
Human and animal models in BioRobotics

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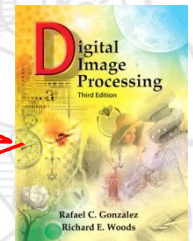
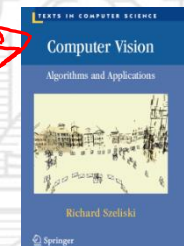
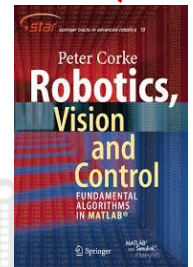
The BioRobotics Institute
Scuola Superiore Sant'Anna

Pontedera, 01 October 2019
Scuola Superiore Sant'Anna

Reference materials and credits

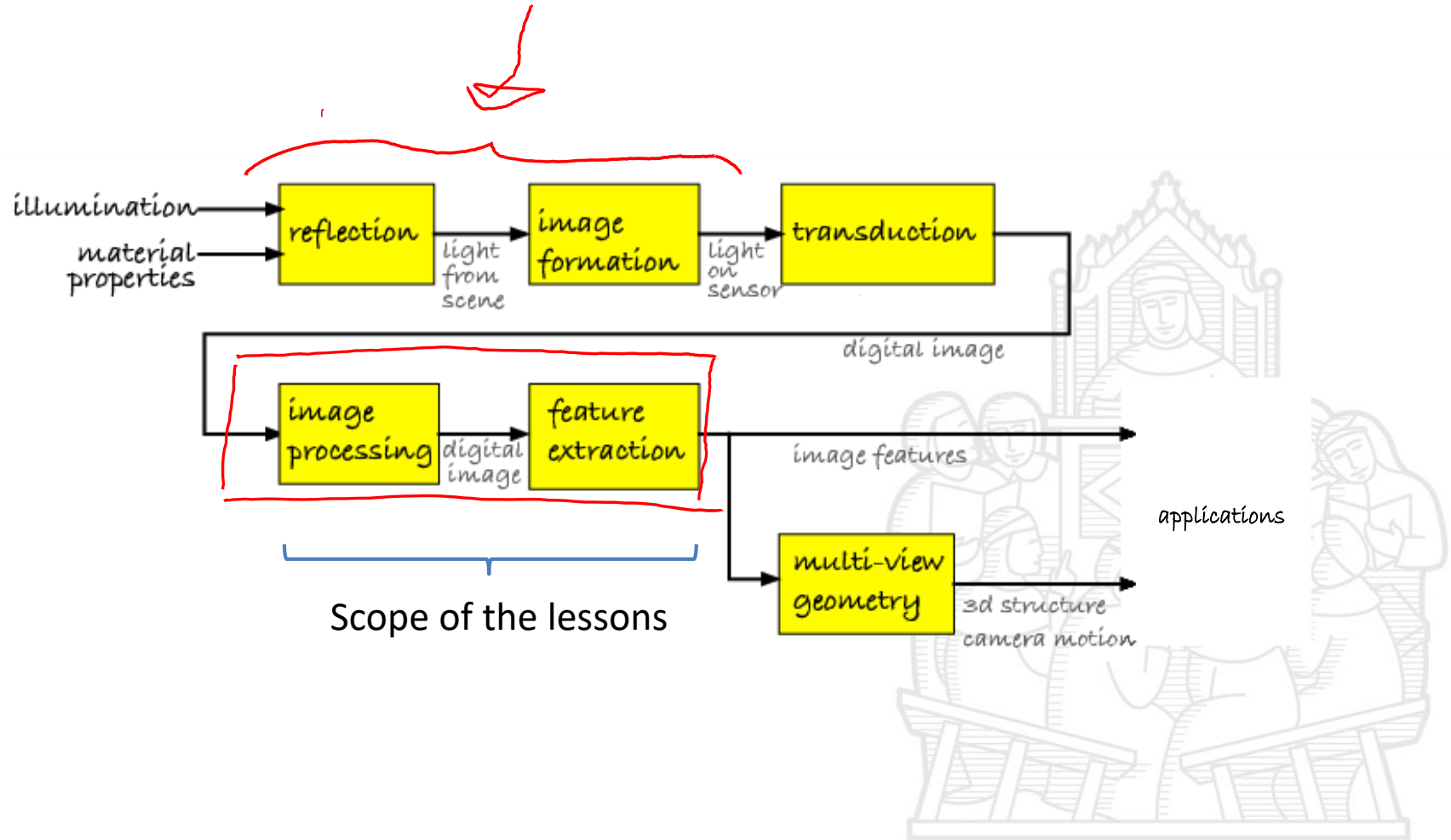
Most of the material presented in these lessons can be find on the brilliant, seminal books on robotics and image analysis reported hereafter:

1. P.I. Corke, “Robotics, Vision & Control”, Springer 2011, ISBN 978-3-642-20143-1
2. R. Szeliski, “Computer Vision: Algorithms and Applications”, Springer-Verlag New York, 2010
3. R.C. Gonzalez & R.E. Woods, “Digital Image Processing (3rd edition)”, Prentice-Hall, 2006



Most of the images of these lessons are downloaded from RVC website <http://www.petercorke.com/RVC/index.php> and, despite they are free to use, they belong to the author of the book.

Overall computer vision process and scope of the lessons



What's a digital image?

320×240 , LR

8-bit = $[0, 255]$

1080×920

fundamental element:
the pixel

Digital images are **mosaics**
made of **pixels**

$I[600,516]=213$

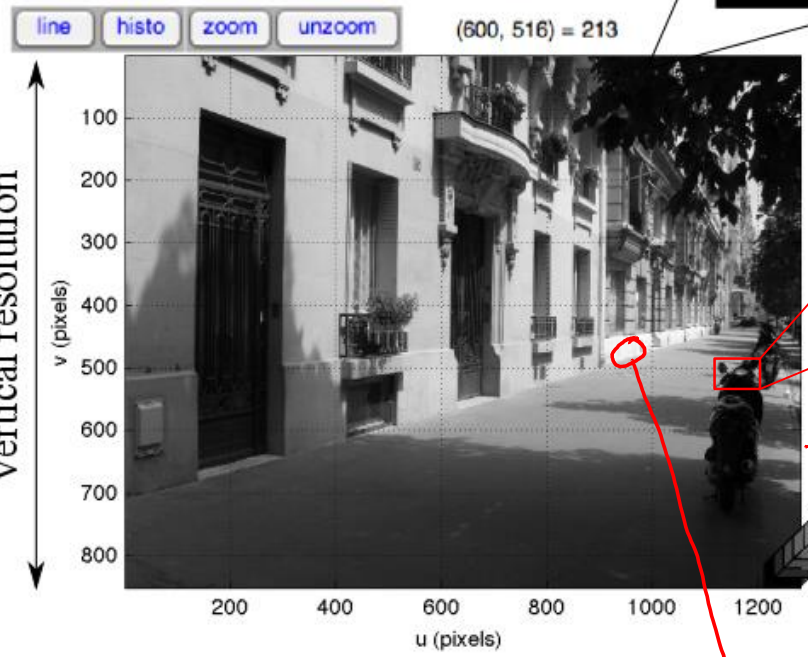


image depth:
8-bit = $[0, 255]$

Image resolution is the
number of pieces (pixel)
used to build the mosaic
(image)

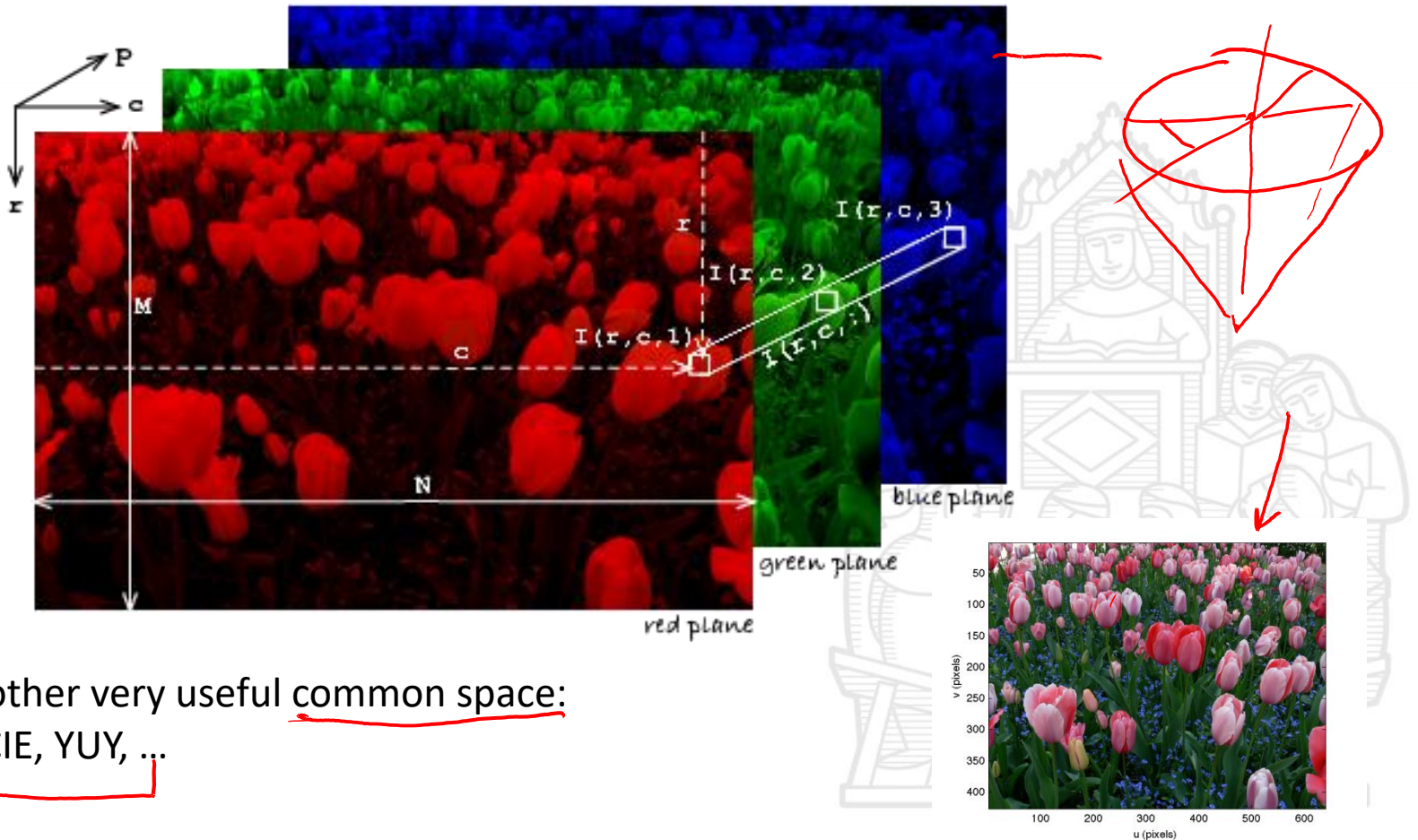
Image depth is the number
of colours (levels) of mosaic
pieces

255 B -

Colour images

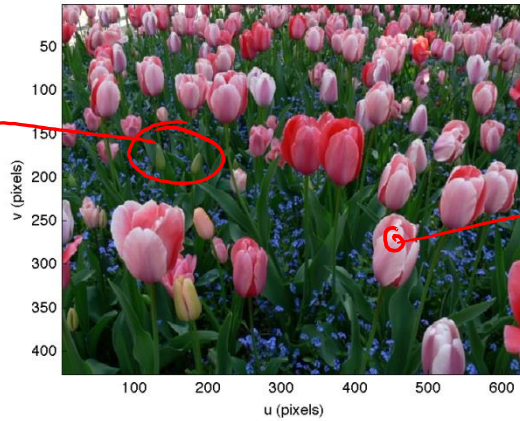
$$I(x, y, c)$$

Colour images have three channels: the most common triplet is the **R-G-B**

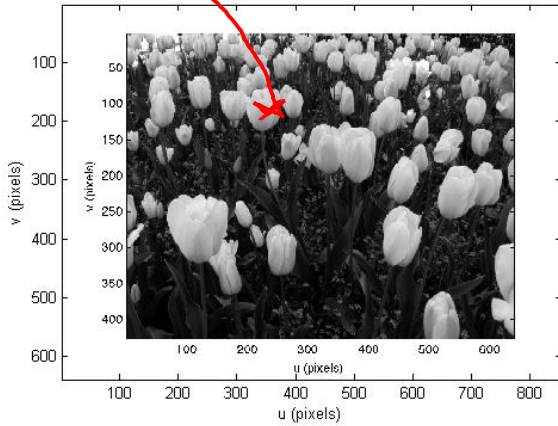


There are other very useful common space:
HSV, XYZ, CIE, YUY, ...

