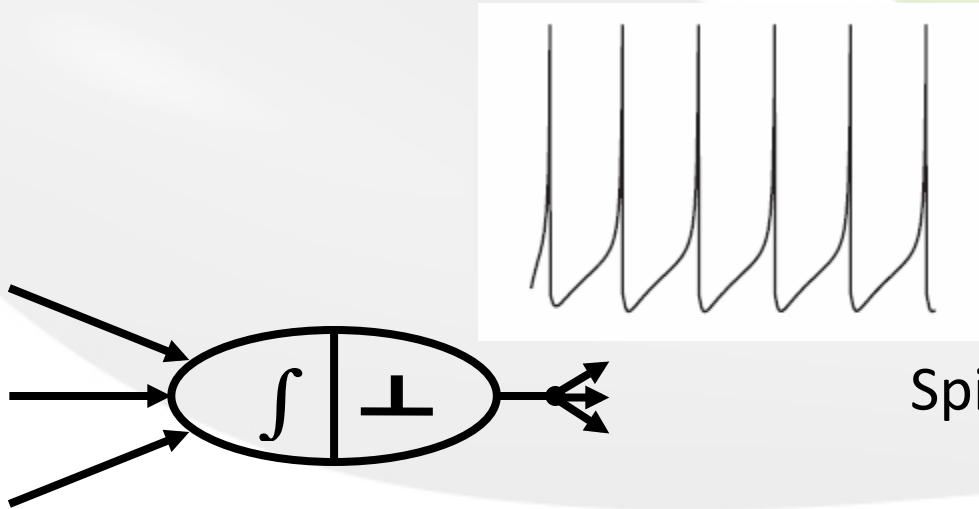
# Other thresholding functions

Type of function	Graphical represent.	Mathematical formula	MATLAB implementation
Linear		$g^{\text{lin}}(x) = x$	<code>x</code>
Step		$g^{\text{step}}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{elsewhere} \end{cases}$	<code>floor(0.5*(1+sign(x)))</code>
Threshold-linear		$g^{\text{theta}}(x) = x \Theta(x)$	<code>x.*floor(0.5*(1+sign(x)))</code>
Sigmoid		$g^{\text{sig}}(x) = \frac{1}{1+\exp(-x)}$	<code>1./(1+exp(-x))</code>
Radial-basis		$g^{\text{gauss}}(x) = \exp(-x^2)$	<code>exp(-x.^2)</code>



# Modelling Neural Dynamics

## Spiking neuron model



Spiking neural networks are

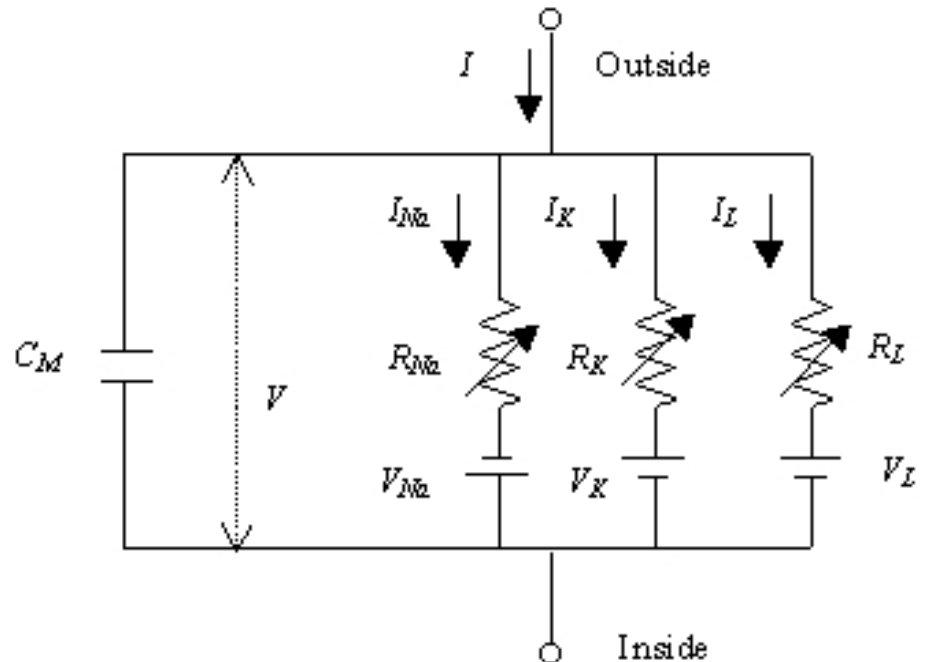
- biologically **more plausible**,
- computationally **more powerful**,
- considerably **faster**

than networks of the second generation

# Modelling Neural Dynamics

## Hodgkin-Huxley (first biologically-plausible neural model - 1952)

$$C \dot{V} = I - \overbrace{\bar{g}_K n^4 (V - E_K)}^{I_K} - \overbrace{\bar{g}_{Na} m^3 h (V - E_{Na})}^{I_{Na}} - \overbrace{g_L (V - E_L)}^{I_L}$$
$$\dot{n} = \alpha_n(V)(1 - n) - \beta_n(V)n$$
$$\dot{m} = \alpha_m(V)(1 - m) - \beta_m(V)m$$
$$\dot{h} = \alpha_h(V)(1 - h) - \beta_h(V)h ,$$



# Hodgkin-Huxley model

$$\frac{dv}{dt} = \frac{I_{external} - (I_K + I_{Na} + I_{leak})}{C}$$

$$I_K = g_K n^4 (v - V_K)$$

$$I_{Na} = g_{Na} m^3 h (v - V_{Na})$$

$$I_{leak} = g_{leak} (v - V_{leak})$$

$$\frac{dm}{dt} = \alpha_m(v)(1 - m) - \beta_m(v)m$$

$$\frac{dn}{dt} = \alpha_n(v)(1 - n) - \beta_n(v)n$$

$$\frac{dh}{dt} = \alpha_h(v)(1 - h) - \beta_h(v)h$$

$$\alpha_m(v) = 0.1(v + 25) / (e^{(v+25)/10} - 1)$$

$$\alpha_n(v) = 0.01(v + 10) / (e^{(v+10)/10} - 1)$$

$$\alpha_h(v) = 0.07 e^{v/20}$$

$$\beta_m(v) = 4 e^{v/18}$$

$$\beta_n(v) = 0.125 e^{v/80}$$

$$\beta_h(v) = 1 / (e^{(v+30)/10} + 1)$$

Sign is wrong  
in the paper  
from 1952!

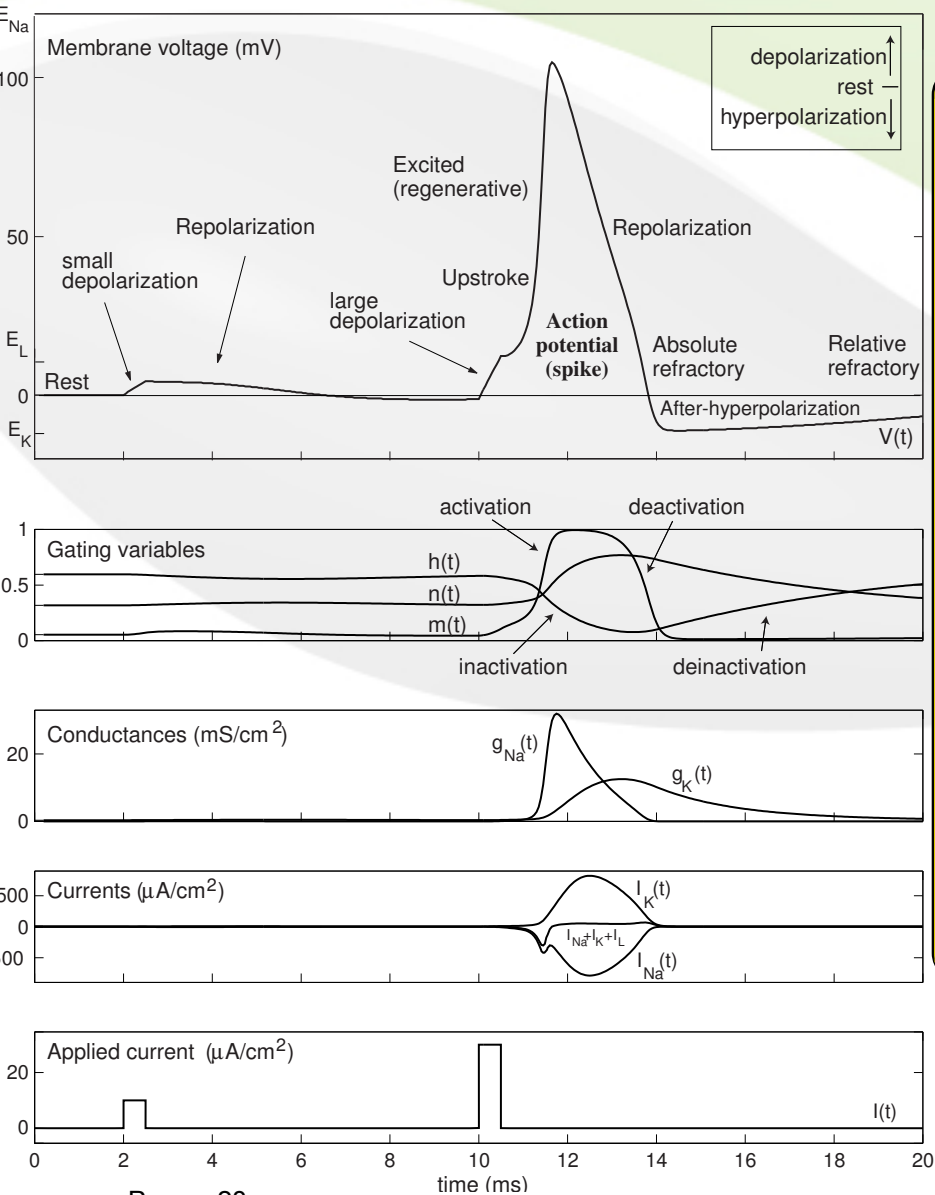
K conductance:	$g_K$	= 36
Na conductance:	$g_{Na}$	= 120
Leak conductance:	$g_{leak}$	= 0.3
Membrane Capacitance:	$C$	= 1
K equilibrium:	$V_K$	= 12
Na equilibrium:	$V_{Na}$	= -115
Leak equilibrium:	$V_{leak}$	= -10.6

Initial and Rest potential  $v_0 = 0$

Initial channel activations  $m_0, n_0, h_0 = 0$



# Hodgkin-Huxley model



## Review of Important Concepts

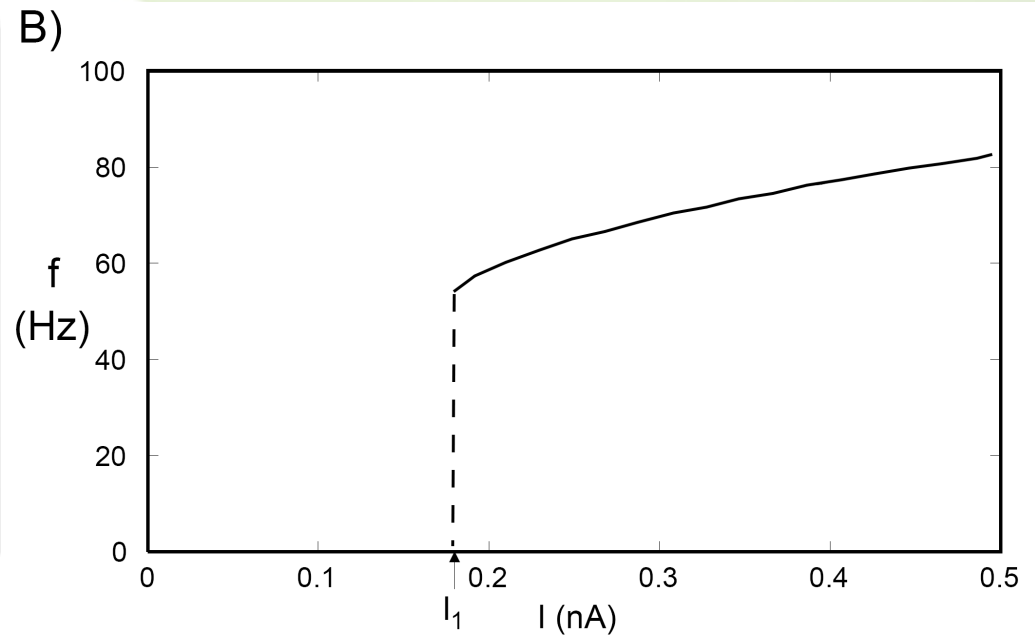
- Electrical signals in neurons are carried by Na<sup>+</sup>, Ca<sup>2+</sup>, K<sup>+</sup>, and Cl<sup>-</sup> ions, which move through membrane channels according to their electrochemical gradients.
- Membrane potential  $V$  is determined by the membrane conductances  $g_i$  and corresponding reversal potentials  $E_i$

$$C \dot{V} = I - \sum_i g_i \cdot (V - E_i) .$$

- Neurons are excitable because the conductances depend on the membrane potential and time.
- The most accepted description of kinetics of voltage-sensitive conductances is the Hodgkin-Huxley gate model.
- Voltage-gated activation of inward Na<sup>+</sup> or Ca<sup>2+</sup> current depolarizes (increases) the membrane potential.
- Voltage-gated activation of outward K<sup>+</sup> or Cl<sup>-</sup> current hyperpolarizes (decreases) the membrane potential.
- An action potential or spike is a brief regenerative depolarization of the membrane potential followed by its repolarization and possibly hyperpolarization, as in Fig. 2.16.

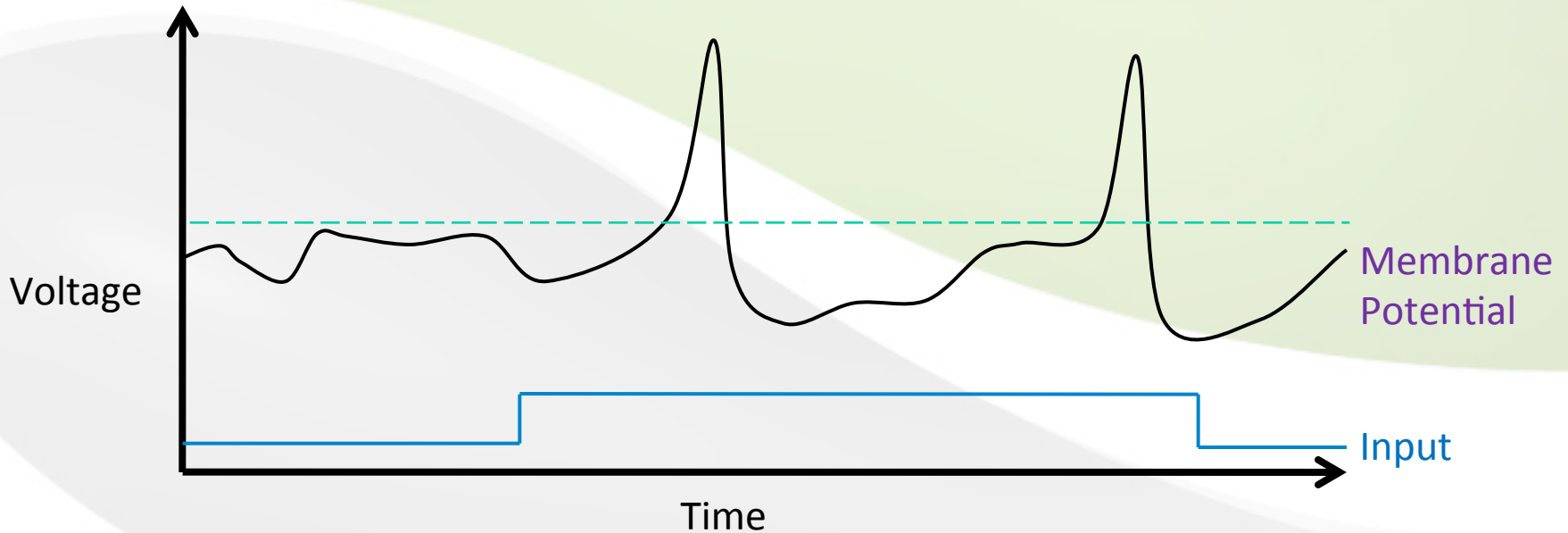
# Hodgkin-Huxley f-I curve

Rate coding: firing rate response ( $f$ ) to input current ( $I$ ), steady state

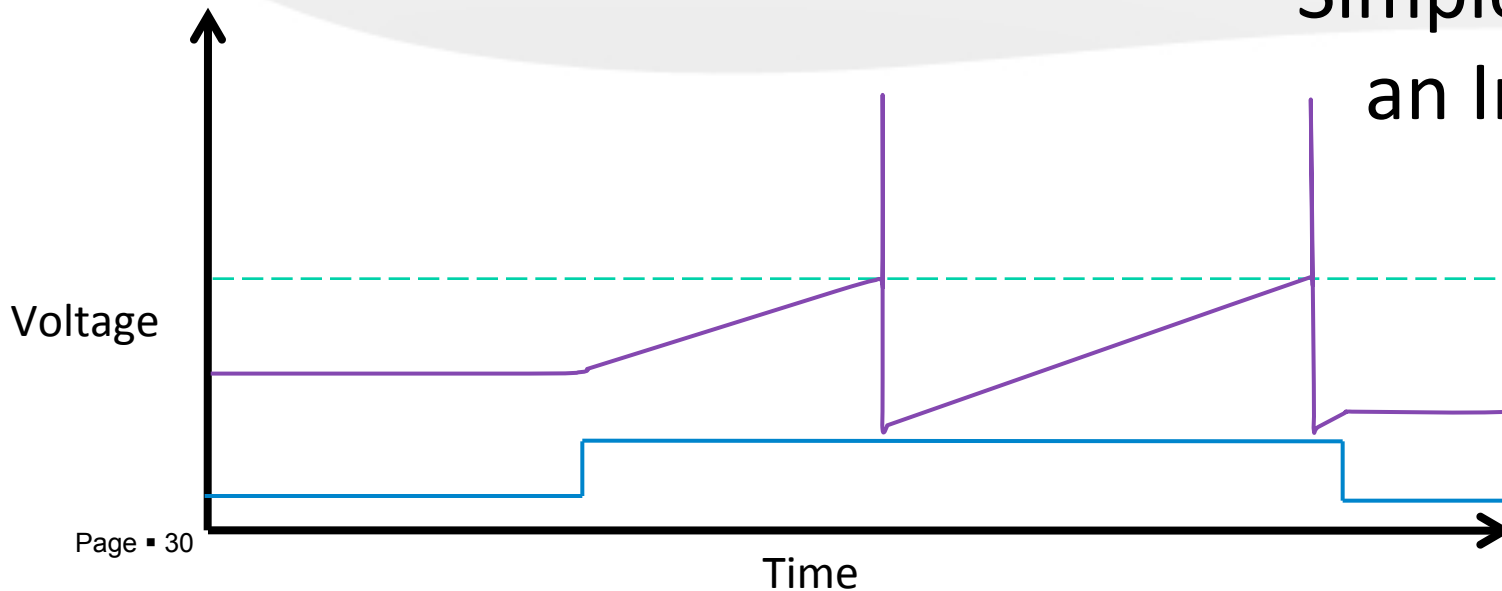


There is a minimum firing rate (58 Hz)