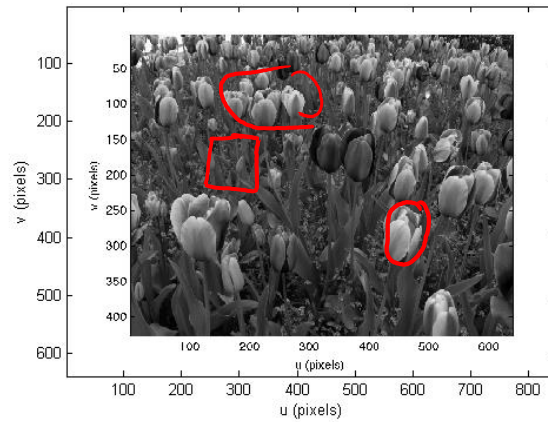
Colour images



Red channel



Green channel



640x854x1 uint8

0 - 255
640x854x3 uint8
R G B
255, 255, 255

Blue channel

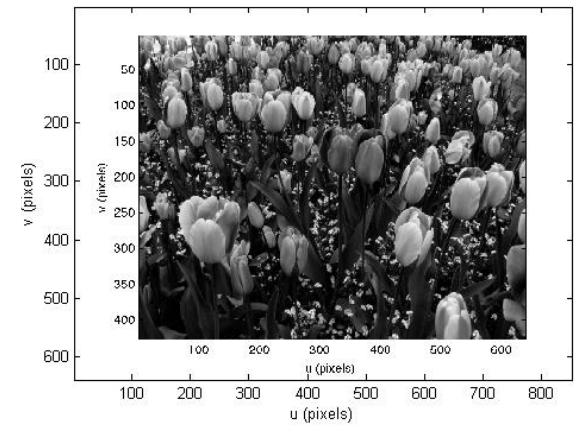
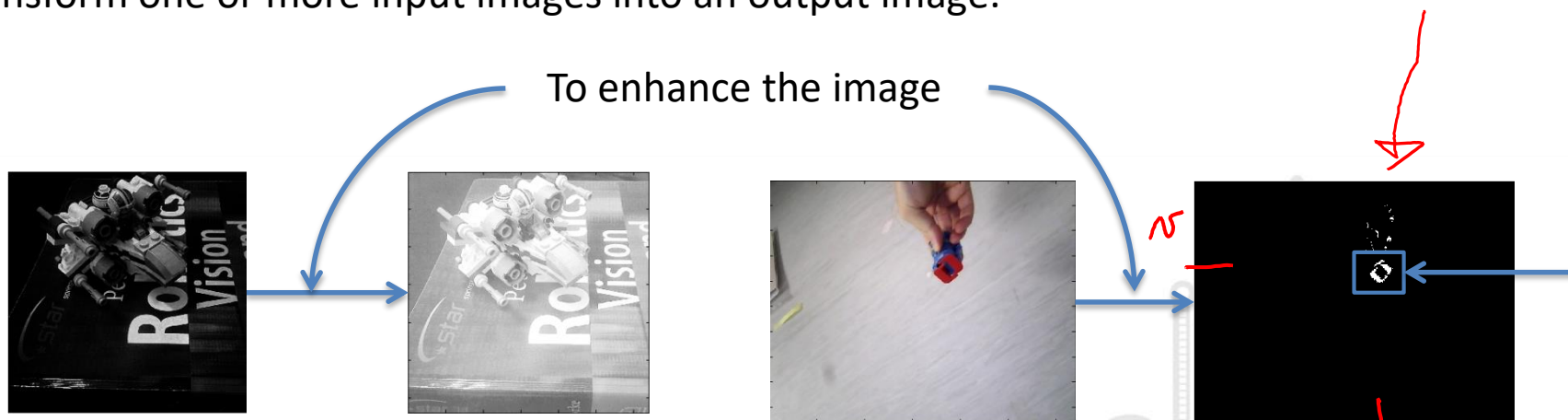
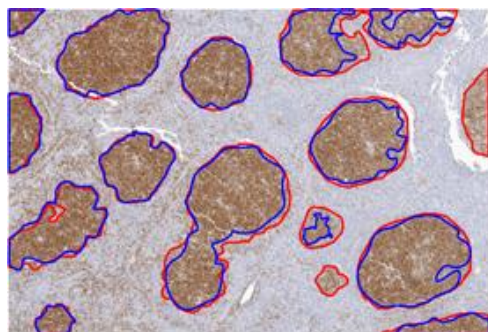


Image processing

Transform one or more input images into an output image.



Human interpretation



Computer aided diagnosis
Metin Gurcan, Ph.D – Ohio State



Objects recognition
HERB robot butler – Carnegie Mellon

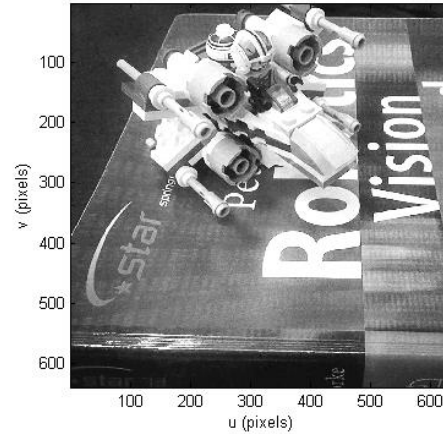
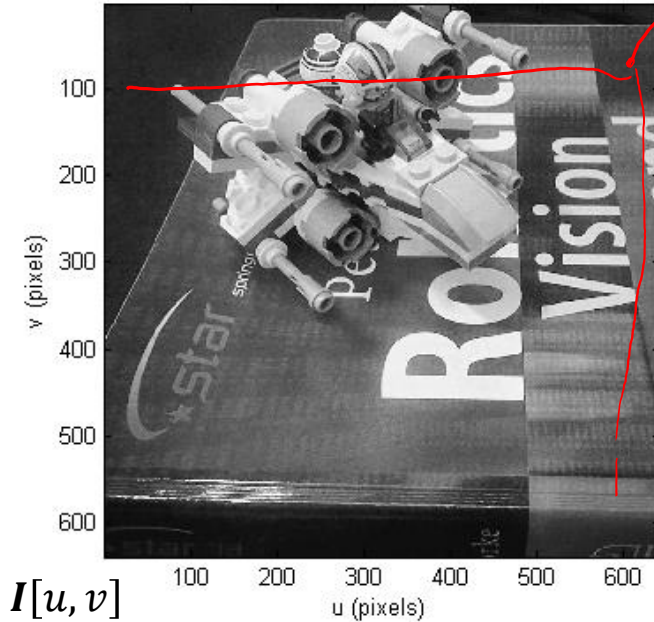
Features extraction

Lightening and darkening

$$8 - 60 + \rightarrow \frac{278}{255}$$

$$I[600, 100] = 120$$

$$255$$

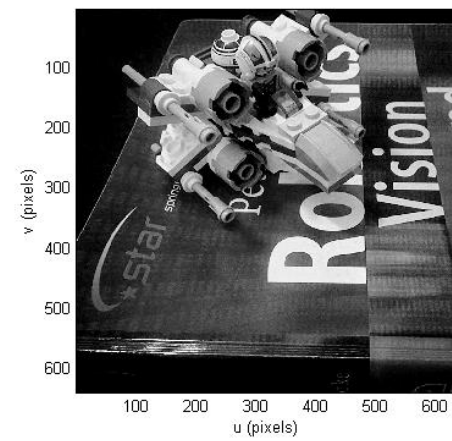


$$O[u, v] = I[u, v] + 50$$

$$205 \uparrow$$

$$\downarrow$$

$$\hookrightarrow 255$$

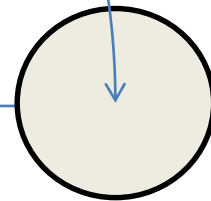
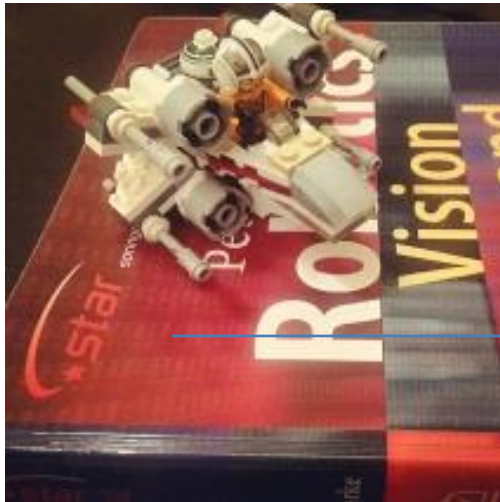


$$O[u, v] = I[u, v] - 50$$

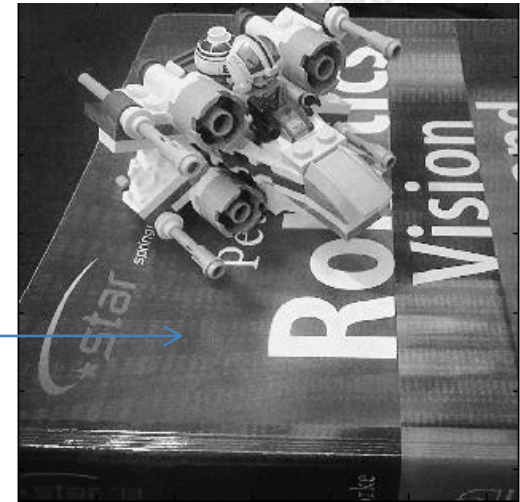
Monadic operations change the distribution of grey levels on images

Simple monadic operation (more channel):

Gray-scale conversion with International Telecommunication Unit (ITU) recommendation 709