# What does a neuron do?



Simplest idea –  
an Integrator



# A neuron as an Integrator

Input current:

$I$

Membrane Capacitance:

$C$

Spike threshold:

$V_{thresh}$

Reset voltage:

$V_{reset}$

Membrane Voltage:

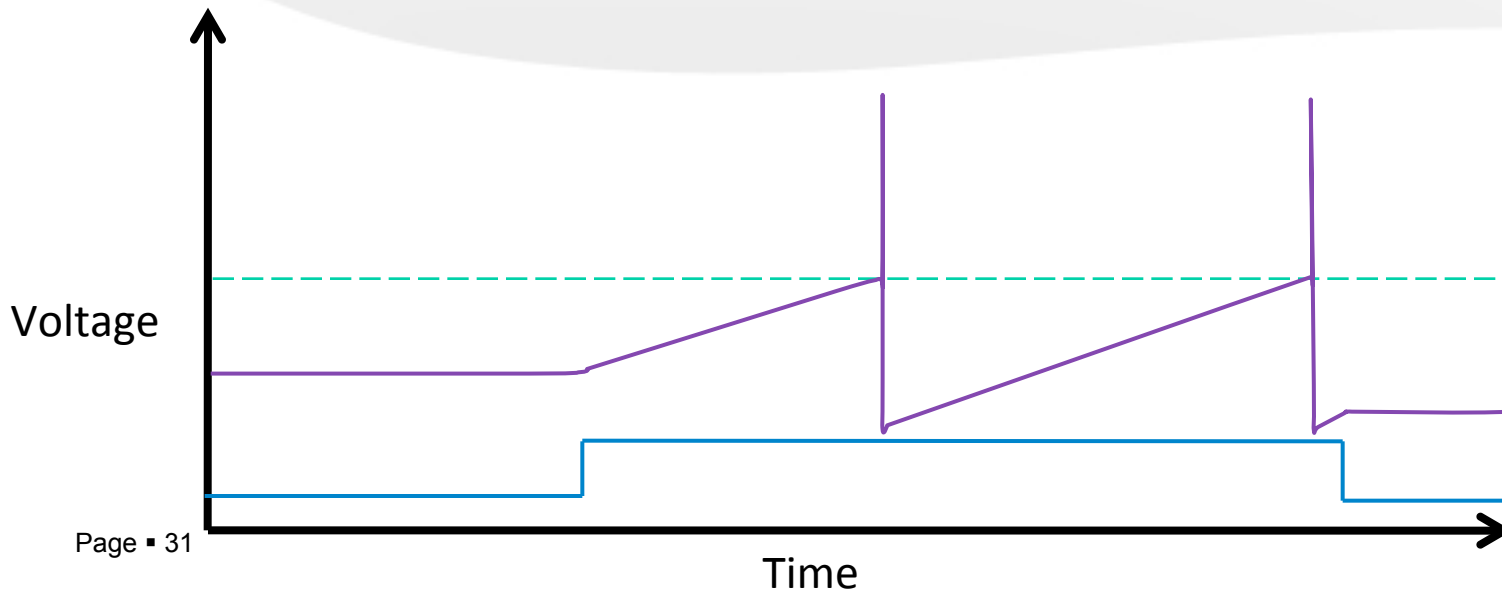
$$\frac{dv}{dt} = \frac{I}{C}$$

if  $v > V_{thresh}$   
 $\rightarrow v = V_{reset}$

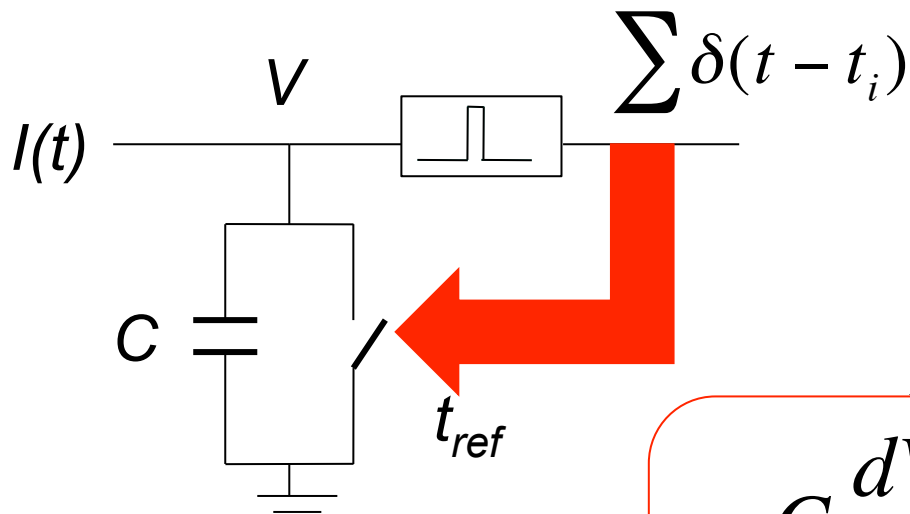
- Firing rate is unlimited
- Integration is “perfect”



Neuron response is linear



# A neuron as an Integrator



$$C \frac{dV(t)}{dt} = I(t)$$

*linear*

$$V(t) = V_{Thr} \Rightarrow \text{Fire+reset } \textit{threshold}$$

$$\int_{t_i}^{t_{i+1}} I(t) dt = CV_{th}$$



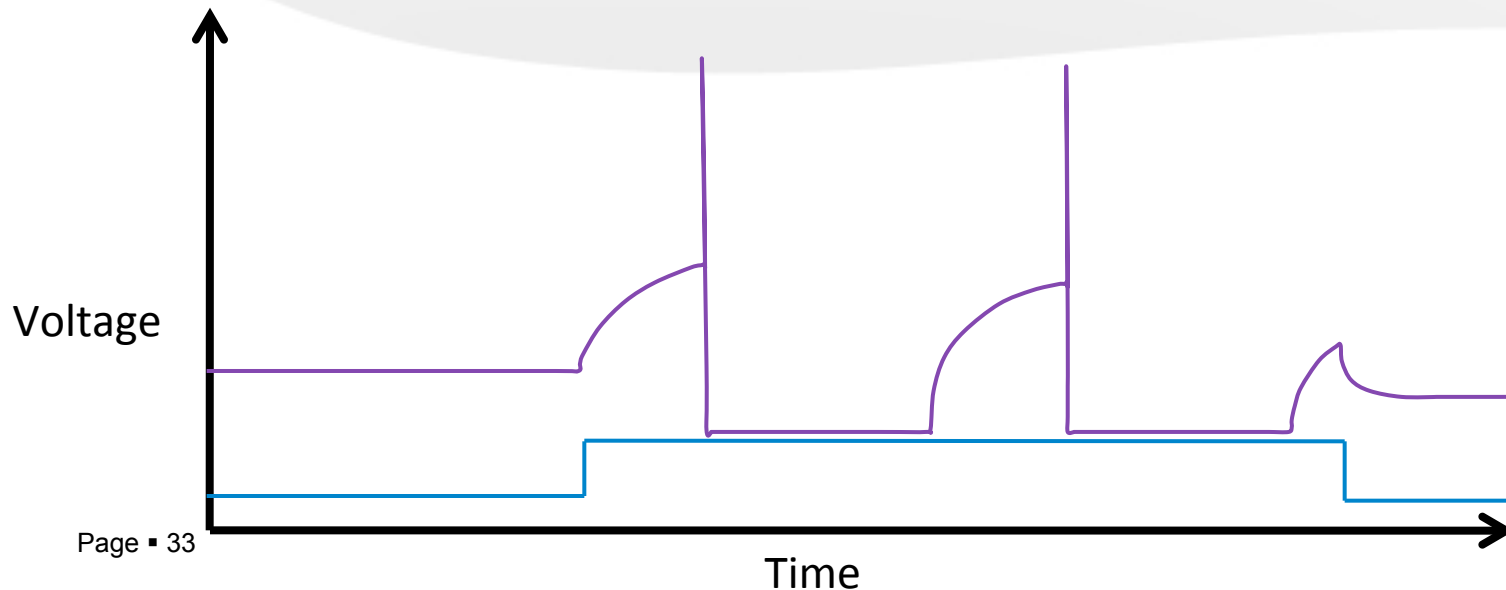
# Leaky Integrate and Fire (LIF) model

Input current:  $I$   
Membrane Capacitance:  $C$

Spike threshold:  $V_{thresh}$   
Reset potential:  $V_{reset}$   
Resting potential:  $V_{rest}$   
Membrane Time Constant:  $\tau$   
Refractory period:  $t_{refrac}$

Membrane Voltage: 
$$\frac{dv}{dt} = \frac{I}{C} - \frac{v - V_{rest}}{\tau}$$

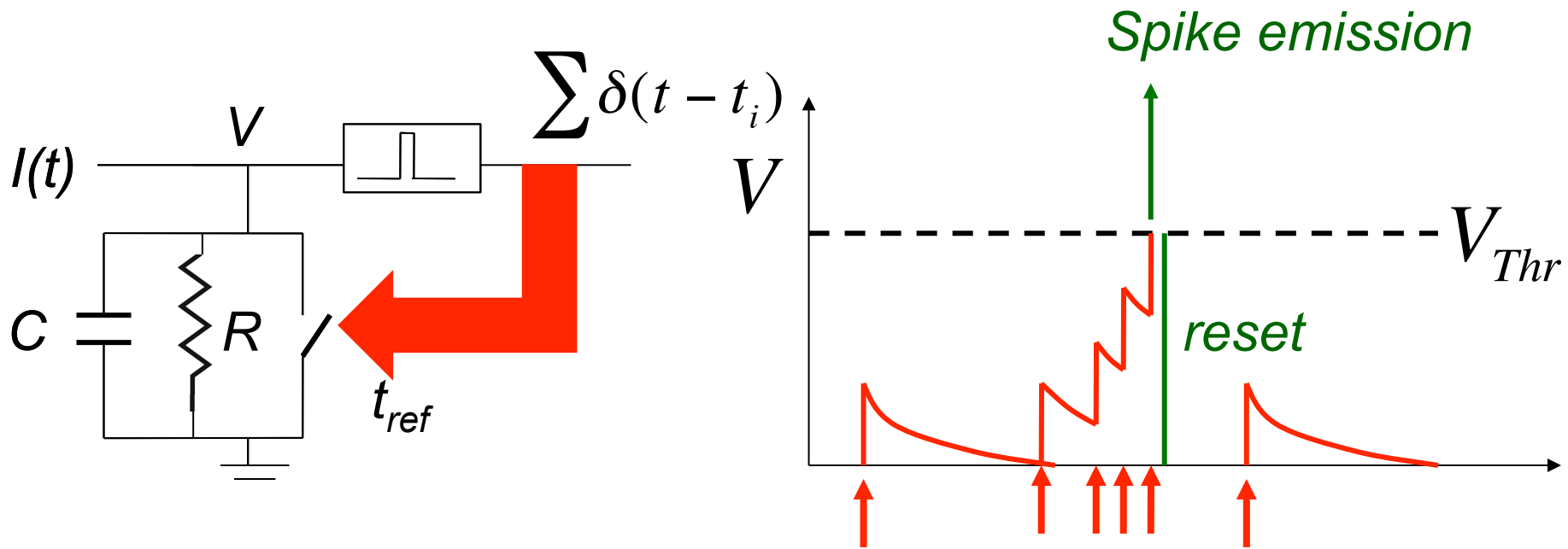
if  $v > V_{thresh}$   
 $\rightarrow v = V_{reset}$  held for  $t_{refrac}$



Stein, 1967



# Leaky Integrate and Fire (LIF) model

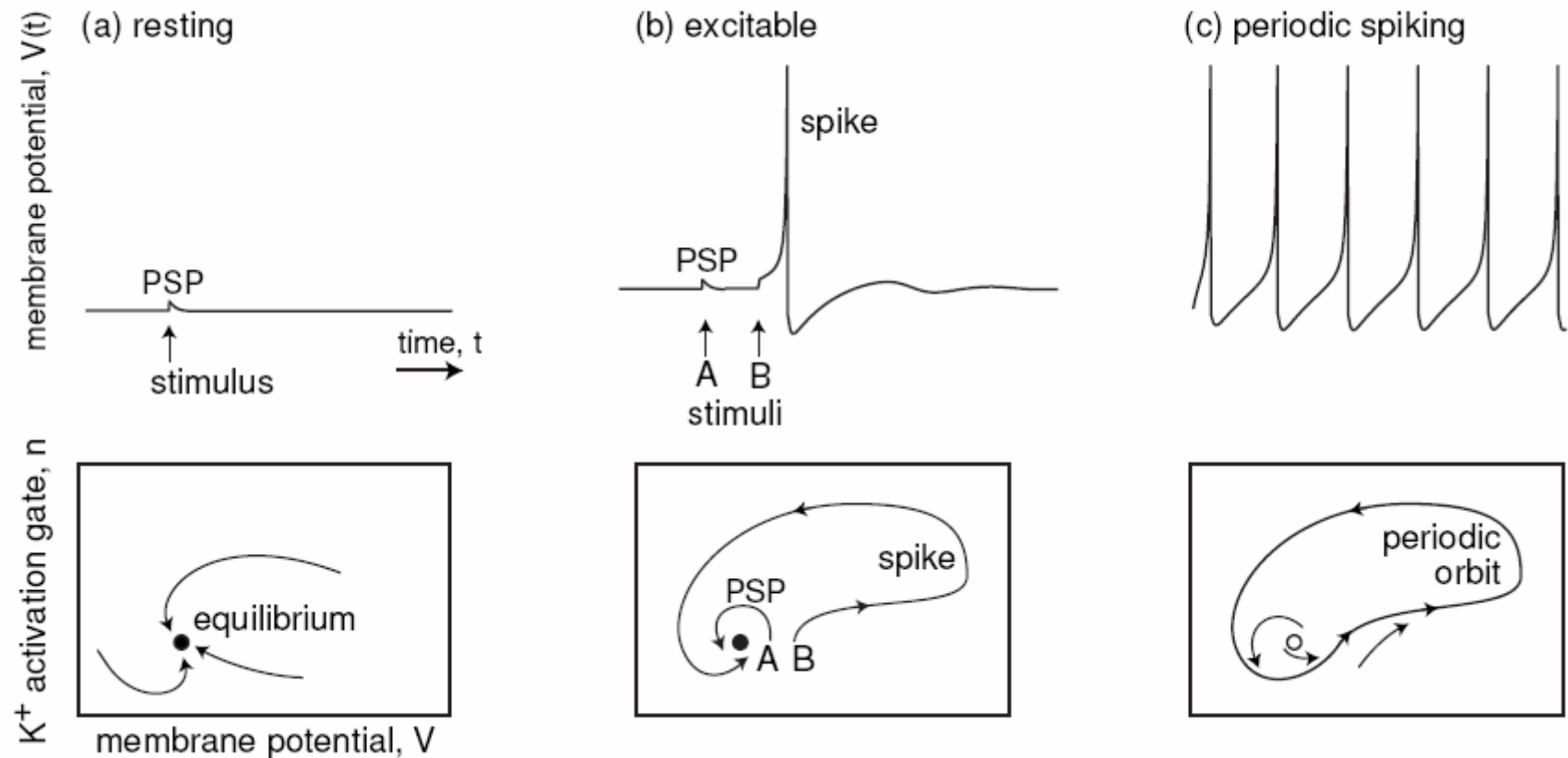


$$C \frac{dV(t)}{dt} + \frac{V(t)}{R} = I(t) \quad \text{linear}$$

$$V(t) = V_{Thr} \Rightarrow \text{Fire+reset threshold}$$

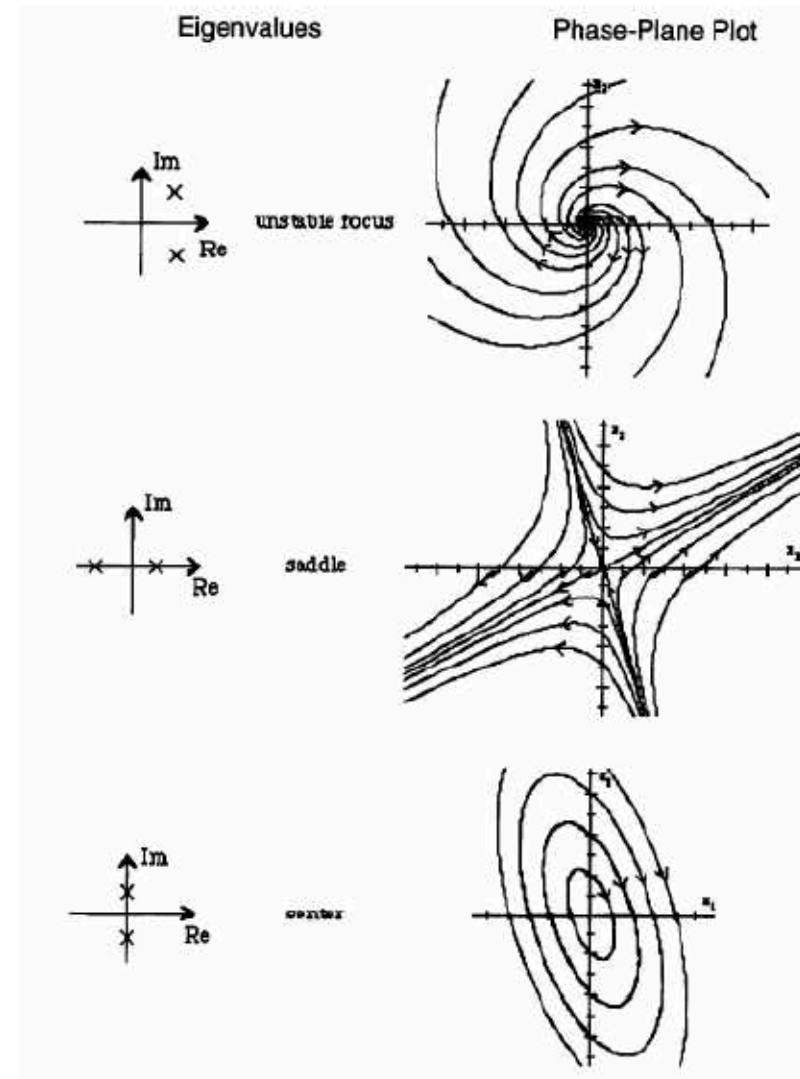
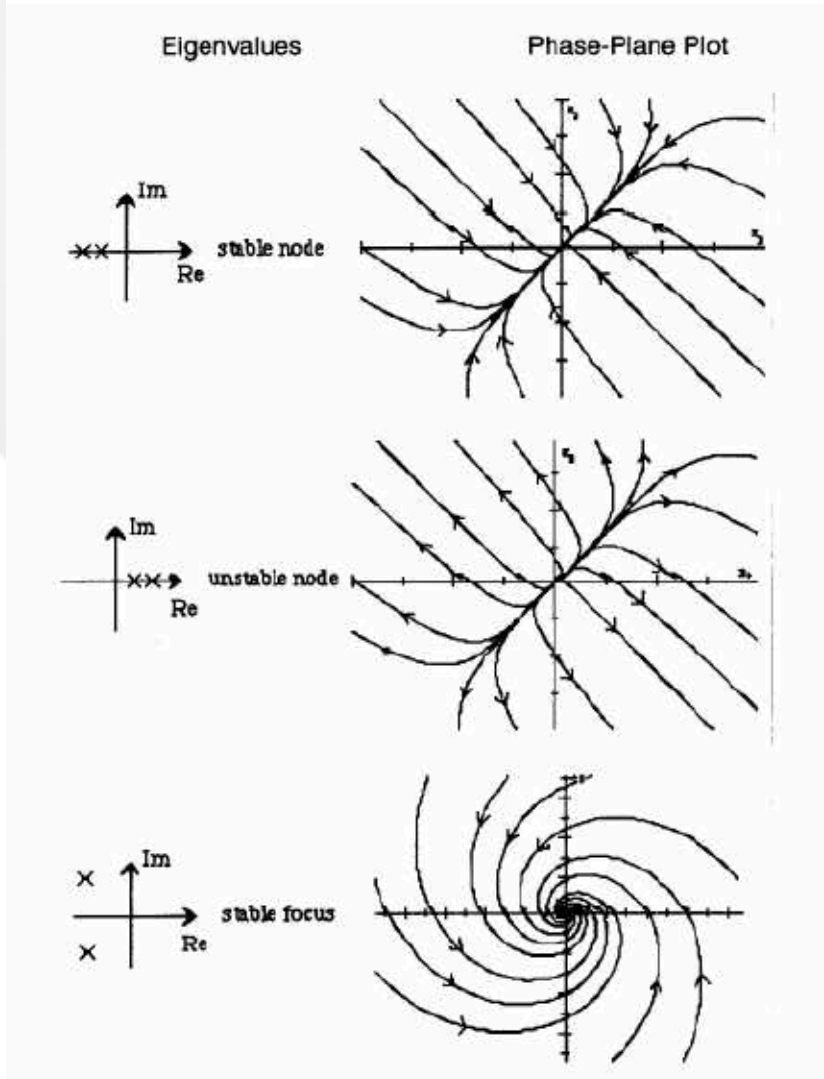
# Neural Modeling and Dynamics

## Neurons as dynamical systems: phase space



- Neurons are dynamical systems.
- Resting state of neurons corresponds to a stable equilibrium, tonic spiking state corresponds to a limit cycle attractor.
- Neurons are excitable because the equilibrium is near a bifurcation.

# Exemplary Phase Space Analysis



# Neural Excitability

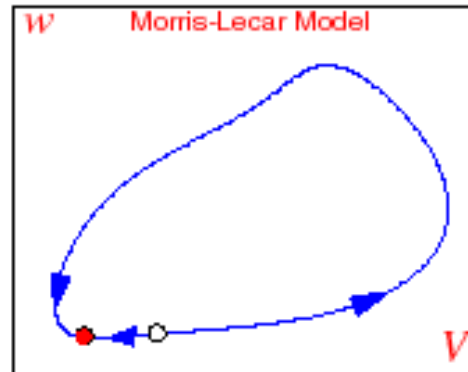
Excitability is the **most fundamental** property of **neurons** allowing **communication via action potentials or spikes**.

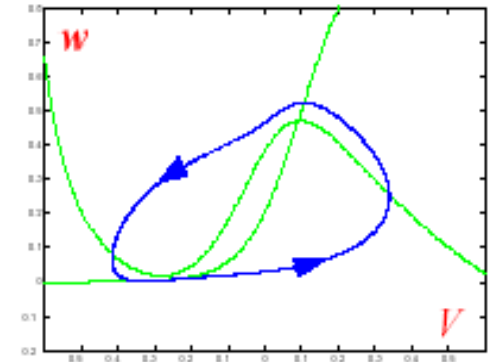
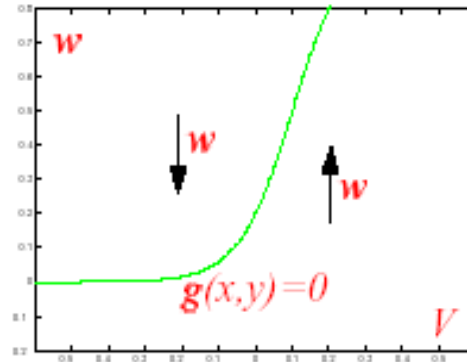
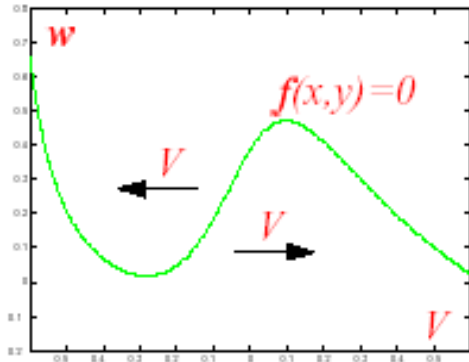
From **mathematical point of view** a system is **excitable** when small **perturbations** near a rest state can cause large **excursions** for the solution before it returns to the rest.

**Systems are excitable** because they are *near bifurcations* from **rest to oscillatory** dynamics.

The **type of bifurcation** determines **excitable properties** and hence **neuro-computational** features of the brain cells. Revealing these features is the most important goal of **mathematical neuroscience**.

The neuron produce spikes periodically when there is a **large amplitude limit cycle attractor**, which may **coexist** with the **quiescent state**.





Most of the bifurcations discussed here can be illustrated using a two-dimensional (planar) system of the form

$$\mu \cdot x' = f(x, y)$$

$$y' = g(x, y)$$

Much insight into the behavior of such systems can be gained by considering their nullclines.

the sets determined by the conditions  $f(x, y) = 0$  or  $g(x, y) = 0$ .

When  $0 < \mu \ll 1$  nullclines are called fast and slow, respectively. Since the language of nullclines is universal in many areas of applied mathematics

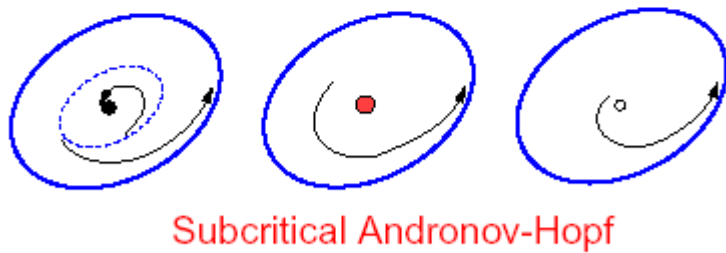
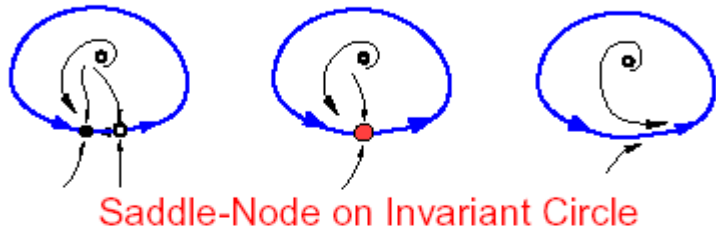
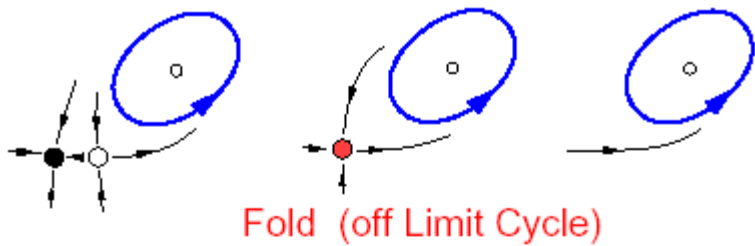


Fig. 7. Codimension 1 bifurcations corresponding to a transition from equilibrium to oscillatory dynamics. Fold bifurcation occurs when the Jacobian matrix at the equilibrium has a zero eigenvalue. We refer to it as the saddle-node on invariant circle bifurcation when the center manifold makes a loop. Andronov-Hopf bifurcation occurs when the matrix has a pair of complex-conjugate eigenvalues with zero real

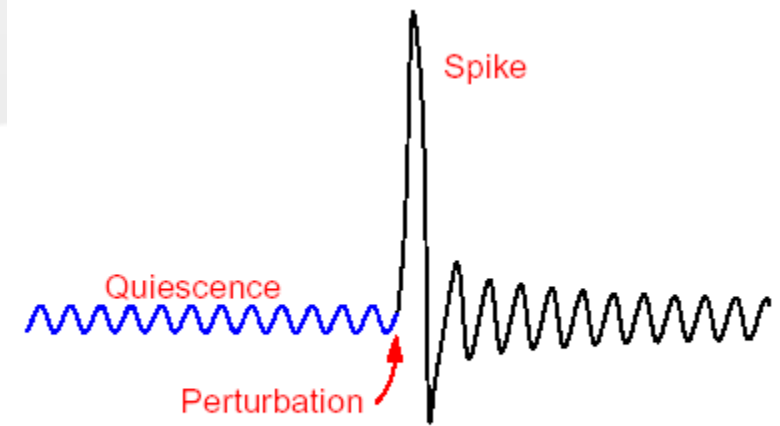
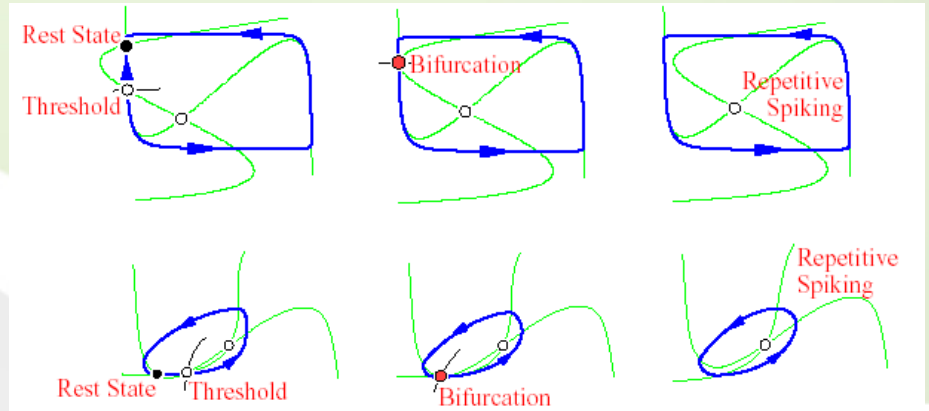


Fig. 26. An example of a small amplitude subthreshold oscillation (blue) corresponding to the quiescent state.

# Bursters

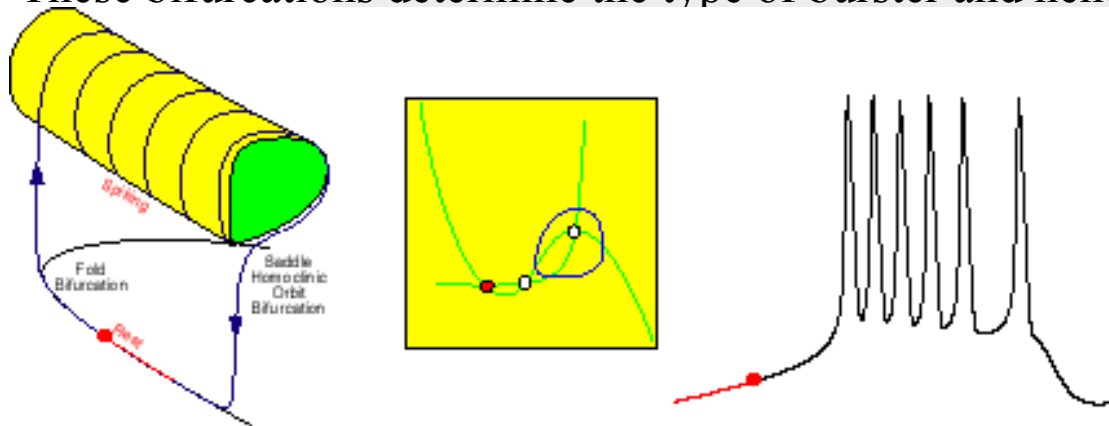
When neuron activity **alternates** between a **quiescent state** and **repetitive spiking**, the neuron activity is said to be **bursting**. It is usually caused by a slow voltage- or calcium-dependent process that can modulate fast spiking activity.

There are **two important bifurcations** associated with bursting:

*Bifurcation of a quiescent state that leads to repetitive spiking.*

*Bifurcation of a spiking attractor that leads to quiescence.*

These bifurcations determine the type of burster and hence its neuro-computational features.





# Modelling Neural Dynamics

Usually they are express in form of ODEs (Ordinary Differential Equations)

## INTEGRATE-AND-FIRE

$$v' = I + a - bv \quad \text{if } v \geq v_{thresold} \quad \text{then } v \leftarrow c$$

$I$   $\longrightarrow$  Input Current  
 $v$   $\longrightarrow$  Membrane Potential  
 $c$   $\longrightarrow$  Reset Value

FLOPS

5 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos	
-	+	-	-	-	-	-	-	-	-	+	-	-	-	-	-	-	-	-	-	-

## IF WITH ADAPTATION

$$v' = I + a - bv + g(d - v)$$

$$g' = (e\delta(t) - g) / \tau$$

$g$   $\longrightarrow$  Conductance  
 $\delta$   $\longrightarrow$  Dirac

FLOPS

10 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos	
-	+	-	-	-	-	+	-	-	-	+	-	-	-	-	+	-	-	-	-	-

# Modelling Neural Dynamics

## QUADRATIC IF (Ermentrout-Koppel)

$$v' = I + a(v - v_{rest})(v - v_{threshold}) \quad \text{se } v = v_{peak} \quad \text{allora } v \leftarrow c$$

$v_{rest}$   $\longrightarrow$  Threshold  
 $v_{threshold}$   $\longrightarrow$  V Threshold

FLOPS 7 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos
-	+	-	-	-	-	-	+	-	-	+	-	-	+	+	-	-	-	-	-

## IF OR BURST

$$v' = I + a - bv + gH(v - v_h)h(v_T - v) \quad \text{se } v = v_{threshold} \quad \text{allora } v \leftarrow c$$

$$h' = \begin{cases} -h/\tau^- & \text{se } v > v_h \\ (1-h)/\tau^+ & \text{se } v < v_h \end{cases}$$

$H$   $\longrightarrow$  Heaviside Function

$h$   $\longrightarrow$  T-current function

FLOPS 13 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos
-	+	+	?	+	-	-	-	-	-	+	+	+	-	+	+	-	-	-	?