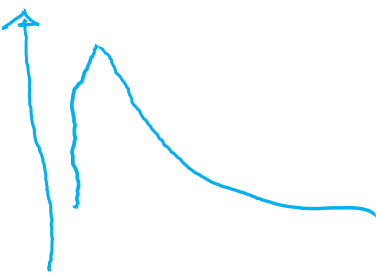
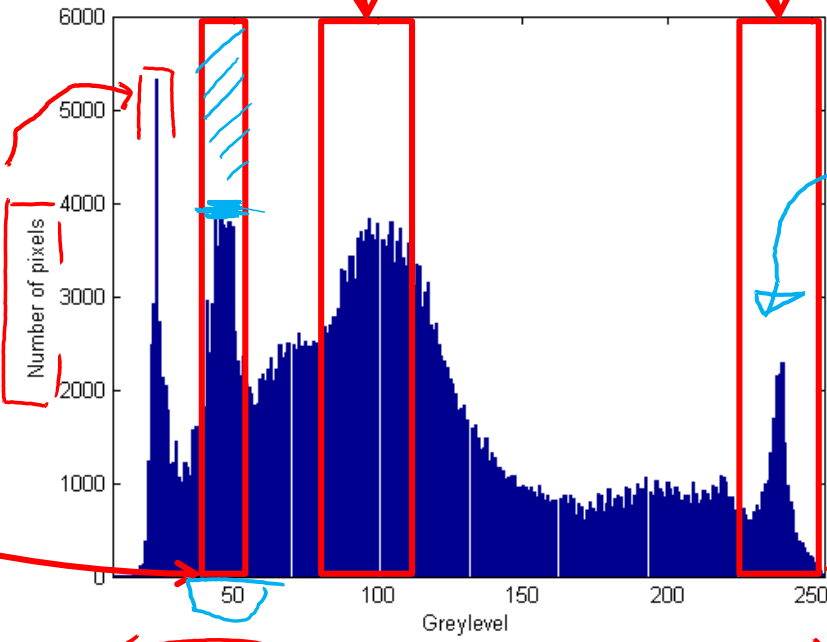
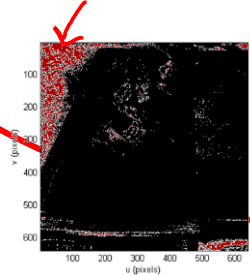
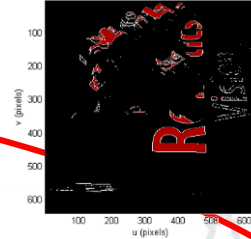
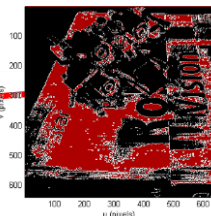
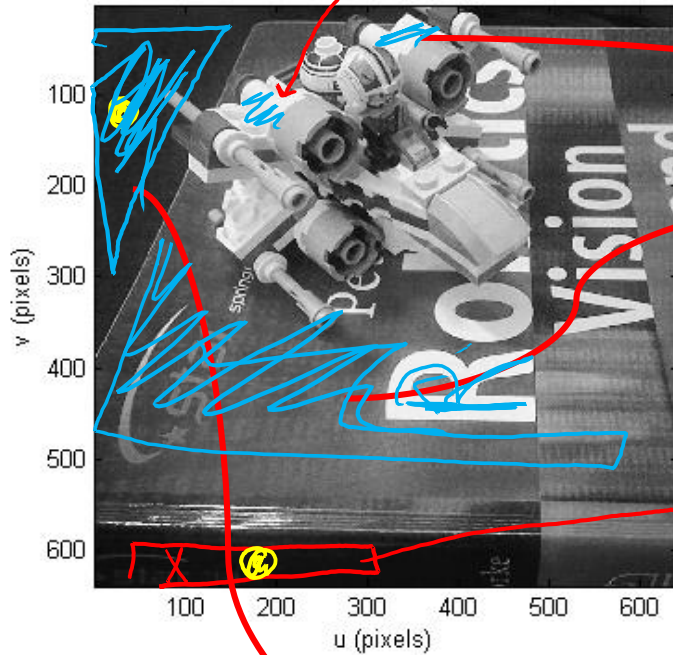
$$Y=0,212R+0,7152G+0,0722B$$



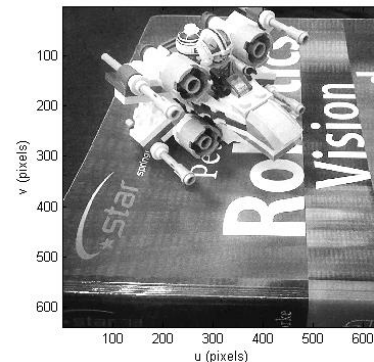
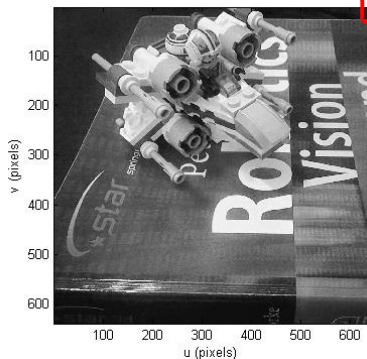
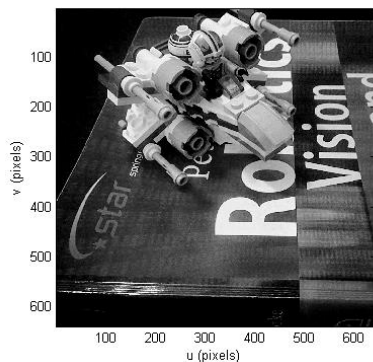
Histogram

Is a graph representing the grey level occurrences of an image.

↑
GREY
→



Histograms and monadic operations



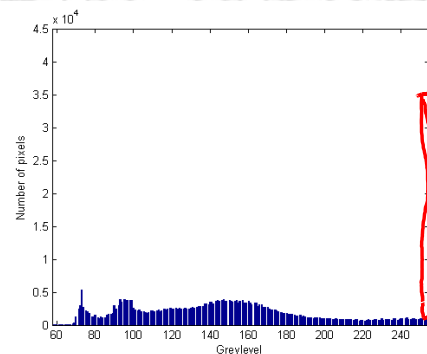
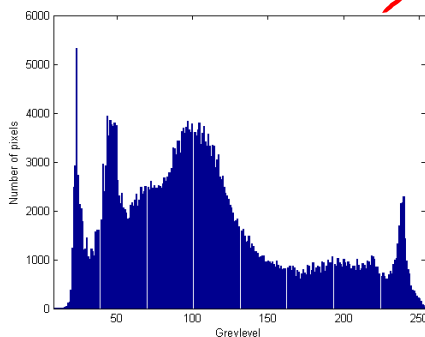
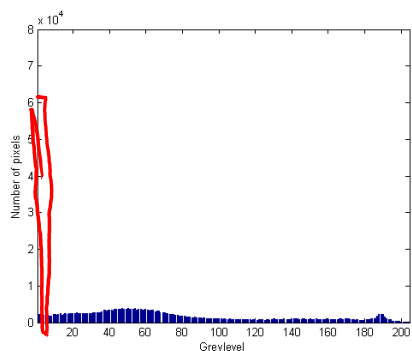
$$O[u, v] = I[u, v] - 50$$



$$I[u, v]$$

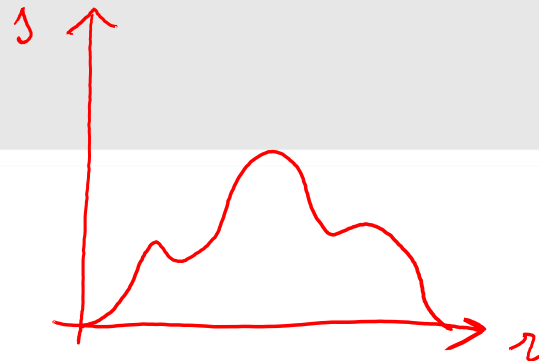


$$O[u, v] = I[u, v] + 50$$



Common operations

$$s(r) = c \cdot r^\gamma$$



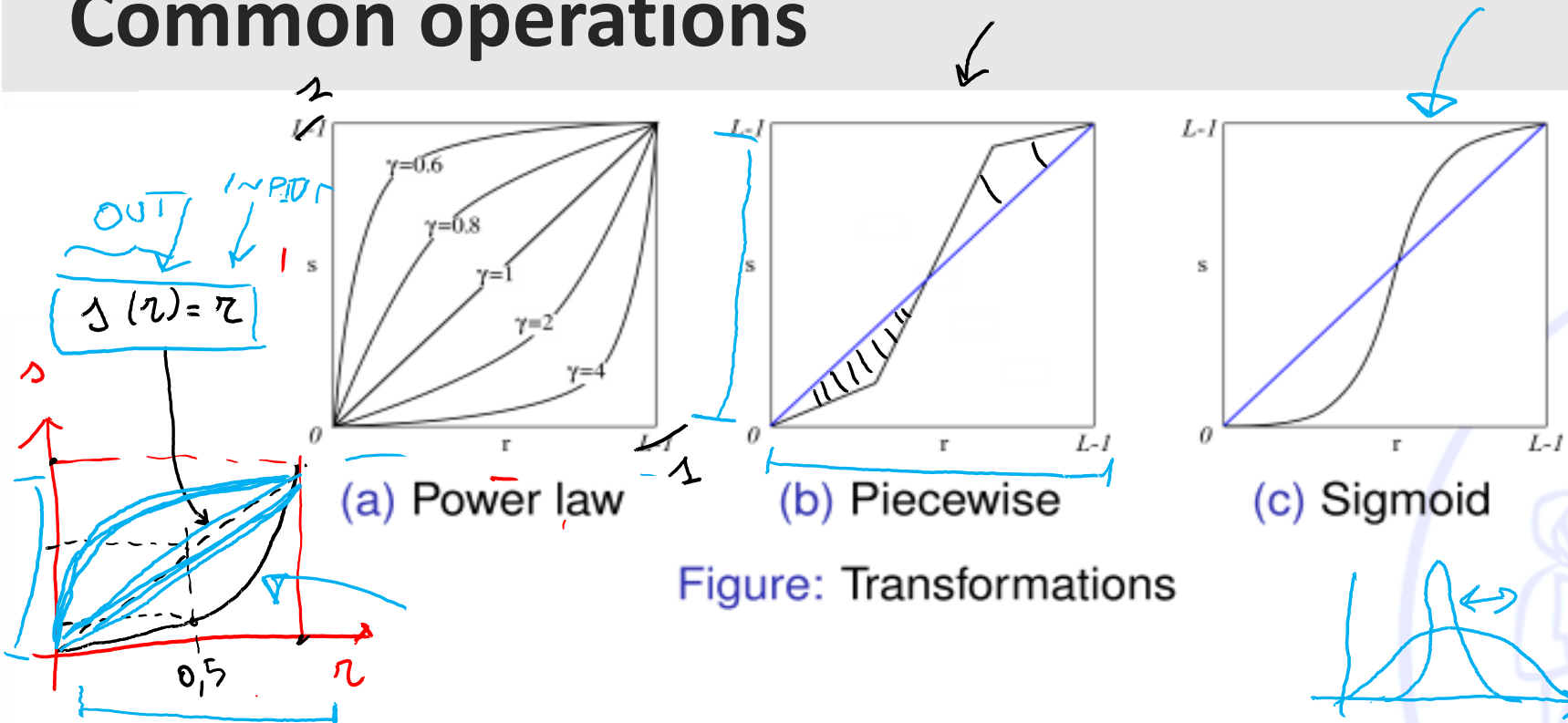
$$s(r) = \begin{cases} c_1 \cdot r & 0 \leq r < r_{min} \\ c_2 \cdot r & r_{min} \leq r < r_{max} \\ c_3 \cdot r & r_{max} \leq r < L - 1 \end{cases}$$

$$s(r) = \frac{c_1}{1 + e^{-r}}$$

- power law
- piecewise
- sigmoid

Each function requires parameters definition

Common operations



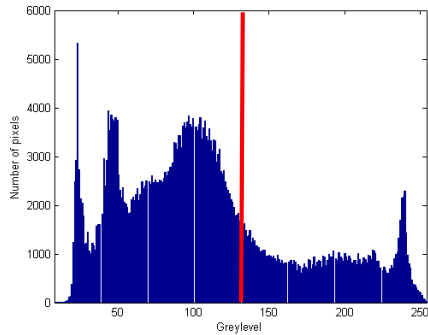
Power law lighten or darken

Piecewise flexible

Sigmoid enhance the contrast



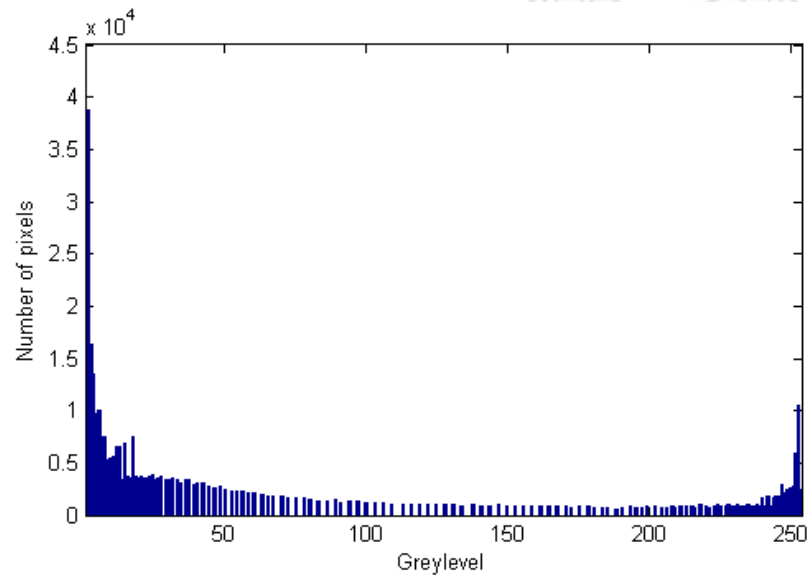
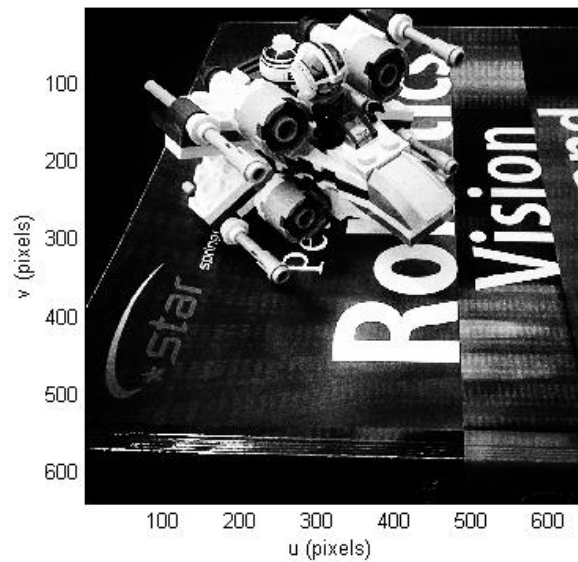
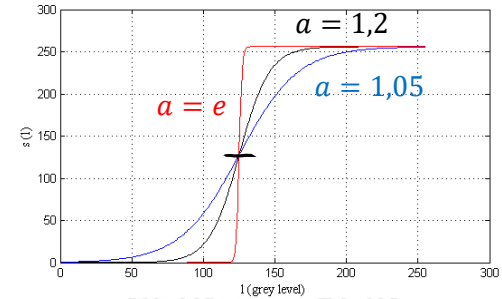
Contrast enhancement



Sigmoid function

$$s(l) = \frac{256}{1 + a^{-(l-125)}}$$

↙
↘



Pay attention

$$\begin{aligned}L &= 21 \\c &= 1 \\ \gamma &= 2 \\ s(r) &= c \cdot r^\gamma\end{aligned}$$

$$\begin{aligned}s(1) &= 1 \\s(2) &= 2^2 = 4 \\s(3) &= 3^2 = 9 \\s(4) &= 4^2 = 16 \\s(5) &= 5^2 = 25 \\&\dots = \dots \\s(20) &= 20^2 = 400\end{aligned}$$

???

We have only 21 levels, but:

$$s(5) = 5^2 = 25$$



Monadic operations

Code sample >

```
% lightening/darkening
xwing_light=xwing_grey+50;
idisp(xwing_light);
xwing_dark=xwing_grey-50;
idisp(xwing_dark);
% select areas by levels
level48 = (xwing_grey>=40) & (xwing_grey<=50) ;
idisp(level48);
level225 = (xwing_grey>=225) &
(xwing_grey<=255) ;
idisp(level225);
% contrast enanch
xwing_contrast=zeros(r,c);
    for i=1:r
        for j=1:c
            xwing_contrast(i,j)=256./(1+1.05.^-(
double(xwing_grey(i,j))-150));    % Sigmoid
        end
    end
idisp(xwing_contrast)
```



Pay attention

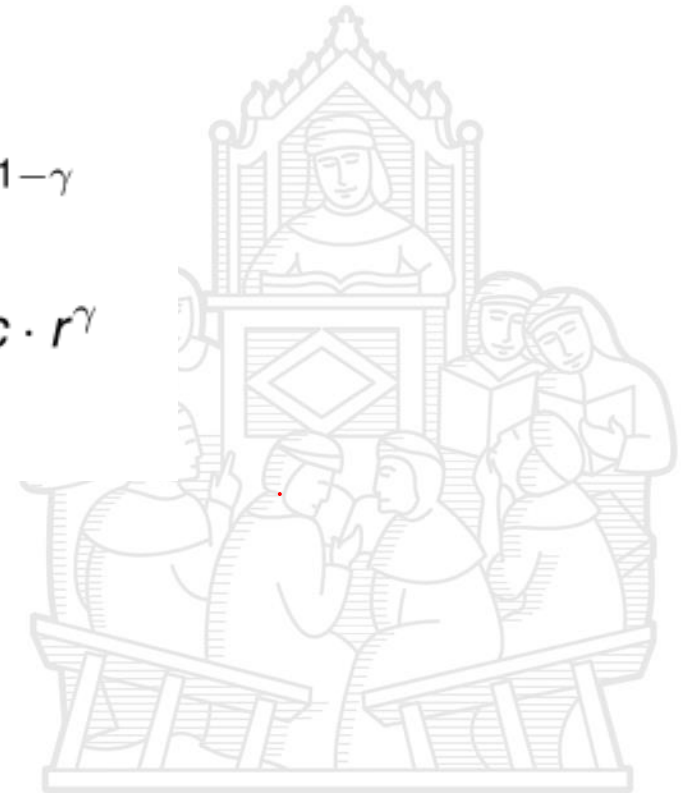
We need to remap the output between $[0, L - 1]$:

$$\frac{s'}{s} = \frac{20}{400}$$

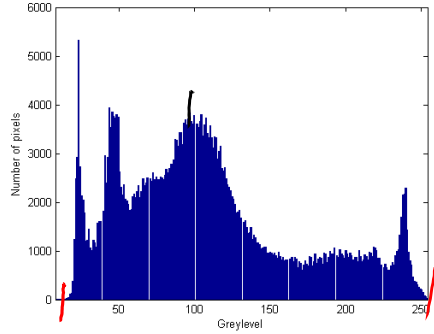
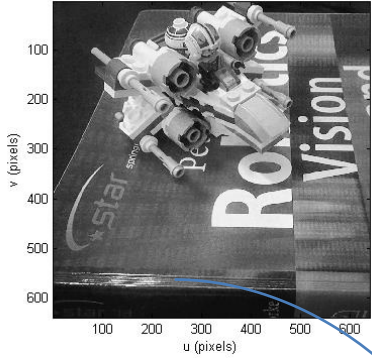
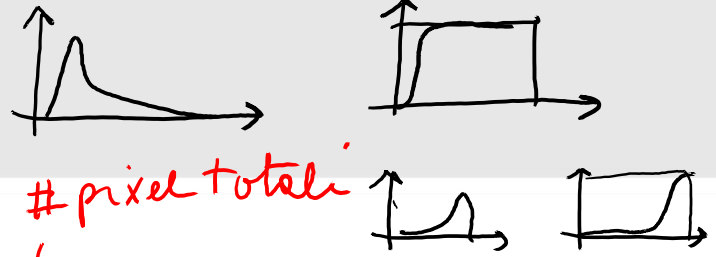
$$\frac{20}{400} = \frac{L - 1}{(L - 1)^\gamma} = (L - 1)^{1-\gamma}$$

$$s' = (L - 1)^{1-\gamma} s = c \cdot s = c \cdot r^\gamma$$

Thus c is related to L and γ .

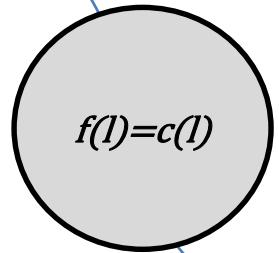
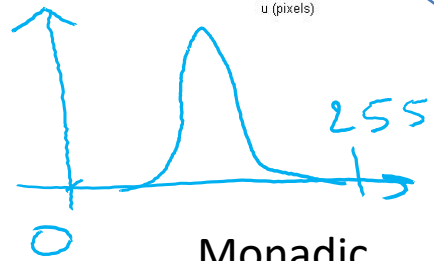


Histogram equalization

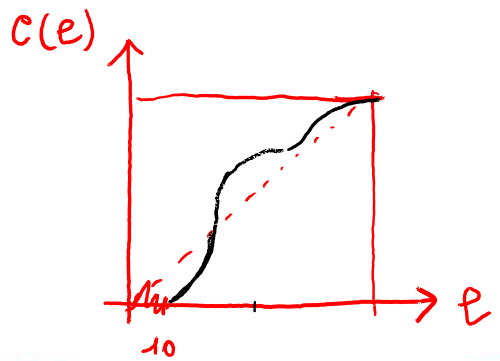


$$c(l) = \frac{1}{N} \sum_{i=0}^l h(i) = c(l-1) + \frac{h(l)}{N}$$

$c(l)$	Cumulative distribution
$h(l)$	histogram
l	Grey level

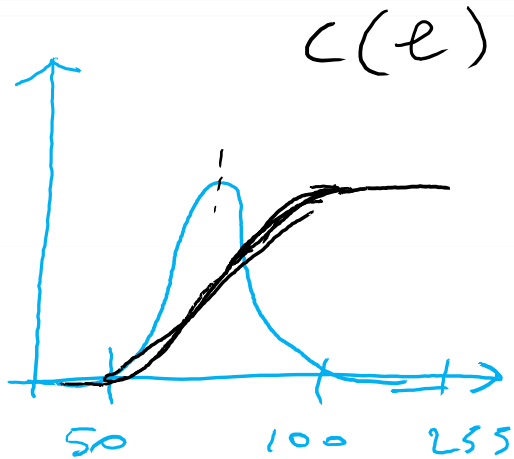


Monadic operation



$l=0$
 $c(0) = 2$
 $l=10$
 $h(10) = 2$ $c(e) = \frac{2}{N}$
 $c(250) = c(251) \dots$
 $O[u, v] = c(I[u, v]), \forall (u, v) \in I$
 $l=11$ $c(l-1)$ 4
 $c(11) = \frac{1}{N} (h(10) + h(11)) = 6$