## RESONATE-AND-FIRE

$$z' = I + (b + i\omega)z \quad \text{se } \text{Im}z \geq a_{threshold} \quad \text{allora } z \leftarrow z_0(z)$$

$z$   $\longrightarrow$  Membrane Potential  
 $z_0(z)$   $\longrightarrow$  Reset Value

FLOPS 10 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos
-	+	+	-	-	-	-	-	+	+	+	+	-	-	+	+	+	-	-	+

## FITZHUGH-NAGUMO

$$v' = a + bv + cv^2 + dv^3 - u$$

$$u' = \varepsilon(ev - u)$$

$u$   $\longrightarrow$  Recovery variable

FLOPS 72 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos
-	+	+	-	?	-	-	+	+	+	-	+	-	+	+	-	+	+	-	-

# Modelling Neural Dynamics

## HINDMARSH-ROSE

$$v' = u - F(v) + I - w$$

$$u' = G(v) - u$$

$$w' = (H(v) - w) / \tau$$

FLOPS

120 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos
-	+	+	+	?	?	+	+	+	+	+	+	+	+	+	+	+	+	?	+

## MORRIS-LECAR

$$C v' = -g_L(v - v_L) - g_{Ca} m_\infty (v - v_{Ca}) - g_K n (v - v_K) + I$$

$$n' = \frac{n_\infty - n}{\tau_n}$$

FLOPS

600 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos
+	+	+	-	?	-	-	+	+	+	+	+	?	+	+	-	+	+	-	-

## MODELLO POLINOMIALE (Wilson)

$$C v' = -m_\infty (v - 0.5) - 26 u (v + 0.95) - g_T T (v - 1.2) - g_H H (v + 0.95) + I$$

$$u' = \frac{1}{\tau_u} (-u + u_\infty(v))$$

$$T' = \frac{1}{14} (-T + T_\infty(v))$$

$$H' = \frac{1}{45} (-H + 3T)$$

FLOPS

180 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos
-	+	+	+	?	?	+	+	+	+	+	+	+	+	?	+	+	?	?	?

## HODGKIN-HUXLEY

$$C v' = -g_K n^4 (v - v_K) - g_{Na} m^3 h (v - v_{Na}) - g_l (v - v_l) + I$$

$$m' = \alpha_m (1 - m) - \beta_m m$$

$$n' = \alpha_n (1 - n) - \beta_n n$$

$$h' = \alpha_h (1 - h) - \beta_h h$$

FLOPS

1200 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos
+	+	+	+	?	?	+	+	+	+	+	+	+	+	+	+	+	+	?	+

# Modelling Neural Dynamics

## IZHIKEVICH

$$v' = 0.04v^2 + 5v + 140 - u + I$$

$$u' = a(bv - u)$$

$$\text{If } v \geq +30 \text{ mV, Then } \begin{cases} v \leftarrow c \\ u \leftarrow u + d. \end{cases}$$

$v$   $\longrightarrow$  Membrane Potential

$u$   $\longrightarrow$  Recovery

FLOPS

13 for 1 ms

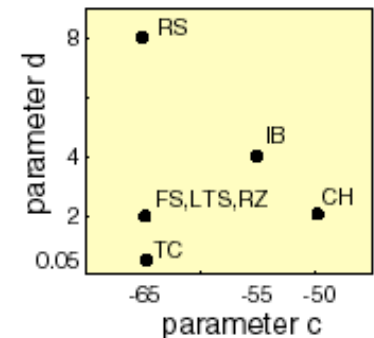
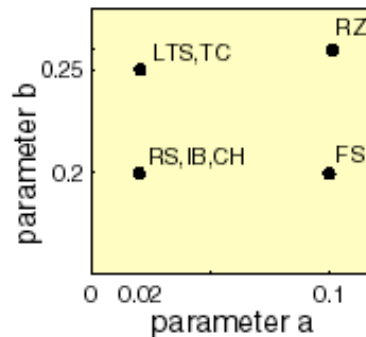
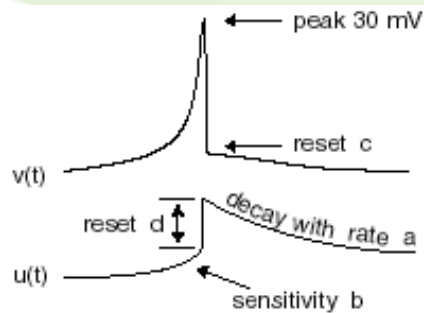
Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos
-	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

# Izhikevich Model

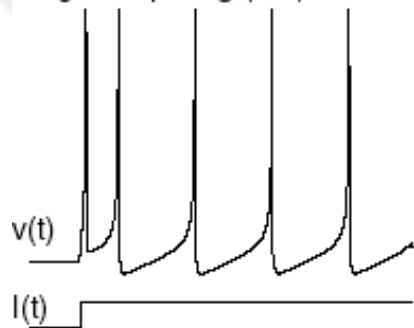
$$v' = 0.04v^2 + 5v + 140 - u + I$$

$$u' = a(bv - u)$$

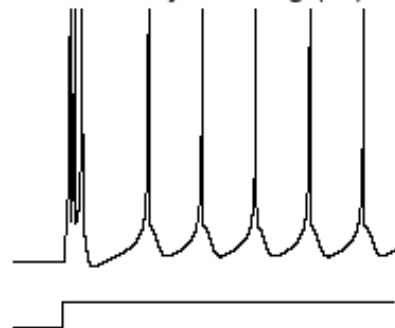
if  $v = 30$  mV,  
then  $v \leftarrow c$ ,  $u \leftarrow u + d$



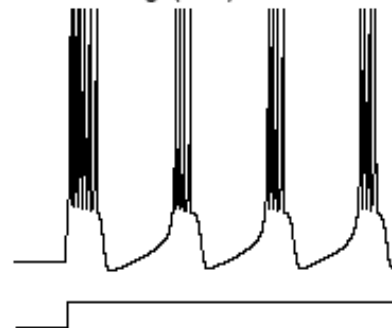
regular spiking (RS)



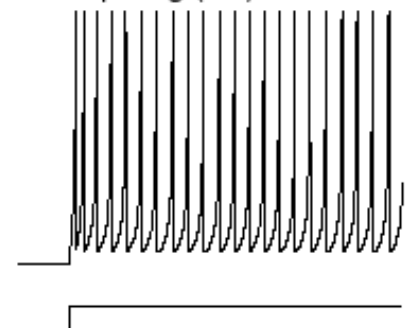
intrinsically bursting (IB)



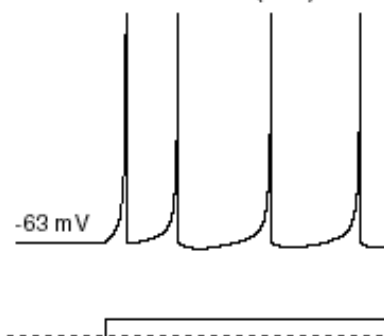
chattering (CH)



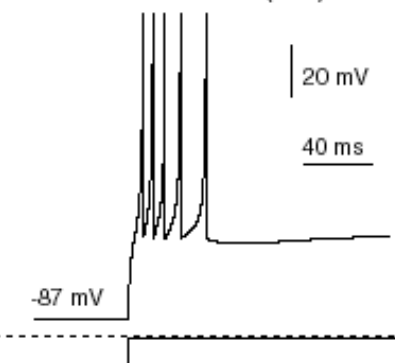
fast spiking (FS)



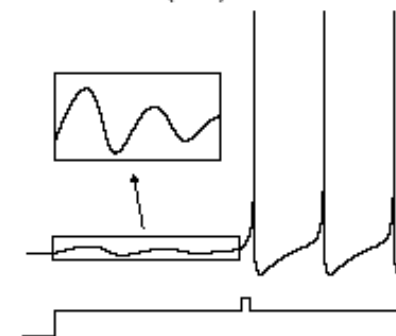
thalamo-cortical (TC)



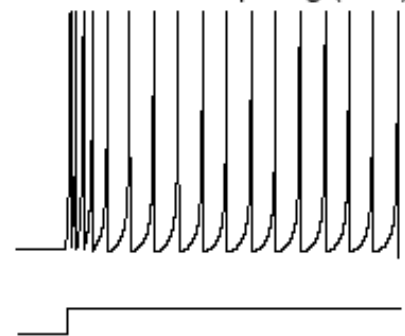
thalamo-cortical (TC)

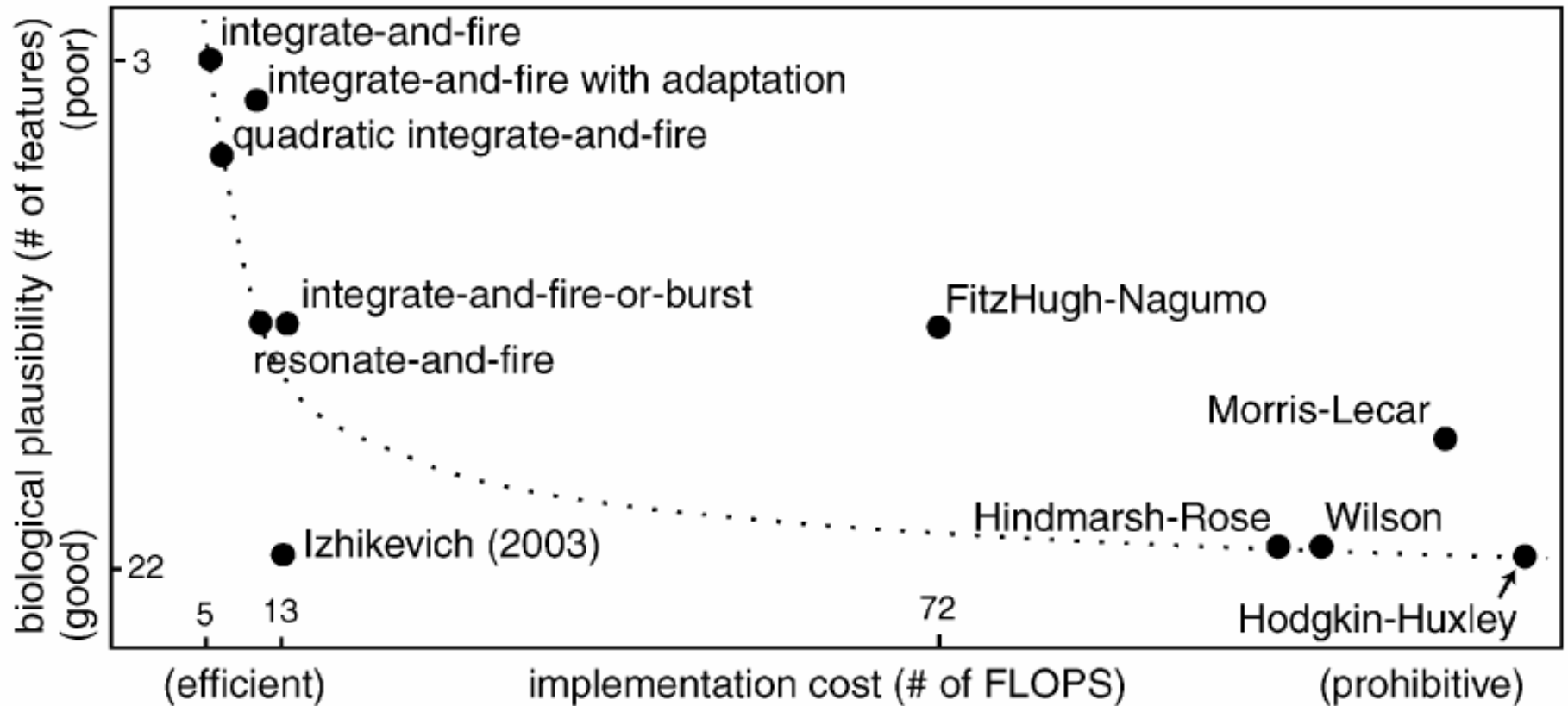


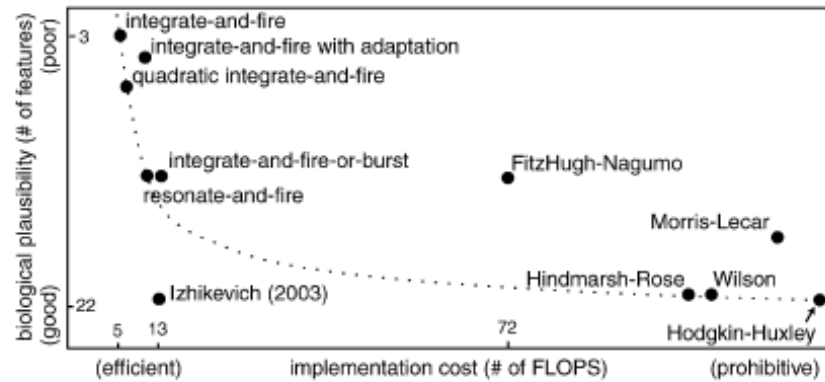
resonator (RZ)



low-threshold spiking (LTS)







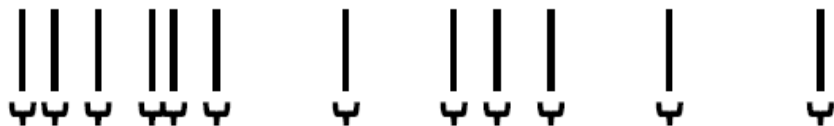
Models	biophysically meaningful	tonic spiking	phasic spiking	tonic bursting	phasic bursting	mixed mode	spike frequency adaptation	class 1 excitable	class 2 excitable	spike latency	subthreshold oscillations	resonator	integrator	rebound spike	rebound burst	threshold variability	bistability	DAP	accommodation	inhibition-induced spiking	inhibition-induced bursting	chaos	# of FLOPS
integrate-and-fire	-	+	-	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	-	-	-	-	5
integrate-and-fire with adapt.	-	+	-	-	-	+	+	-	-	-	-	+	-	-	-	-	+	-	-	-	-	-	10
integrate-and-fire-or-burst	-	+	+		+	-	+	+	-	-	-	+	+	+	-	+	+	-	-	-		13	
resonate-and-fire	-	+	+	-	-	-	+	+	-	+	+	+	+	-	-	+	+	+	-	-	+	10	
quadratic integrate-and-fire	-	+	-	-	-	-	+	-	+	-	-	+	-	-	+	+	-	-	-	-	-	7	
Izhikevich (2003)	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	13	
FitzHugh-Nagumo	-	+	+	-		-	+	-	+	+	+	-	+	-	+	+	-	+	+	-	-	72	
Hindmarsh-Rose	-	+	+	+			+	+	+	+	+	+	+	+	+	+	+	+			+	120	
Morris-Lecar	+	+	+	-		-	+	+	+	+	+	+	+		+	+	-	+	+	-	-	600	
Wilson	-	+	+	+			+	+	+	+	+	+	+	+	+		+	+				180	
Hodgkin-Huxley	+	+	+	+			+	+	+	+	+	+	+	+	+	+	+	+	+		+	1200	

# The Neural Code

## Spike trains



## Where is the Information?

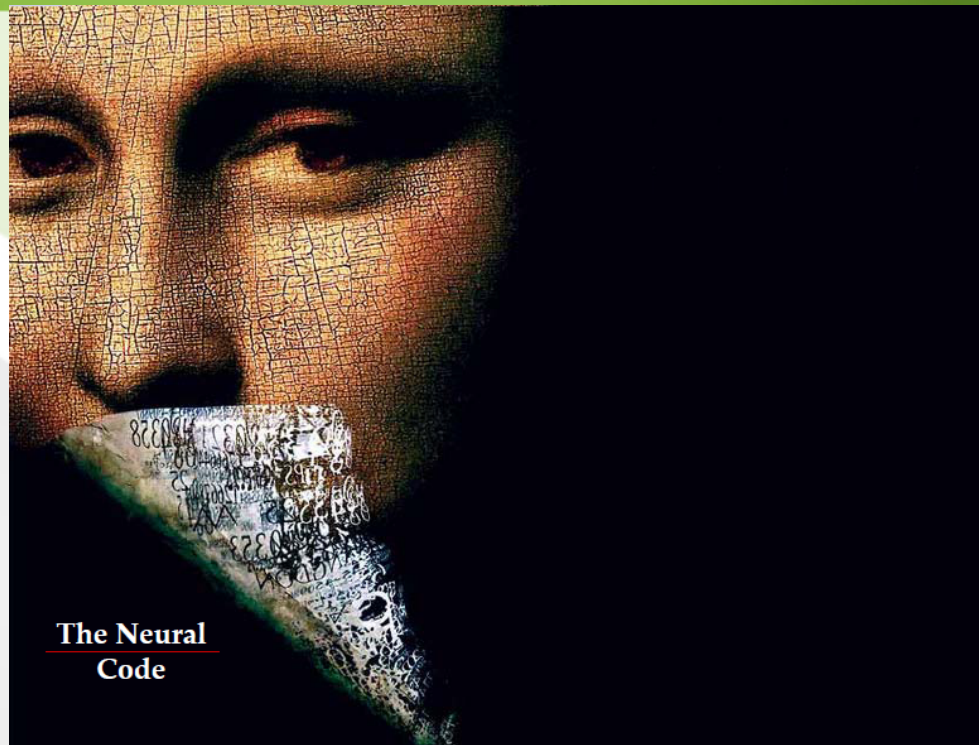


Frequency?

Spikes?