n : total pixel



$$e(e) = \frac{1}{N} \sum_{i=0}^e h(i)$$

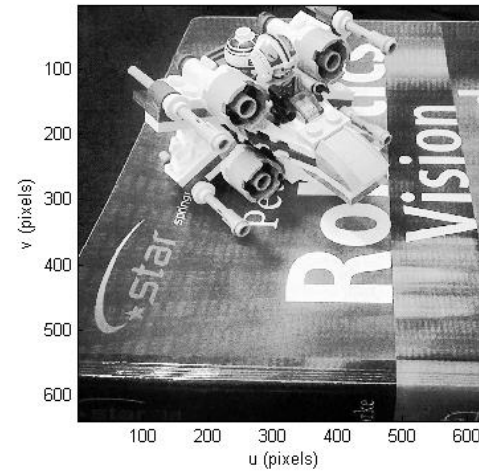
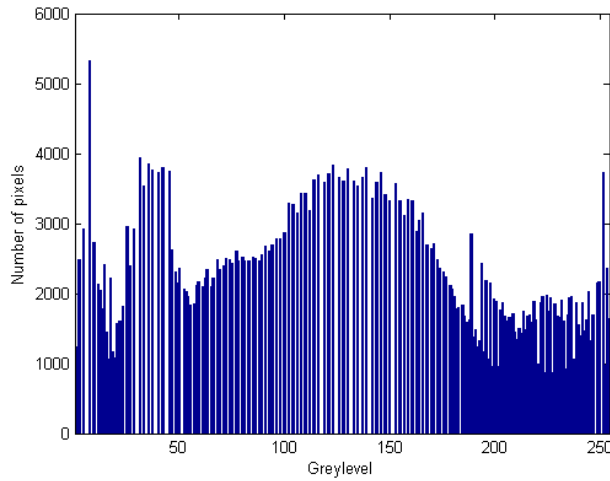
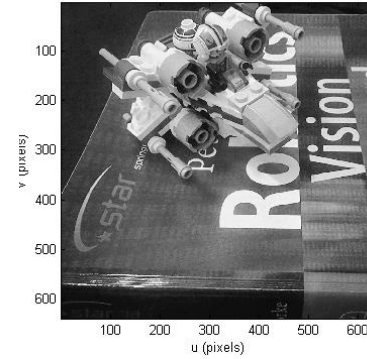
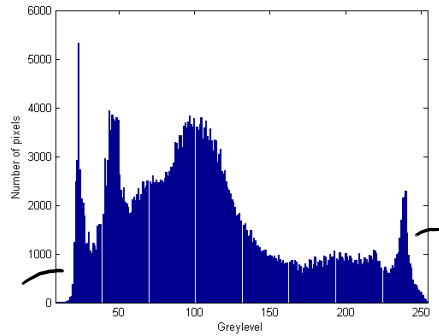
$$c(0) = \frac{1}{N} \quad h(0) = 0$$

$$c(50) = \frac{1}{N} \quad h(50) = \frac{1}{N}$$

$$\begin{aligned} c(51) &= \frac{1}{N} (h(50) + h(51)) \quad \vdots \\ &= c(e-1) + \frac{h(e)}{N} \end{aligned}$$



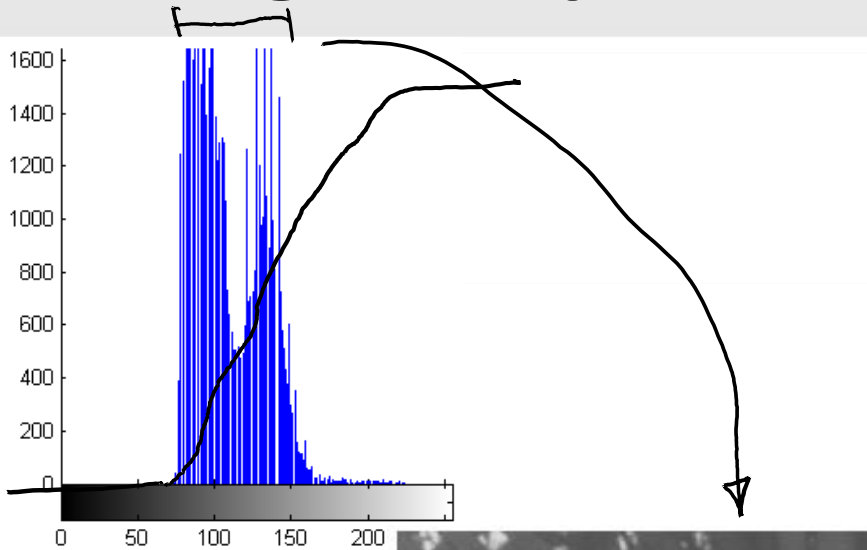
Histogram equalization



After equalization



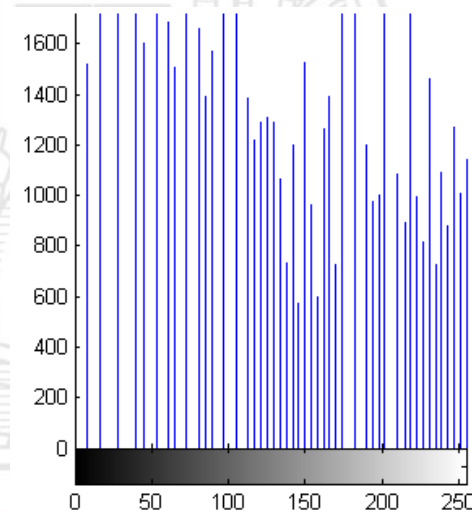
Histogram equalization



before

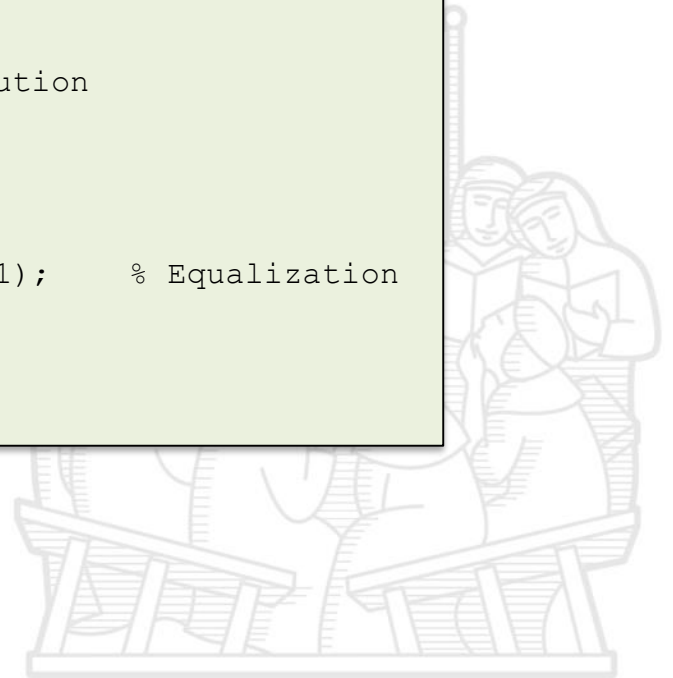


after



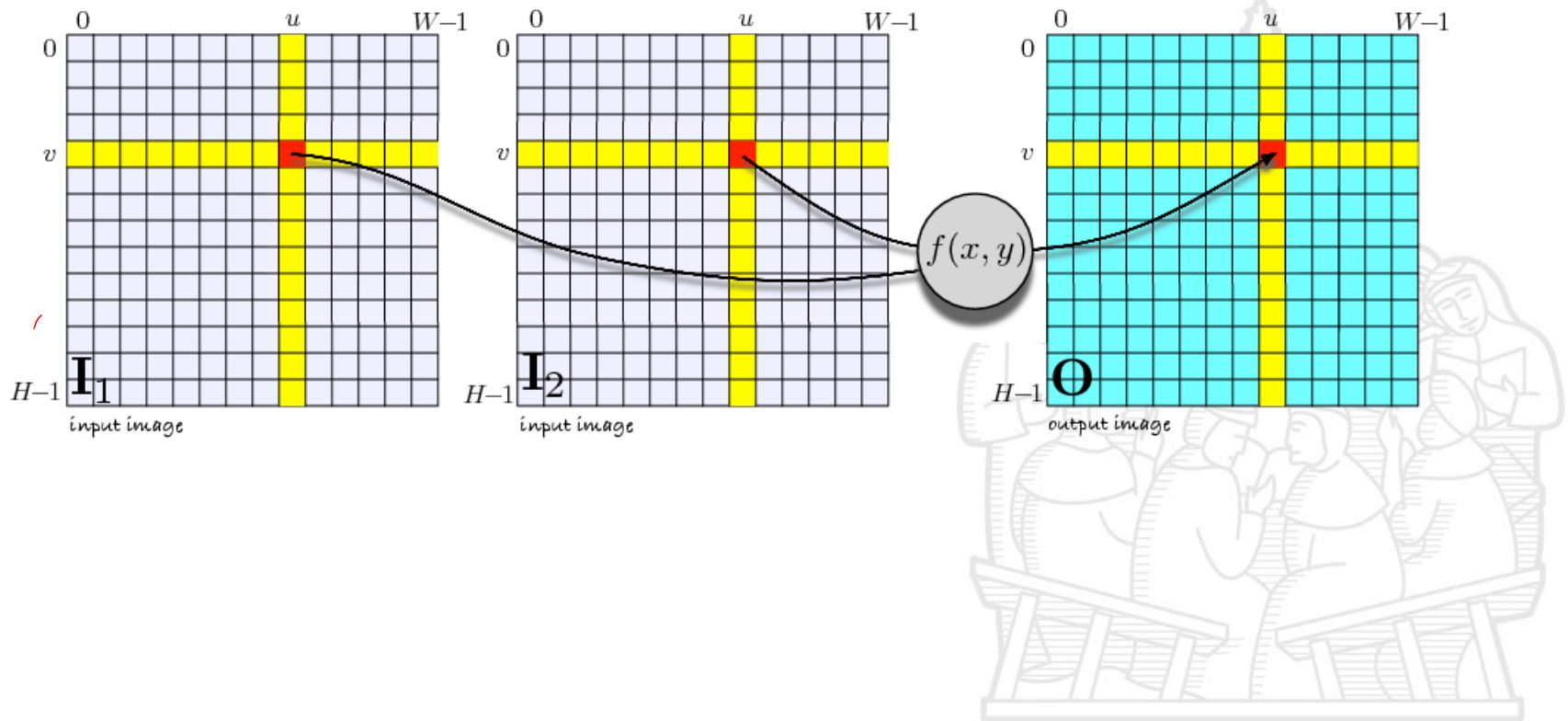
Code sample >

```
%hist equalization
[n,v]=ihist(xwing_grey);
plot(v,n)
cd=zeros(length(v),1);
cd(1)=v(1)/(r*c);
for l=2:length(v)
    cd(l)=cd(l-1)+1/(r*c)*n(l); % cumulative distribution
end
xwing_equalized=zeros(r,c);
for i=1:r
    for j=1:c
        xwing_equalized(i,j)=255*cd(xwing_grey(i,j)+1); % Equalization
    end
end
idisp(xwing_equalized)
```

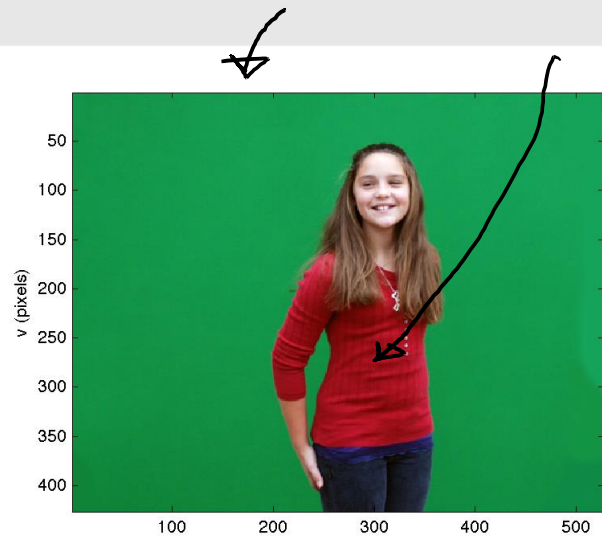


Diadic operations

$$\underline{\underline{O}}[u, v] = f(\underline{\underline{I}}_1[u, v], \underline{\underline{I}}_2[u, v]), \quad \forall (u, v) \in I_1$$