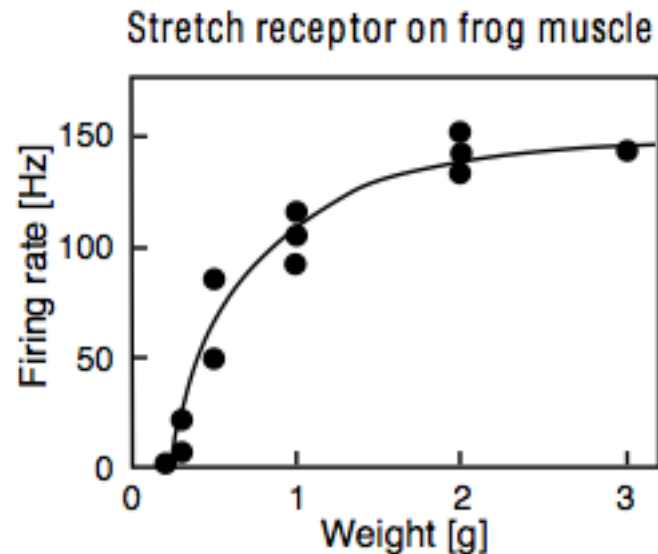
Inter/Intra spike interval?





# The firing rate hypothesis

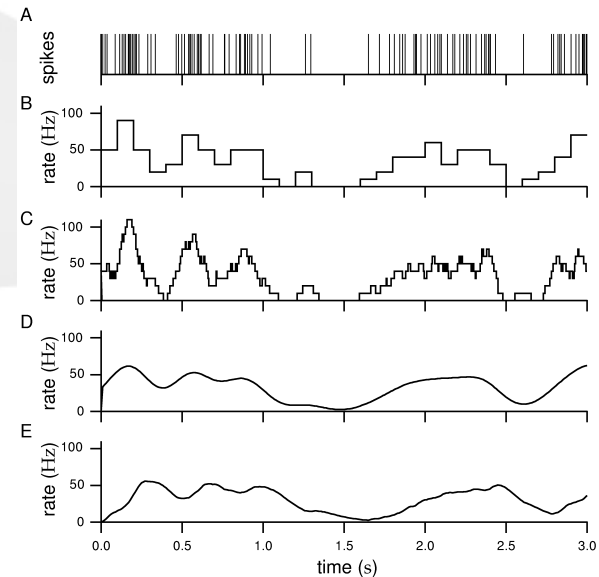
Stimulus features are encoded through the neural firing rate (response curves).



Edgar Adrian  
The Nobel Prize in Physiology or Medicine 1932

Time-dependent firing rate counts number of spikes in a short time interval (averaged over trials):  $r(t) = \frac{1}{\Delta t} \int_t^{t+\Delta t} \rho(\tau) d\tau$ .

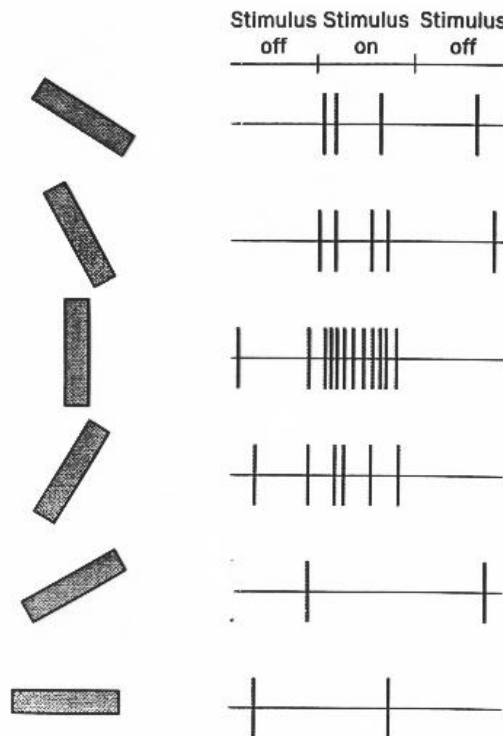
For any  $t > 0$ , each interval contains 0,1 spike.  
Then,  $r(t)$  averaged over trials is  
the probability of any trial firing at time  $t$ .  
B: 100 ms bins



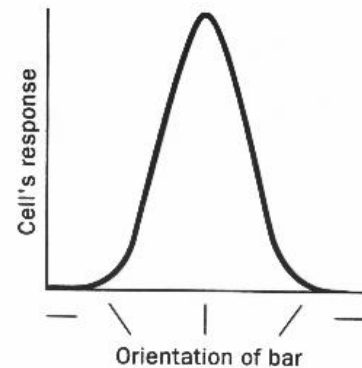
# The firing rate hypothesis

Receptive field: area in the outside/physical world for which a neuron is responsive.

Feature preference

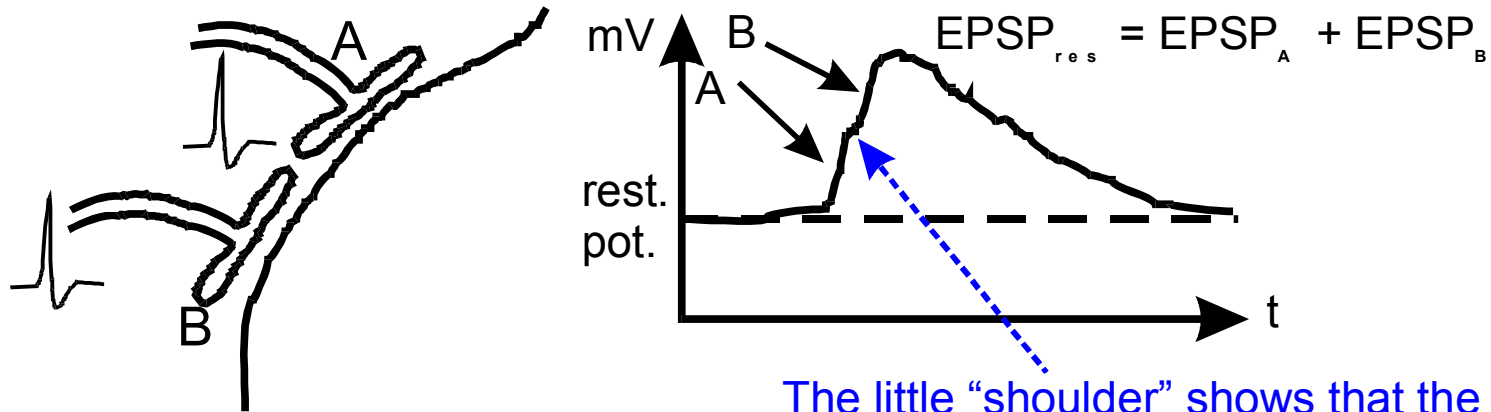


Tuning curve of V1 neuron in cat



## Necessary conditions for optimal summation:

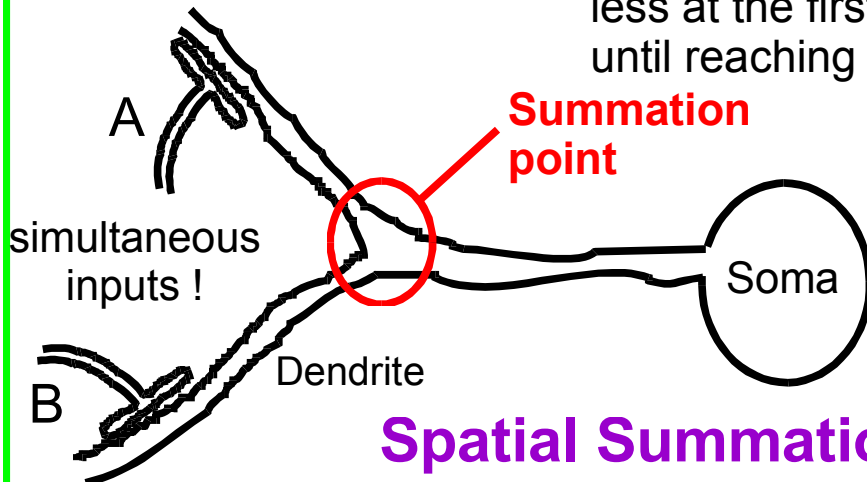
- 1) synapses have to be closely adjacent
- 2) pre-synaptic signals have to arrive simultaneously
- 3) resting potential and reversal potential(s) have to be very different.



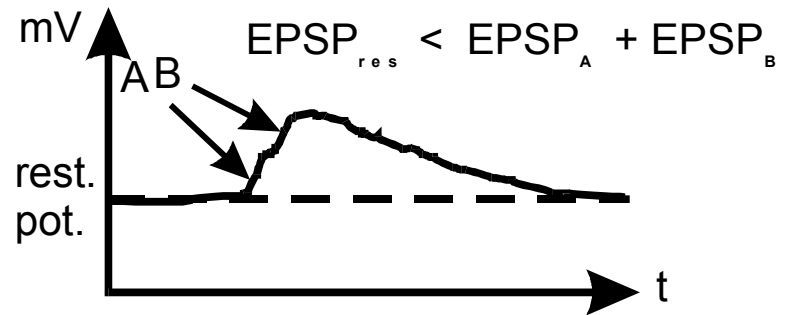
The little "shoulder" shows that the EPSPs were not truly simultaneous.

## Consider 1:

If the synapses are far from each other the amplitude will be less at the first summing point. It will then further decay until reaching the soma.

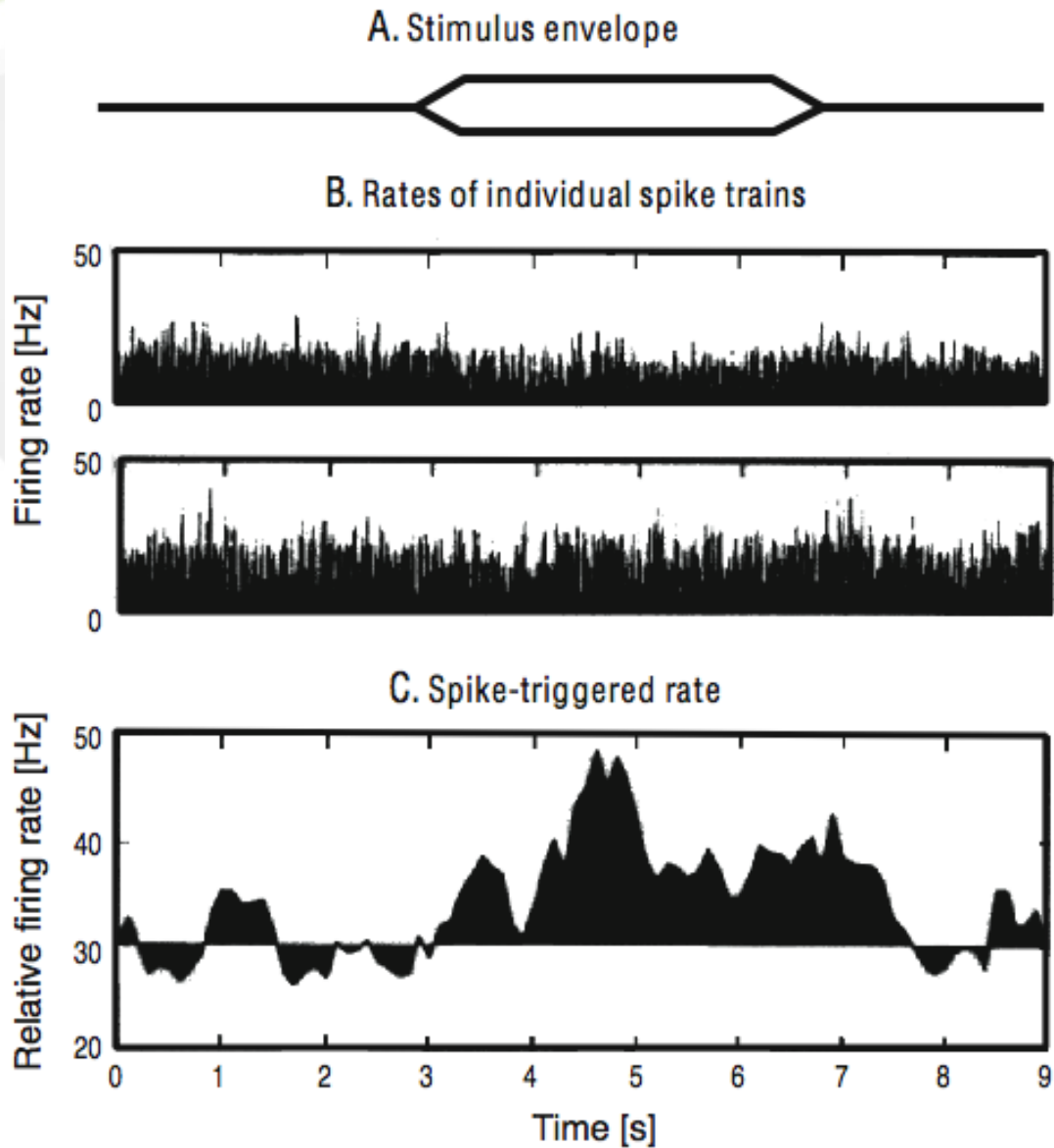


**Spatial Summation**

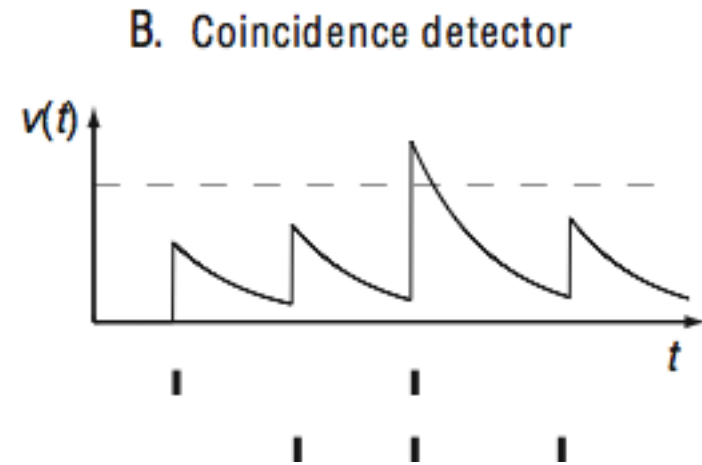
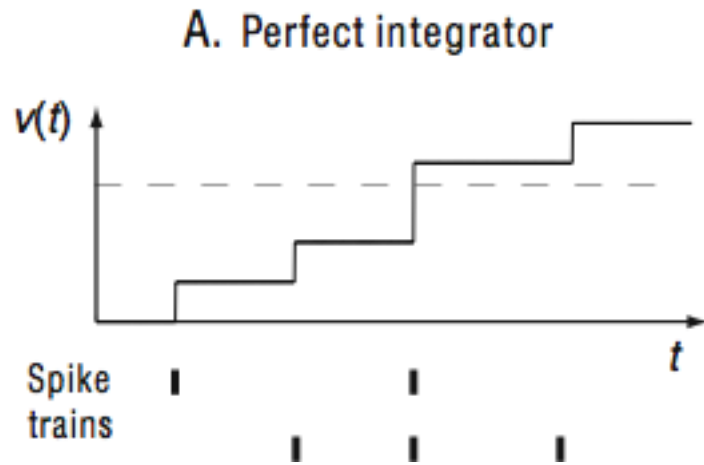


# The correlation code hypothesis

Stimulus features are encoded by neurons firing around the same time

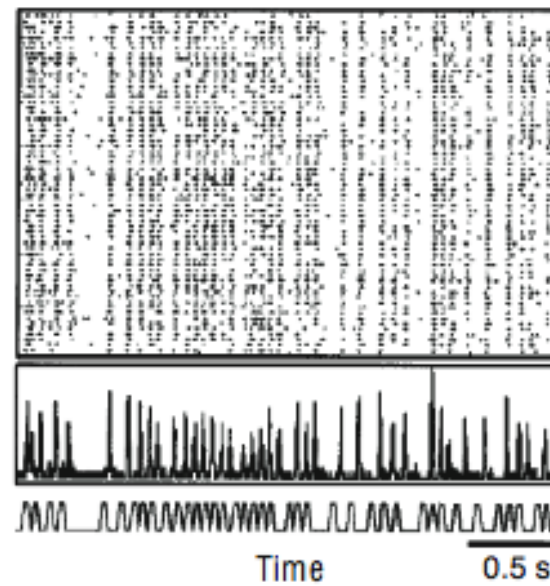
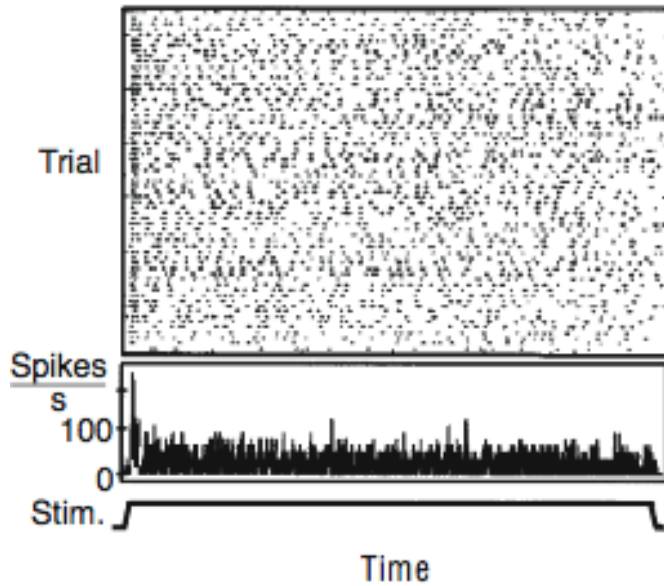