Green screen



$I_1[u,v]$



$I_2[u,v]$

$$I_1[u,v] \geq 250$$

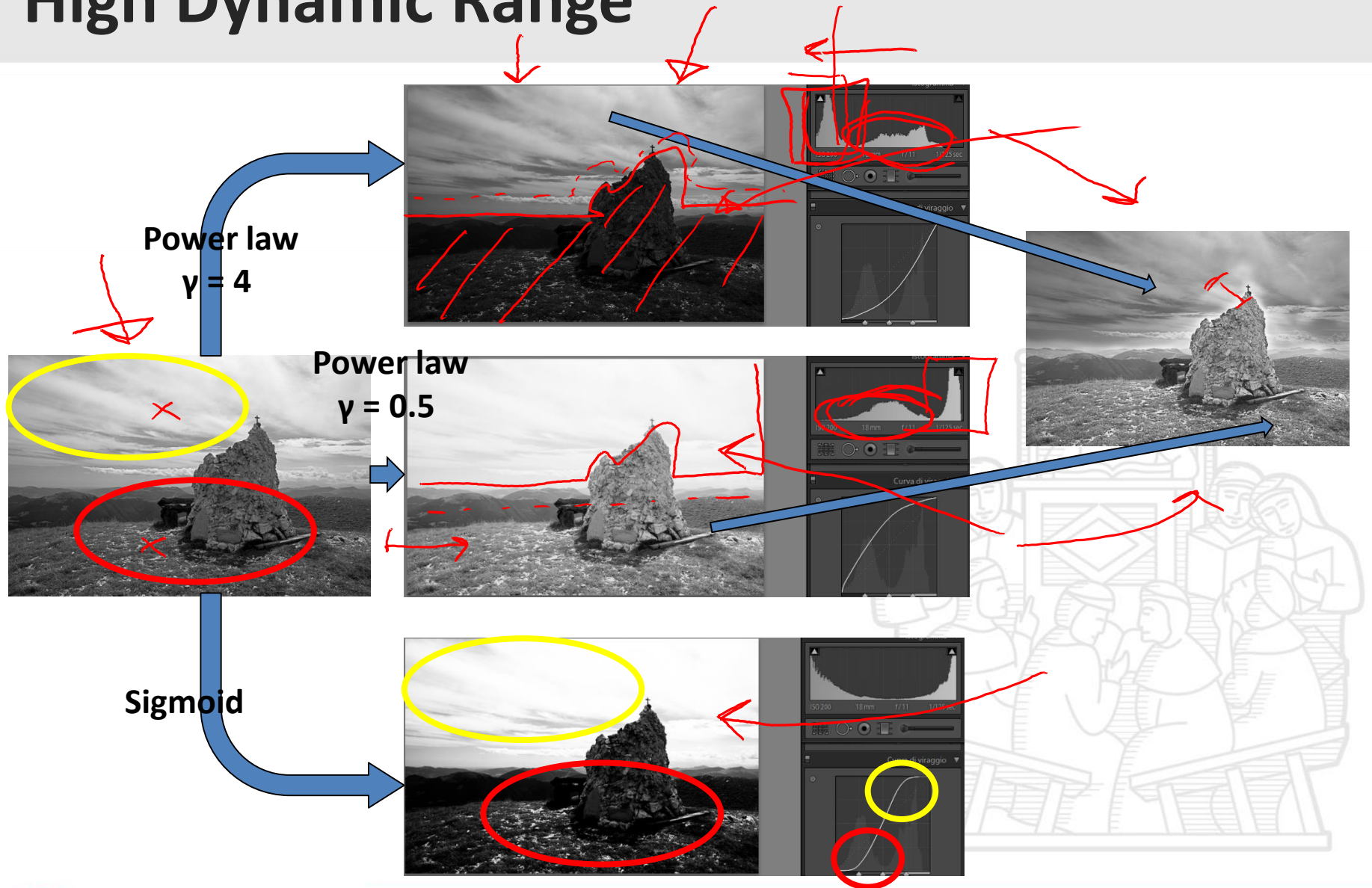
→ If $I_1[u,v]$ is Green
 $O[u,v] = I_2[u,v]$
Else
 $O[u,v] = I_1[u,v]$



$O[u,v]$



High Dynamic Range



Background subtraction

Another important diadic operation is the background subtraction to find novel elements (foreground) of a scene.

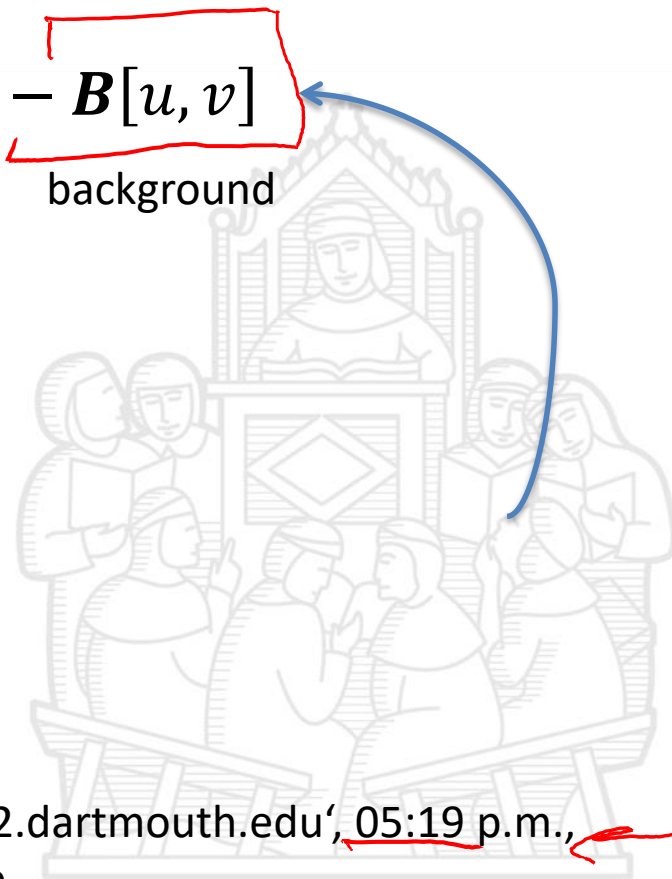
$$\mathbf{O}[u, v] = \mathbf{I}_1[u, v] - \mathbf{I}_2[u, v] = \mathbf{I}_1[u, v] - \mathbf{B}[u, v]$$

background

How we estimate
the background
 $\mathbf{B}[u, v]$?



We can take a
shoot when we
know that only
background is
visible

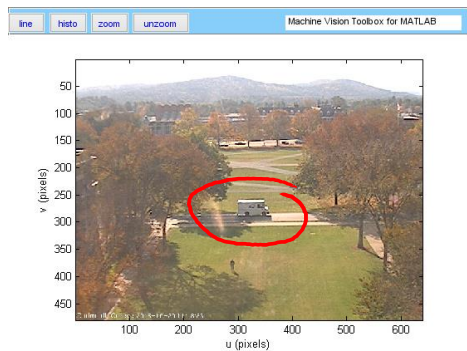


'<http://wc2.dartmouth.edu>', 05:19 p.m.,
Rome time

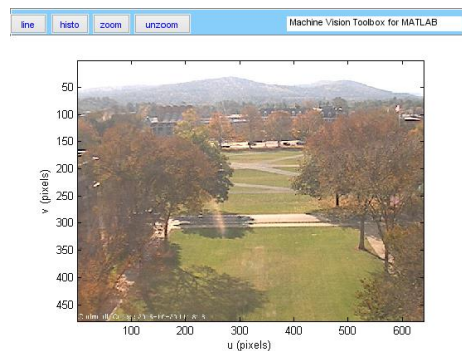
Background subtraction

'http://wc2.dartmouth.edu', 05:19 p.m., Rome time

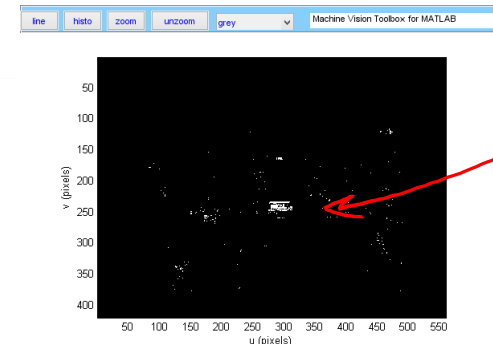
$$I_1[u, v] - B[u, v] = O[u, v]$$



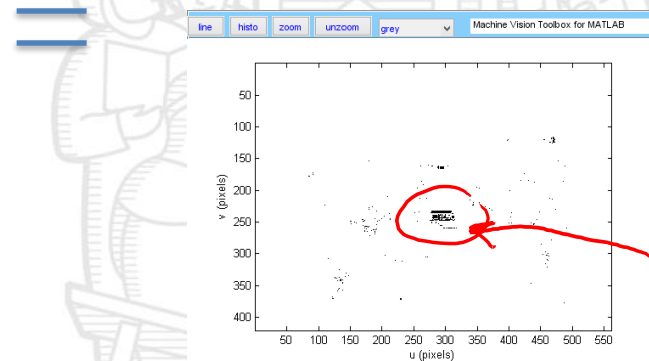
I_1



B_0



foreground

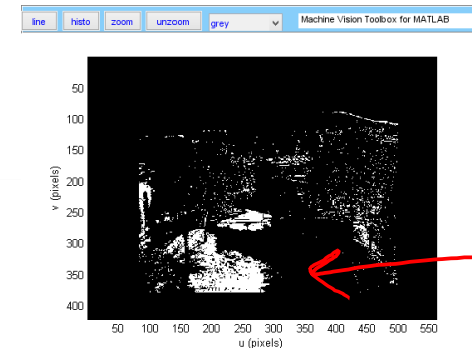
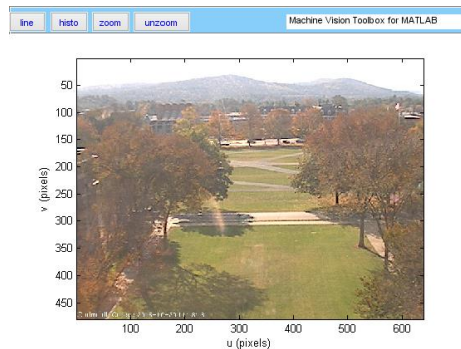
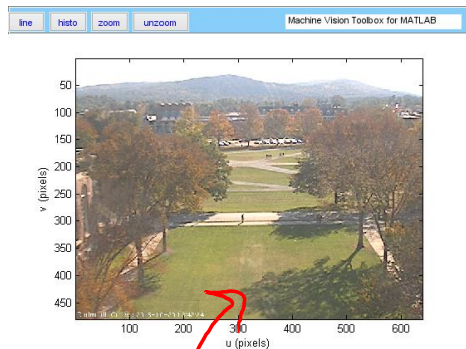


background

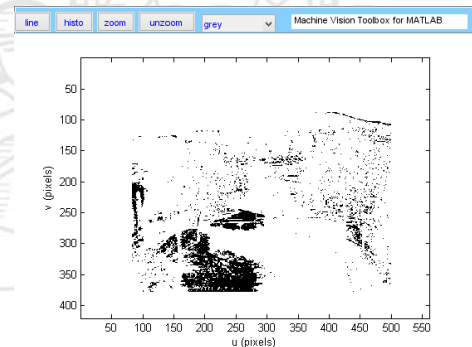
Background subtraction

'http://wc2.dartmouth.edu', 07:48 p.m., Rome time

$$I_1[u, v] - B[u, v] = O[u, v]$$



foreground



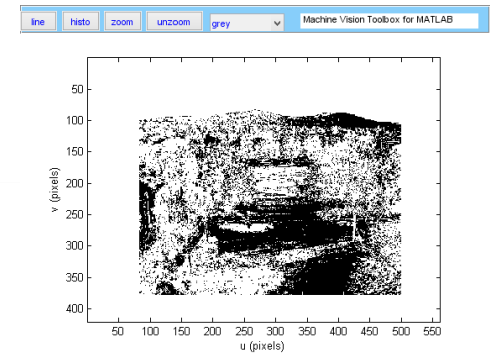
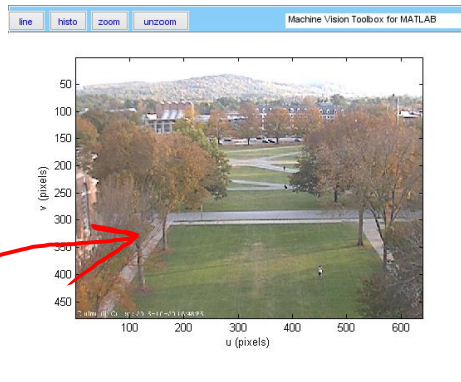
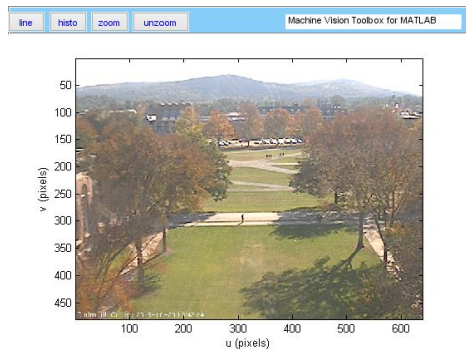
background

What went wrong?