# Integrator or coincidence detector?



**A. Constant stimulus**

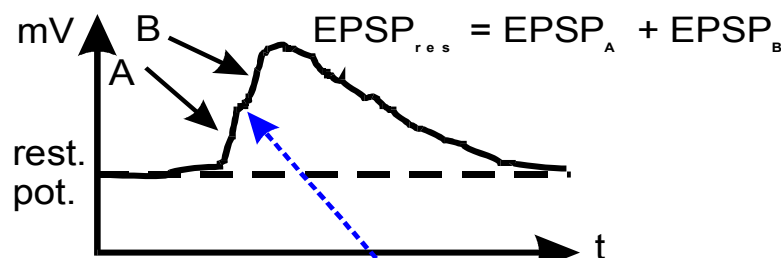
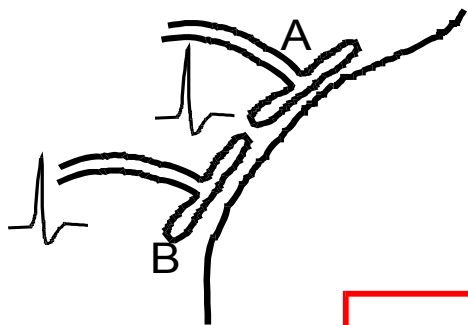
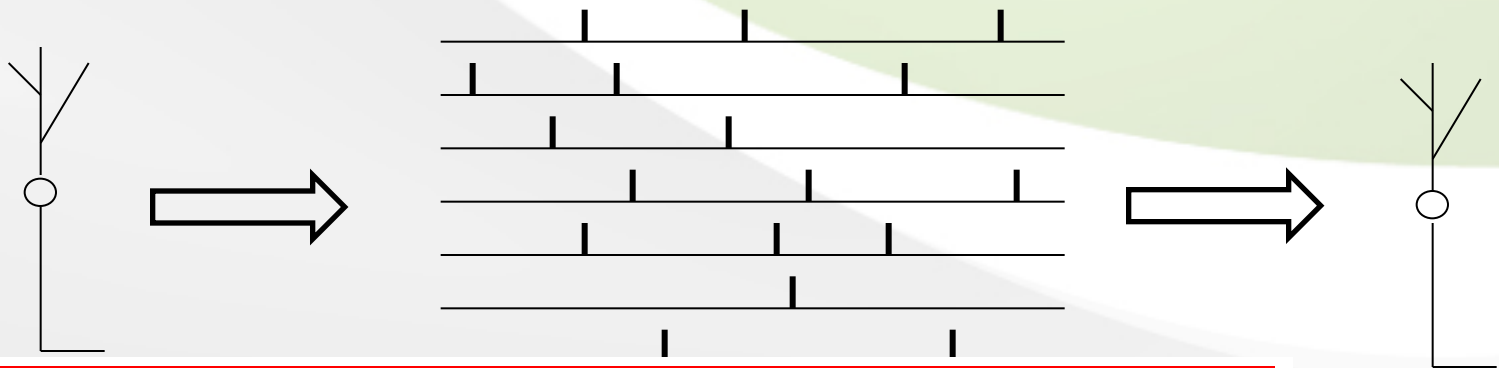
**B. Rapidly changing stimulus**



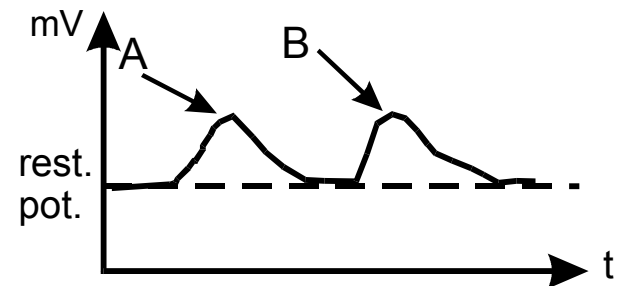
# The Neural Code

Neurons communicate via **exact spike timing**

**Firing rate alone does not carry all the relevant information**



If the difference in arrival times is too large, temporal summation does not occur anymore !



# The Neural Code

**Edelman** (Nobel laureate in Medicine) proposed the **theory of neuronal group selection (TNGS)**, also known as Neural Darwinism,

**Edelman** stated that DNA does not contain all information needed to code all brain connections. DNA provides basic species-related information exclusively.

**Living and dead cells are regulated by stochastic rules, therefore each brain is different from each other.**



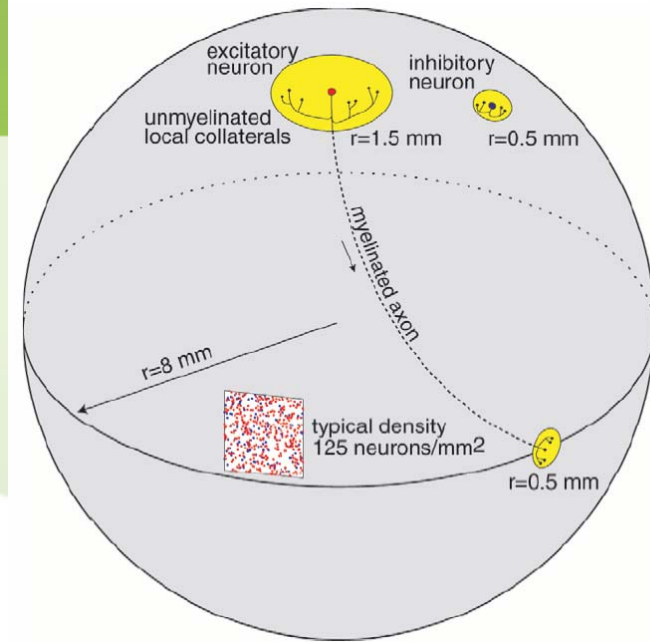
Indeed, in the human brain there are  $10^{11}$  neurons, with  $10^{15}$  synapses. DNA has 109 pairs of nucleotides



# The Neural Code

Neural Groups are characterized by:

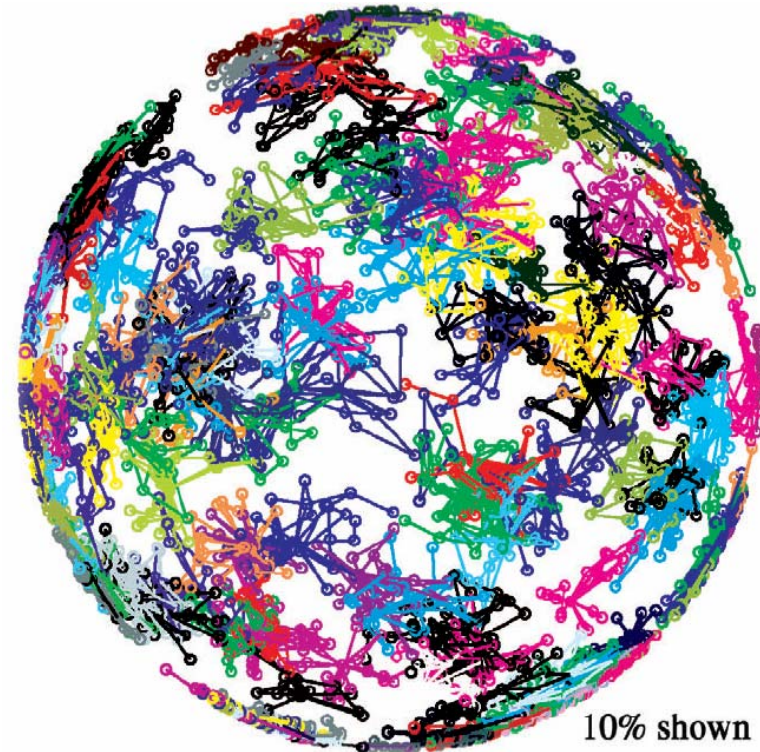
- Biological Selection (DNA)
- Experiential Selection
- **Reentry**



Neural Groups should be considered as the basic processing unit of the brain

How to model Neural Groups in a Spiking Neural Network?

**Time** must be taken into account



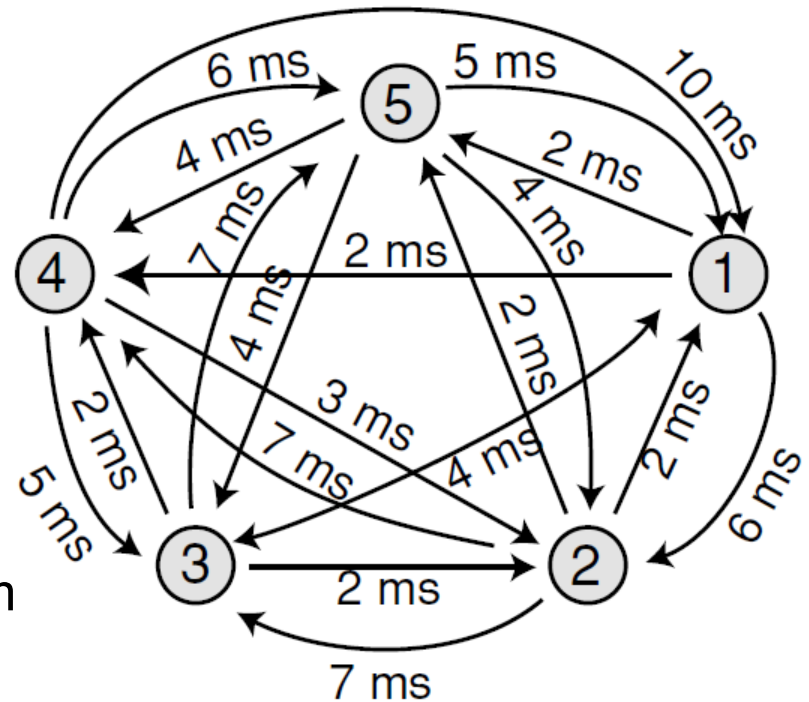
# Spiking neural network

The network consists of cortical spiking neurons with axonal conduction delays and spike timing-dependent plasticity (STDP).

The network is sparse with 0.1 probability of connection between any two neurons.

Neurons are connected to each other randomly

Synaptic connections among neurons have fixed conduction delays, which are random integers between 1 ms and 20 ms.

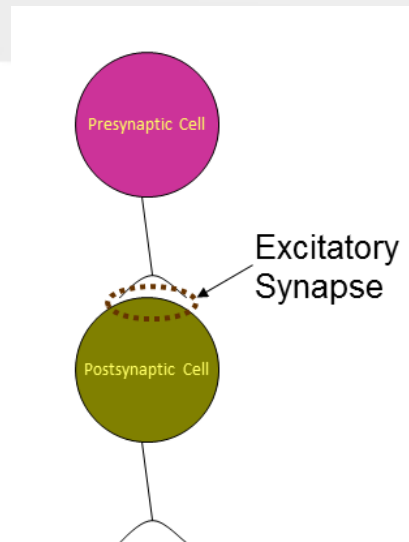


# STDP rule (spike-timing-dependent plasticity)

Initially, all synaptic connections have equal weights.

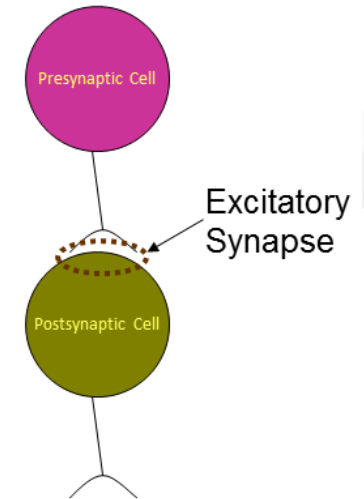
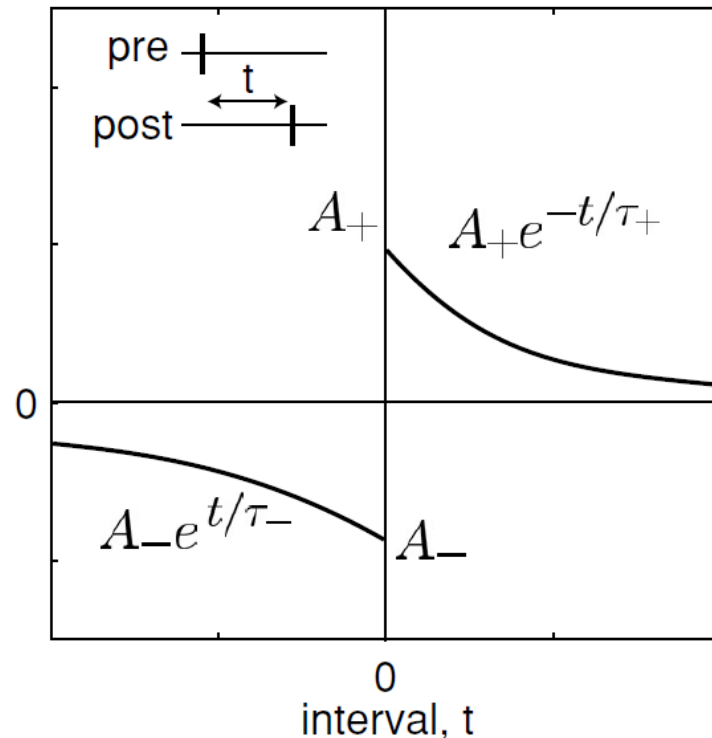
The magnitude of change of synaptic weight

depends on the timing of spikes.



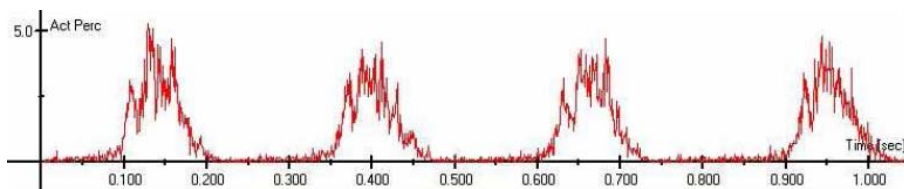
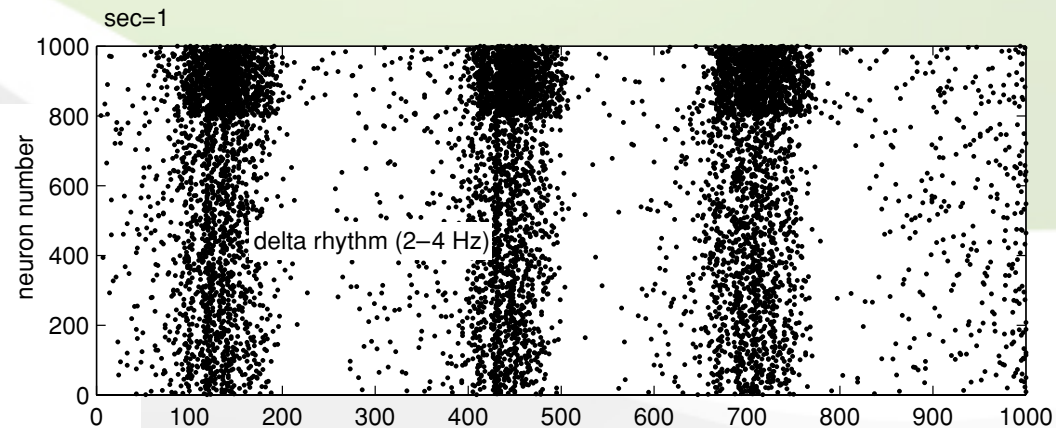
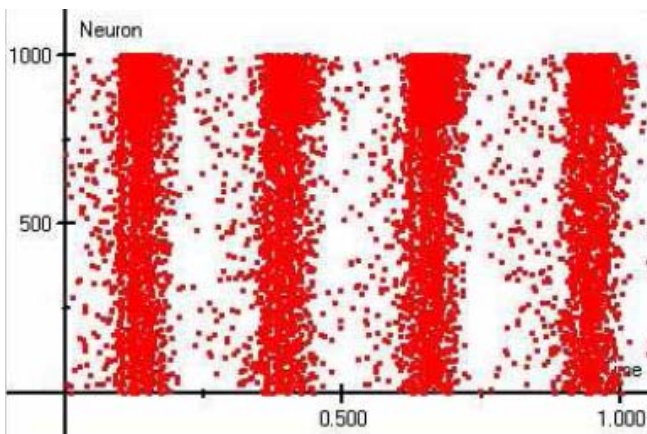
# STDP rule (spike-timing-dependent plasticity)

If the presynaptic spike arrives at the postsynaptic neuron before the postsynaptic neuron fires—for example, it causes the firing—the synapse is potentiated.

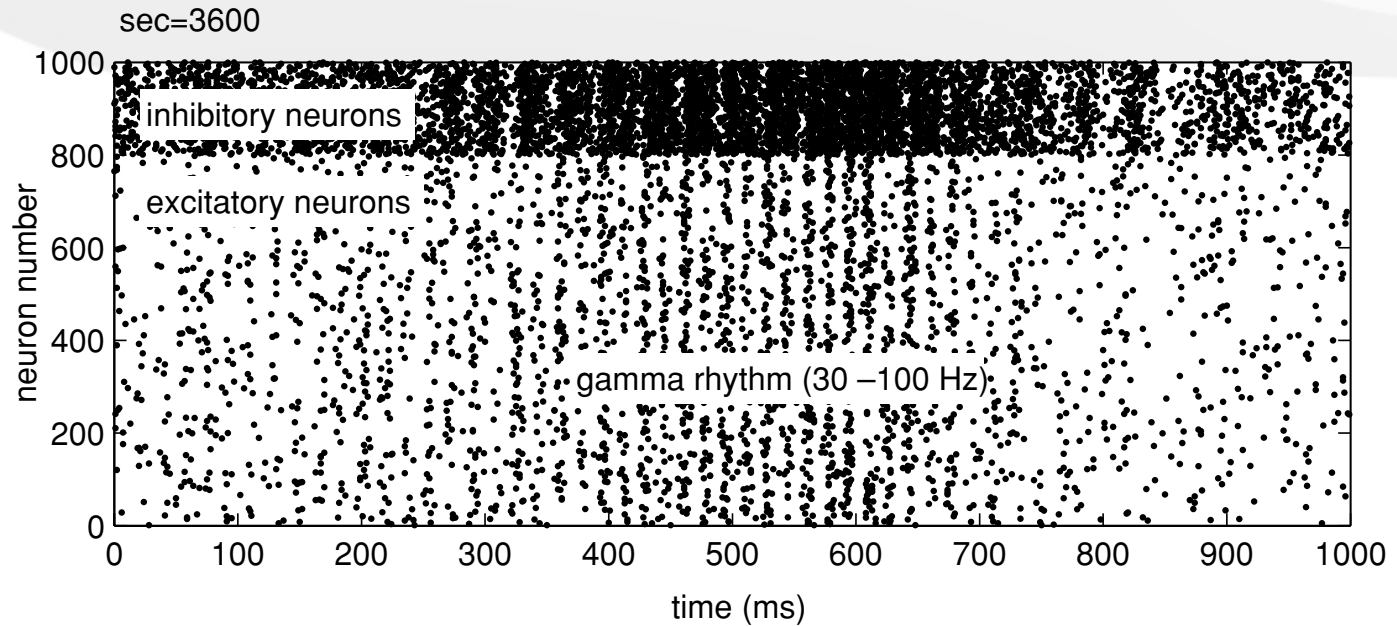
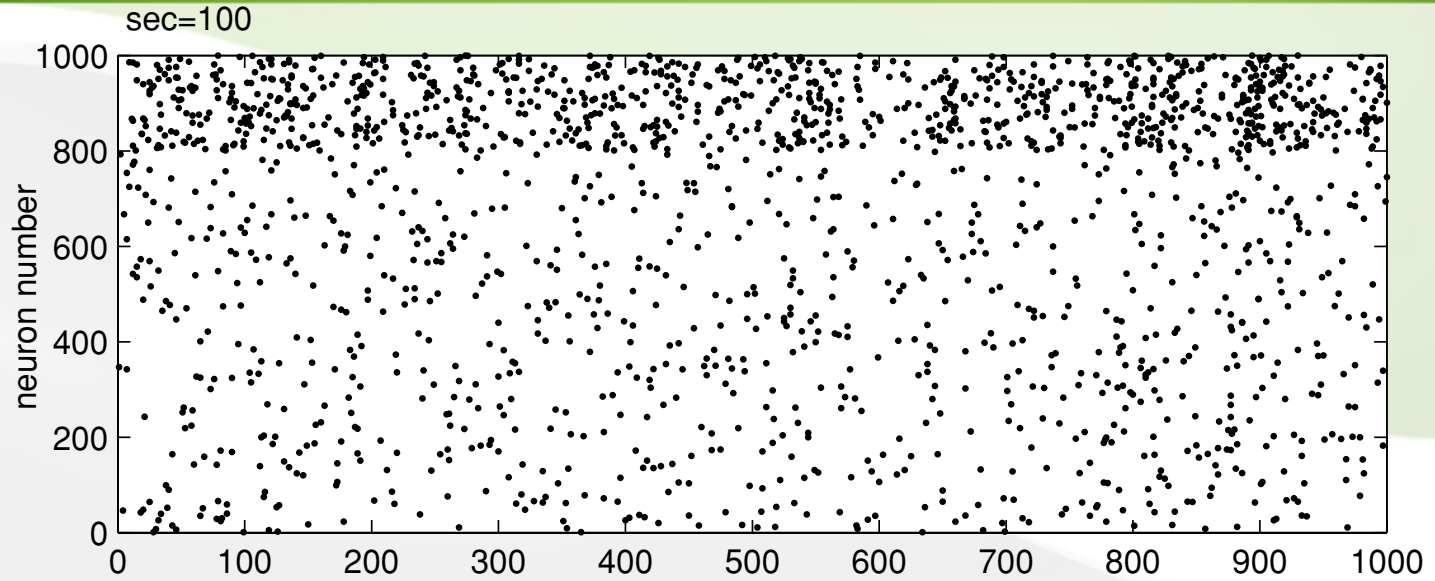


If the presynaptic spike arrives at the postsynaptic neuron after it fired, that is, it brings the news late, the synapse is depressed.

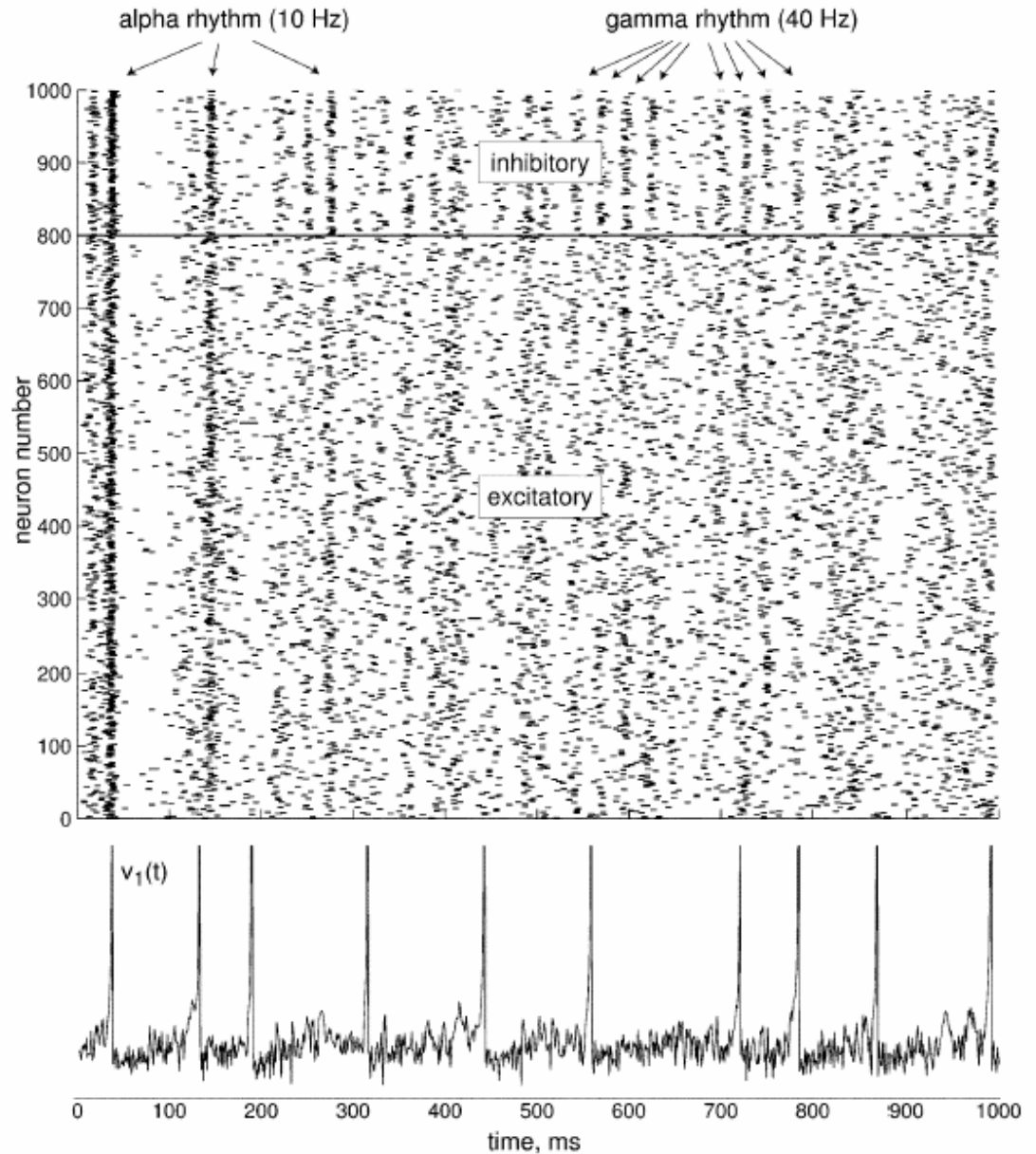
## First Seconds of Simulation



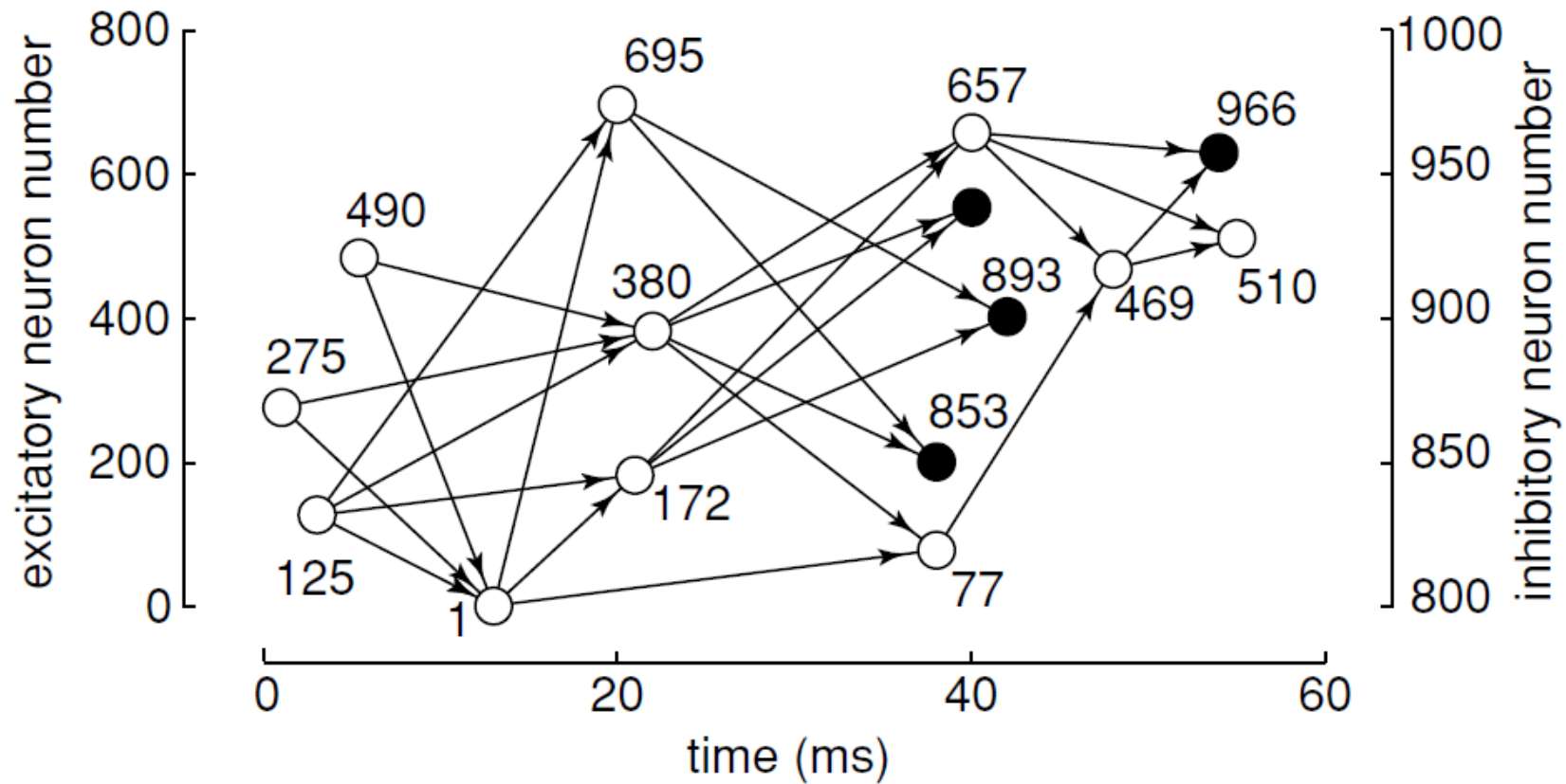
**delta waves (<4 Hz)**



## First Minutes of Simulation



# Polychronous Neural Group (PNG)





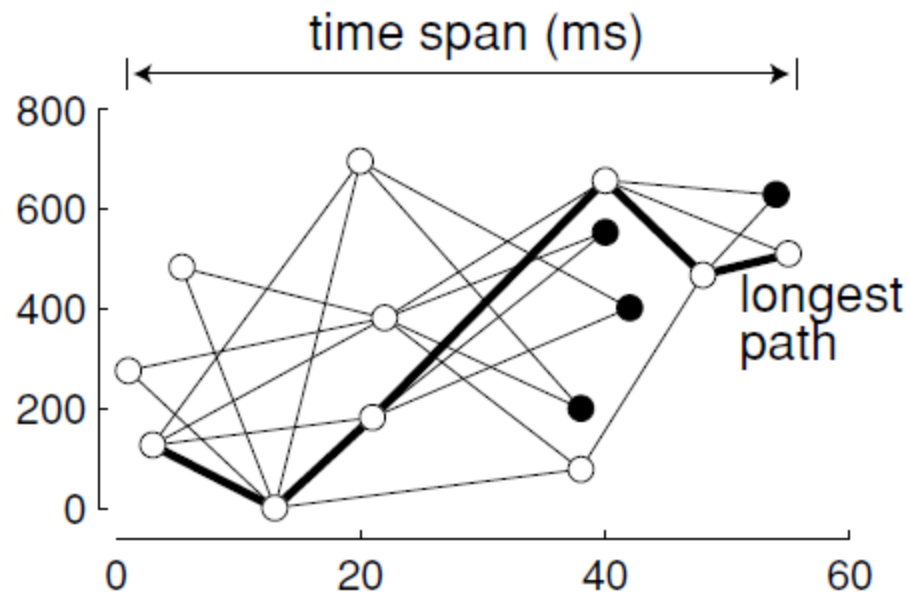
# Characteristics of polychronous groups