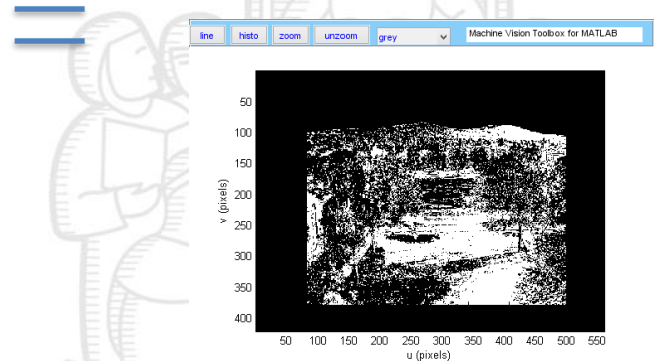
Background subtraction

'http://wc2.dartmouth.edu', 10:55 p.m., Rome time

$$I_1[u, v] - B[u, v] = O[u, v]$$



foreground



background

What went wrong?

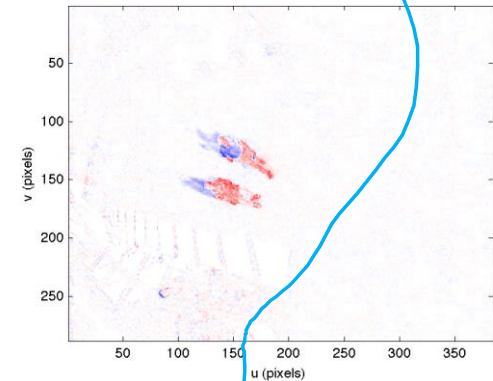
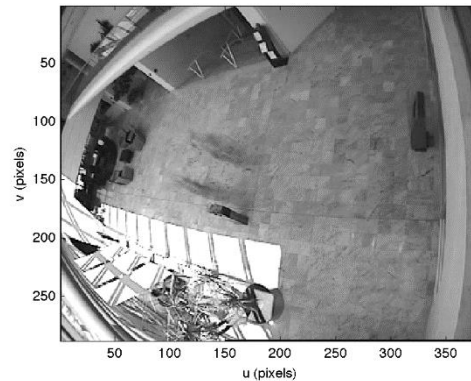
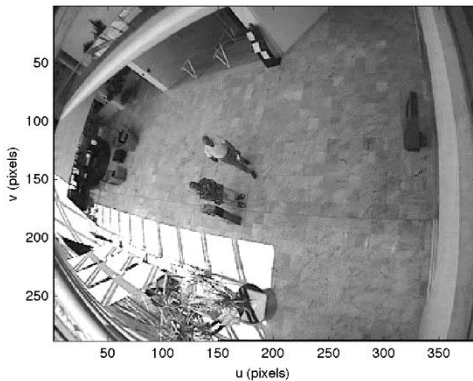
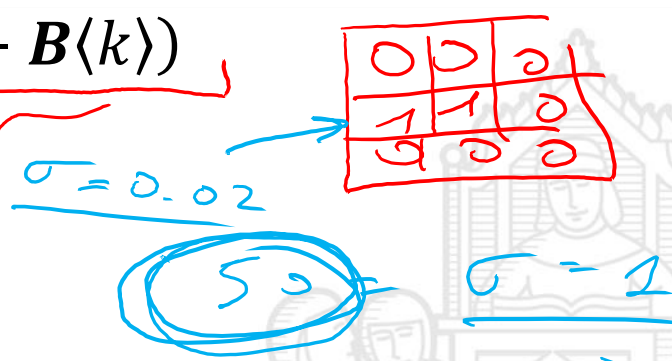
Background estimation

We require a progressive adaptation to small, persistent changes in the background.

Rather than take a static image as background, we estimated it as follow:

$$B\langle k + 1 \rangle = B\langle k \rangle + c(I\langle k \rangle - B\langle k \rangle)$$

$$c(x) = \begin{cases} \sigma, & x > \sigma \\ x, & -\sigma \leq x \leq \sigma \\ -\sigma, & x < -\sigma \end{cases}$$



Background subtraction

Code sample >

```
% background estimation
sigma=0.01;
vid = videoinput('winvideo', 1);
bg=getsnapshot(vid);
bg_small=idouble(imono(bg));
while 1
    img=getsnapshot(vid);
    img_small=idouble(imono(img));
    if isempty(img), break; end
    d=img_small-bg_small;
    d=max(min(d,sigma), -sigma);
    bg_small=bg_small+d;
    idisp(bg_small); drawnow
end
```

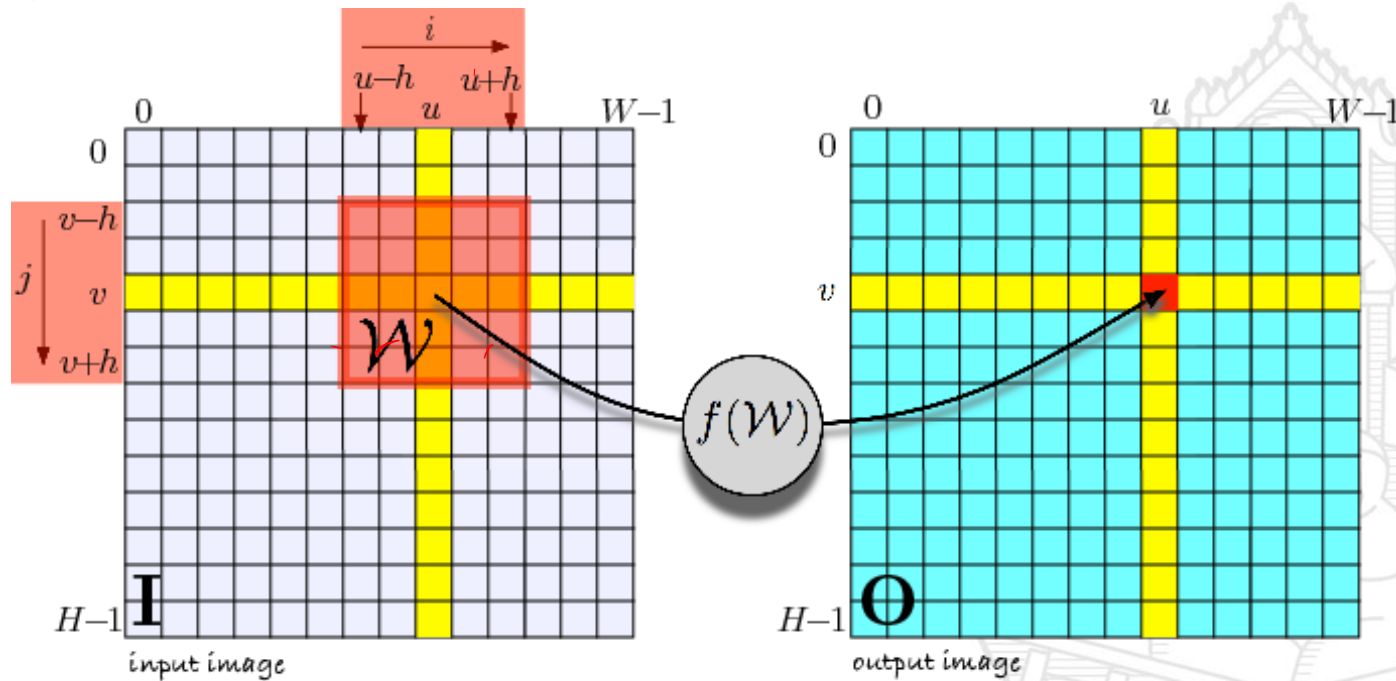


Spatial operation (local operators)

$$O[u, v] = f(I[u + i, v + j]),$$

↑ ↓ ↓ ↓
↖ ↗ ↗ ↘

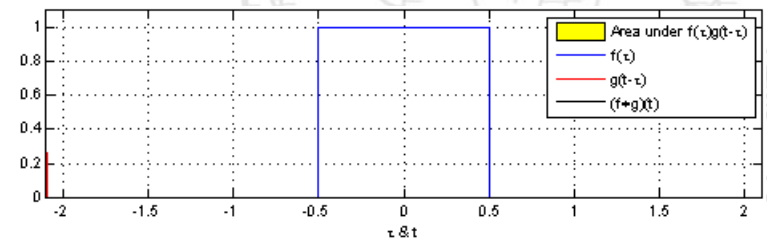
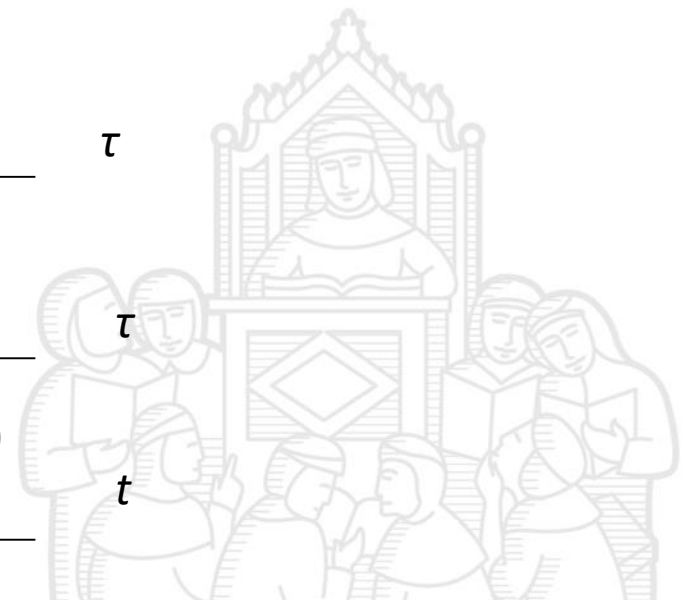
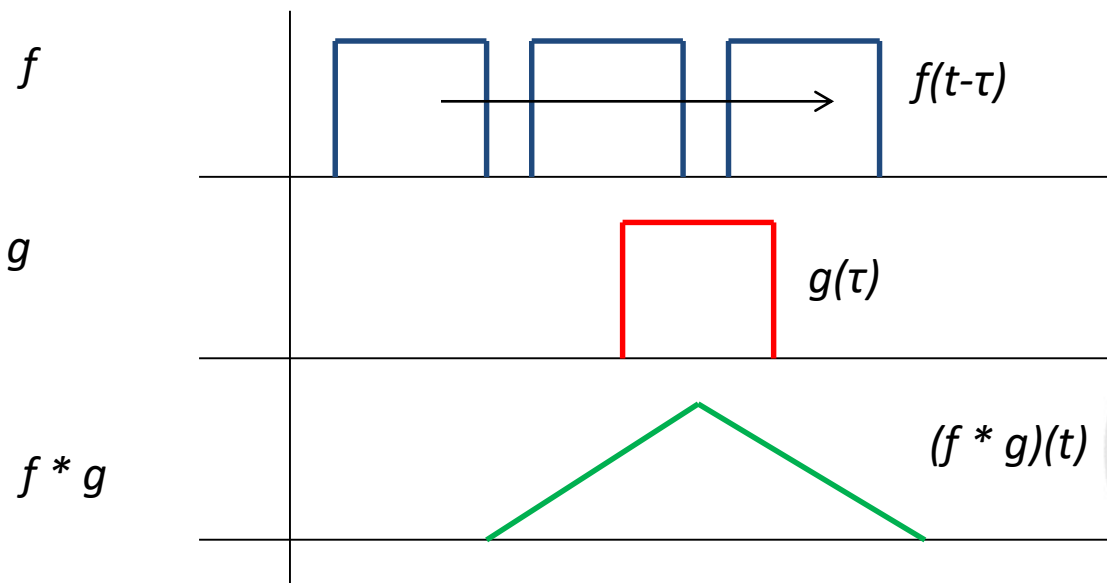
$$\forall (i, j) \in \underline{\underline{W}}, \forall (u, v) \in I$$