The groups have different

Sizes

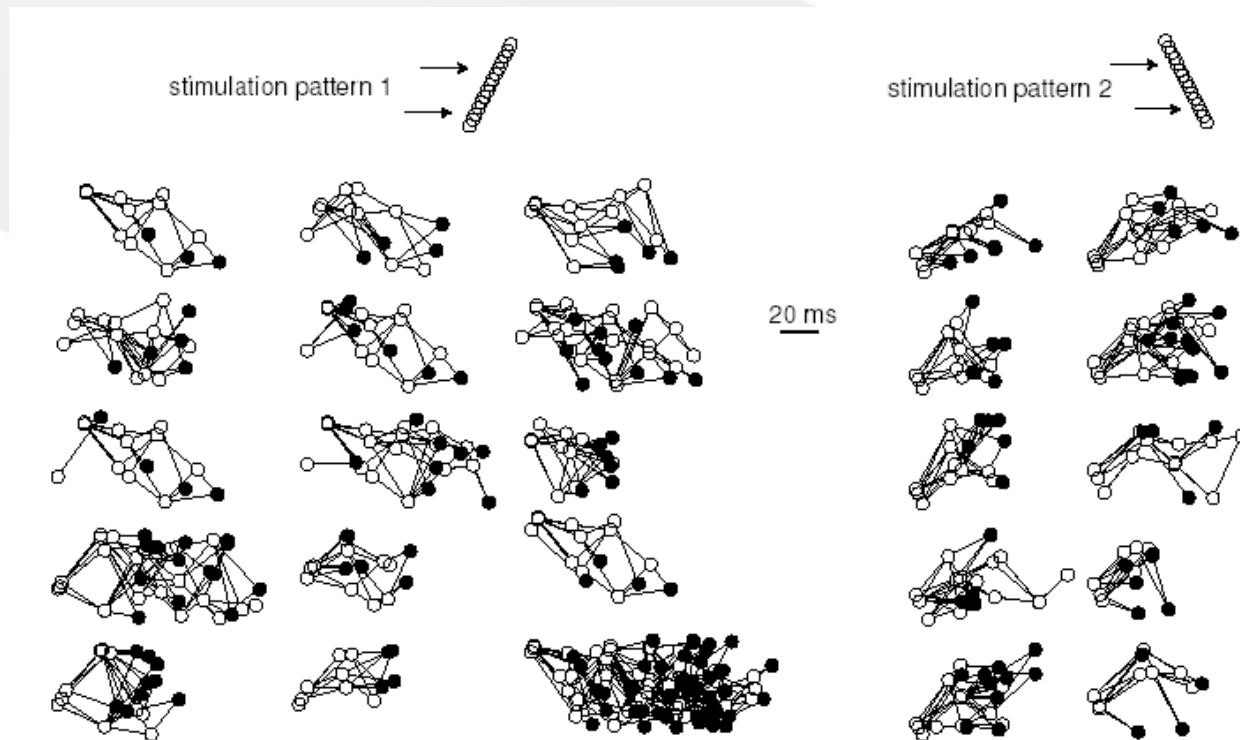
Lengths

Time spans

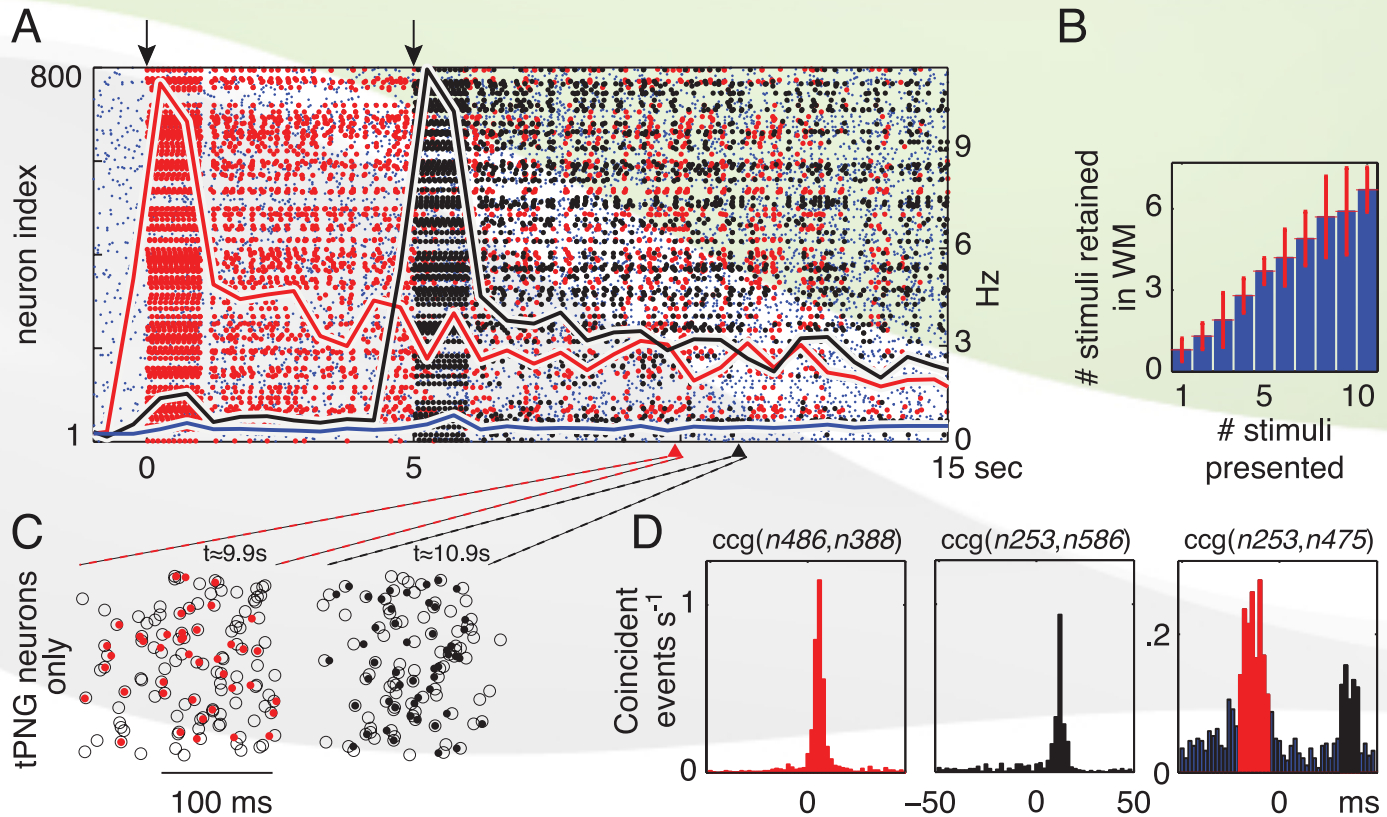


# Representations of Memories and Experience

Persistent stimulation of the network with two **spatio-temporal patterns** result in emergence of polychronous groups that represent the patterns. the **groups activate whenever the patterns are present**.

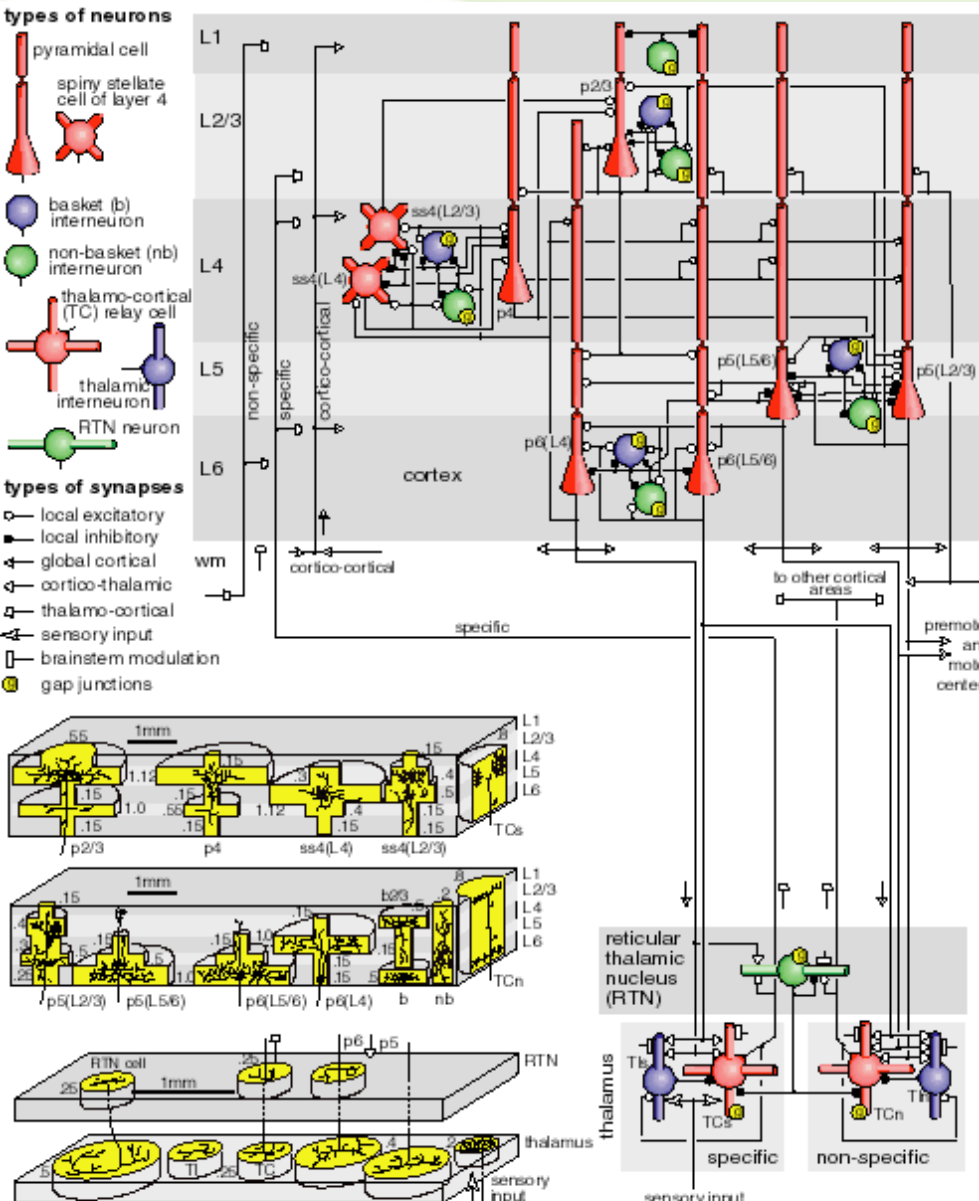


# Spike-Timing Theory of Working Memory



Working memory (WM) provides temporary, storage and manipulation of information necessary for cognition. Using simulations, Szatmary et al (2010) show that large memory content and WM functionality emerge spontaneously if we take the spike-timing nature of neuronal processing into account. Here, memories are represented by extensively overlapping groups of neurons that exhibit stereotypical time-locked spatiotemporal spike-timing patterns, called polychronous patterns

# Simulation of Large-Scale Brain Models



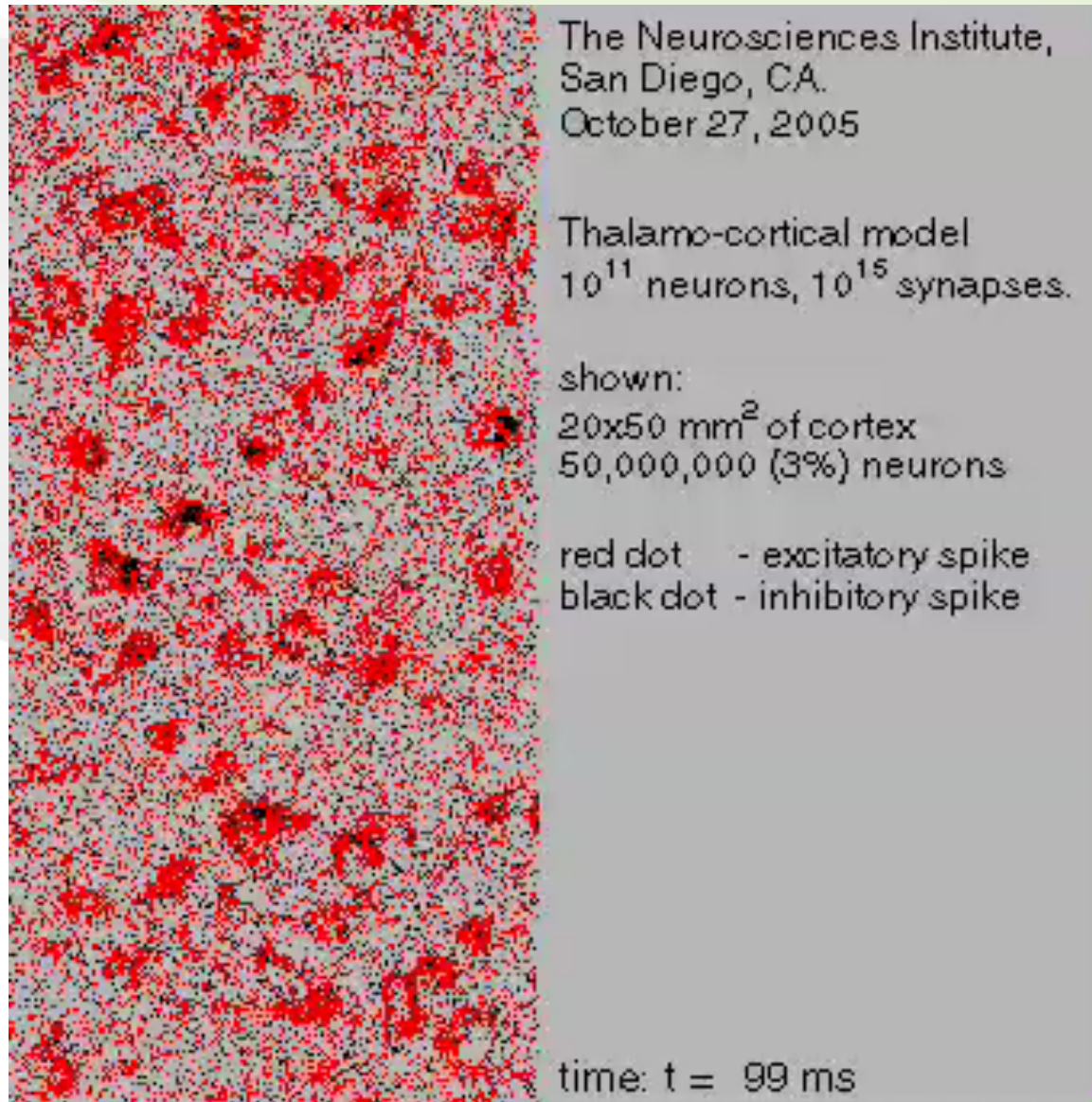
In 2005 Izhikevich finished simulation of a model that has the size of the human brain. The model has 100,000,000,000 neurons (hundred billion or  $10^{11}$ ) and almost 1,000,000,000,000,000 (one quadrillion or  $10^{15}$ ) synapses.

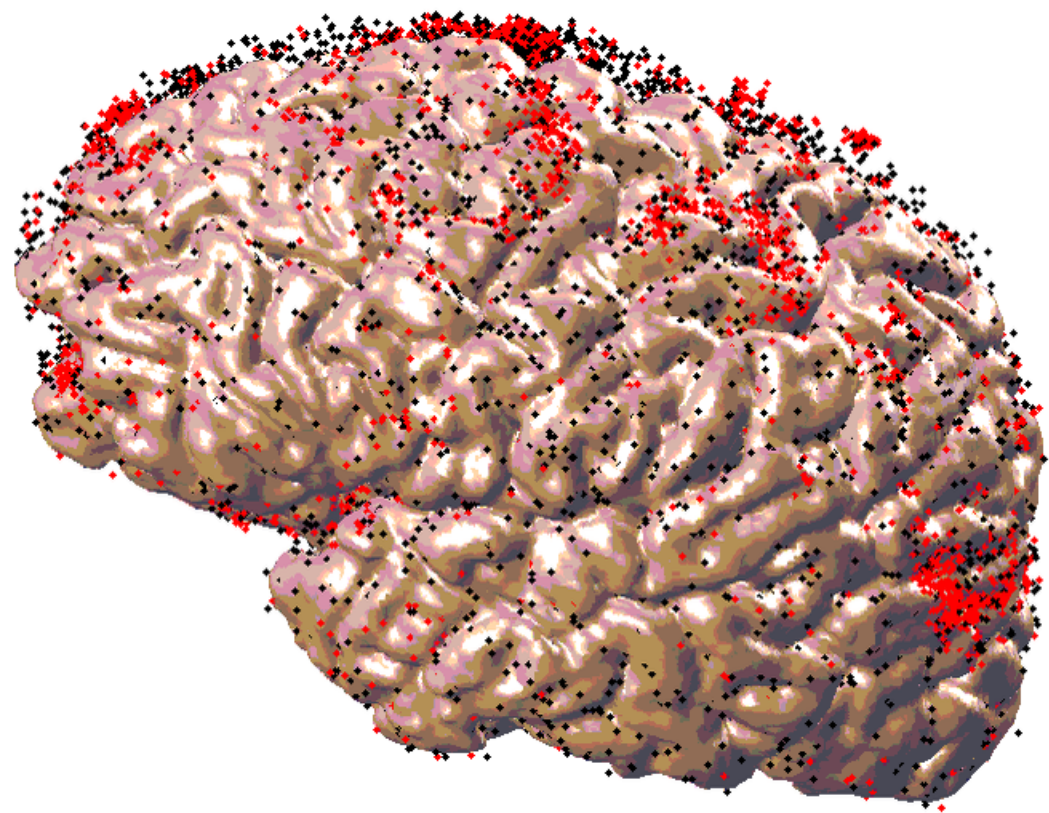
It represents  $300 \times 300 \text{ mm}^2$  of mammalian thalamo-cortical surface, specific, non-specific, and reticular thalamic nuclei, and spiking neurons with firing properties corresponding to those recorded in the mammalian brain.

The model exhibited alpha and gamma rhythms, moving clusters of neurons in up- and down-states, and other interesting phenomena

One second of simulation took 50 days on a beowulf cluster of 27 processors (3GHz each).

# Simulation of Large-Scale Brain Models



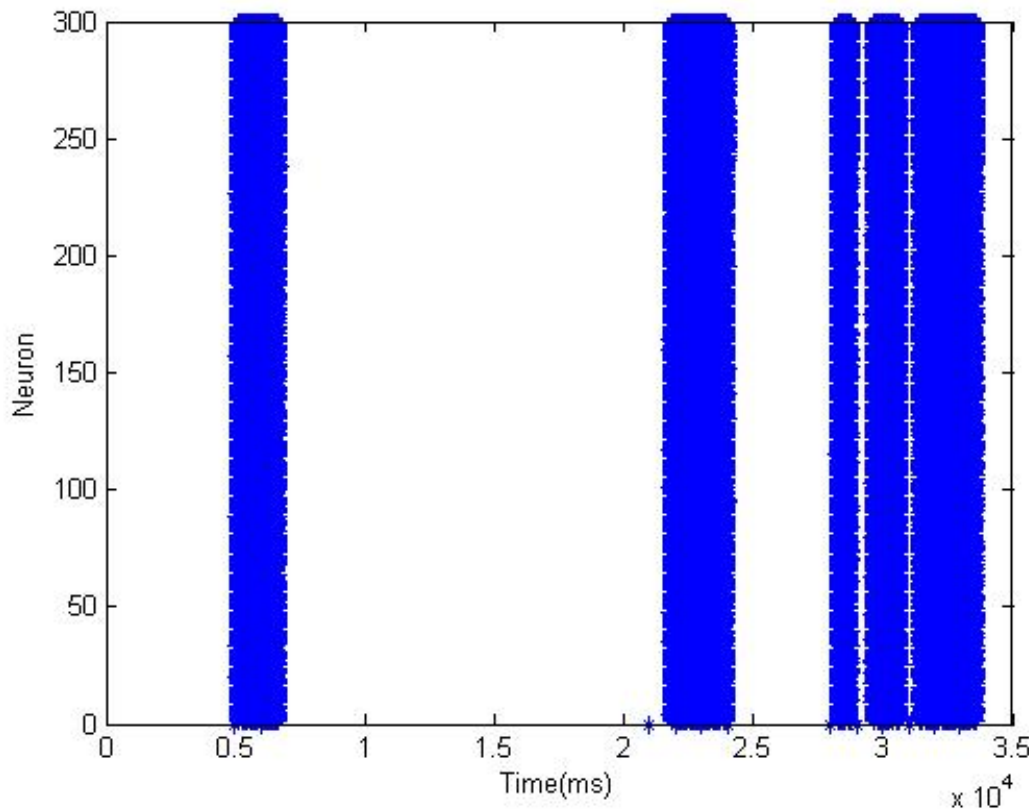


# A stochastic version of Izhichevich Model

$$\frac{dv}{dt} = 0.04v^2 + 5v + 140 - u + \epsilon(\mu, \sigma)$$

$$\frac{du}{dt} = a(bv - u)$$

If  $v \geq 30$   
then  $\{c \rightarrow v, u \rightarrow u+d\}$



Persistent Bursting Activity!