1D Convolution

One important local operator is the convolution:

$$(f * g)(t) := \int_{-\infty}^{\infty} f(t - \tau)g(\tau)d\tau$$



wikipedia

2D Convolution

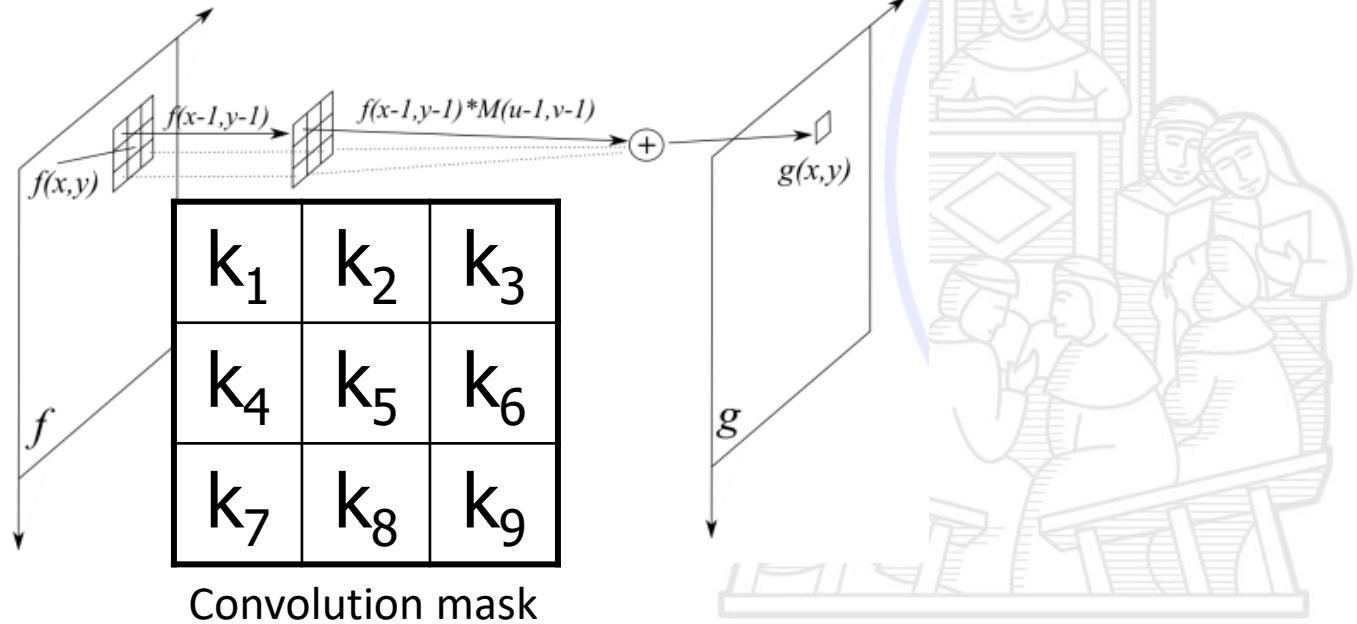
$$(K_1 \otimes K_2) \otimes I$$

$8 \times 6 \times 2$

$$O[u, v] = \sum_{i, j \in K} K[i, j] I[u - i, v - j], \quad \forall (u, v) \in I$$

$$O = K \otimes I$$

$K \times I$



2D Convolution

kernel

·1+	·1+	·1+
·1+	·1+	·1+
·1+	·1+	·1

Input image

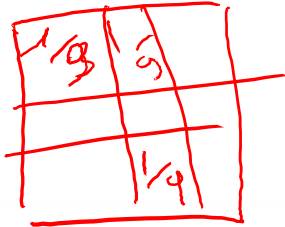
0	1	2	0	12	5	0	1
5	2	6	0	0	1	1	1
5	0	0	4	5	6	1	0
12	25	0	24	56	8	2	3
1	2	6	0	0	1	5	2
1	2	0	2	1	2	1	0
12	0	12	25	3	5	0	1
1	1	1	35	57	5	3	1

Output image

	21						



Convolution



Input image

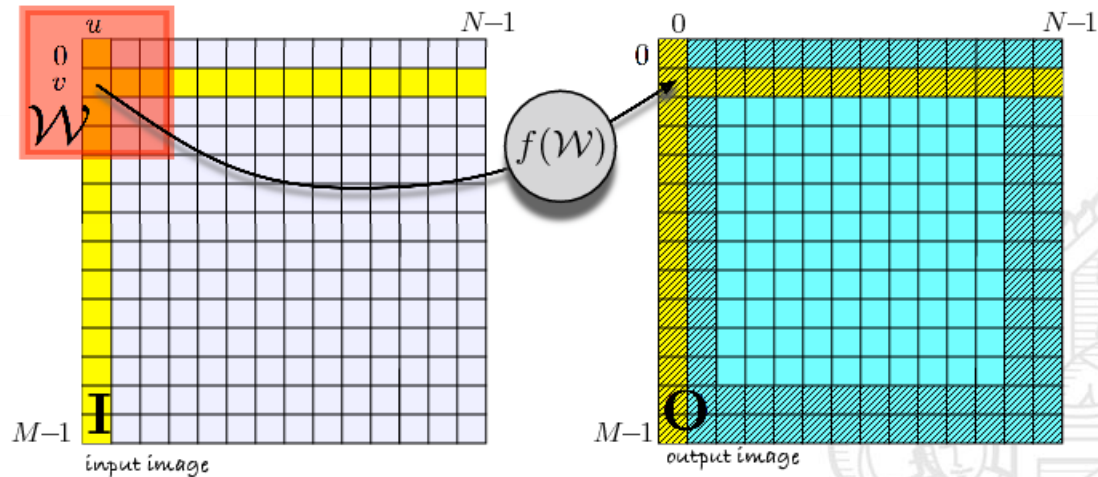
0·1+	1·1+	2·1+	0	12	5	0	1
5·1+	2·1+	6·1+	0	0	1	1	1
5·1+	0·1+	0·1	4	5	6	1	0
12	25	0	24	56	8	2	3
1	2	6	0	0	1	5	2
1	2	0	2	1	2	1	0
12	0	12	25	3	5	0	1
1	1	1	35	57	5	3	1

Output image

	21	15					

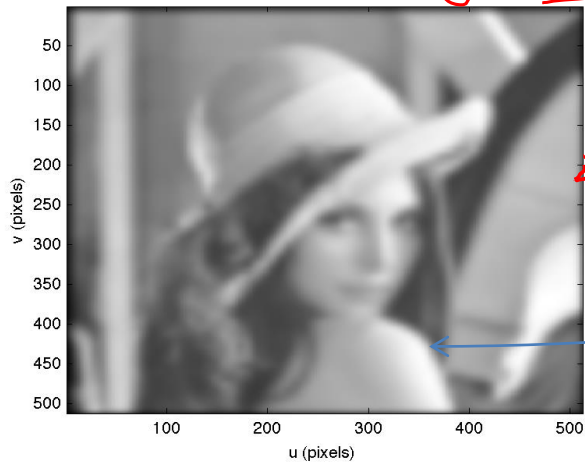
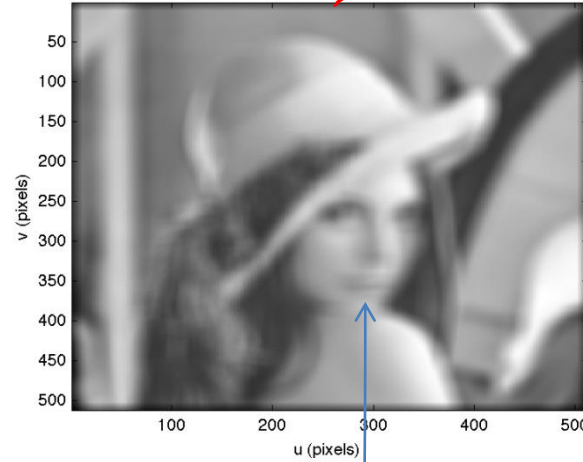


Boundary effect



- Duplicate
- All black
- Reduce size
- ...

Smoothing



gauss

mean

$$K = \text{ones}(21, 21) / 21^2$$

$$O = K \otimes I$$

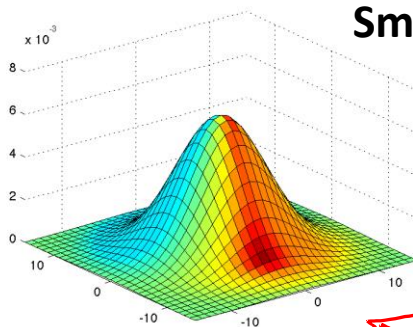
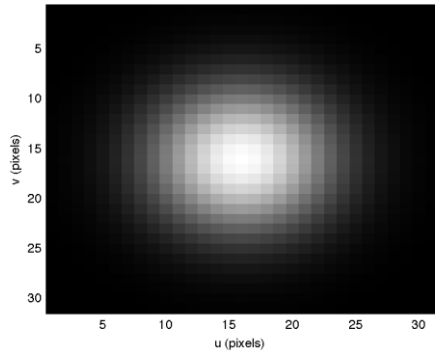
$21 \cdot 21 / 21^2$

$\frac{M \times M}{M^2}$

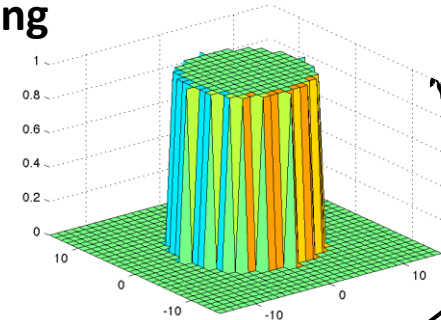
$$G[u, v] = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



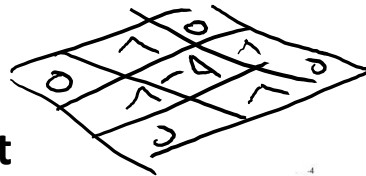
Kernel examples



Smoothing

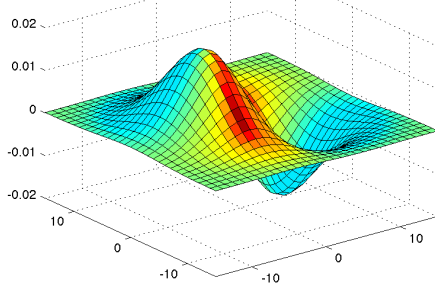


Top hat



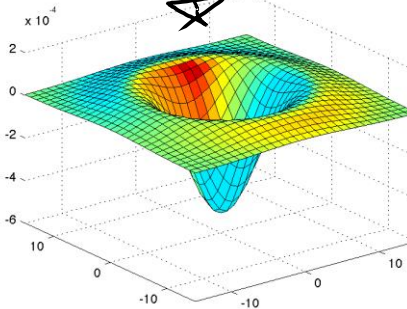
Gaussian

Gradient

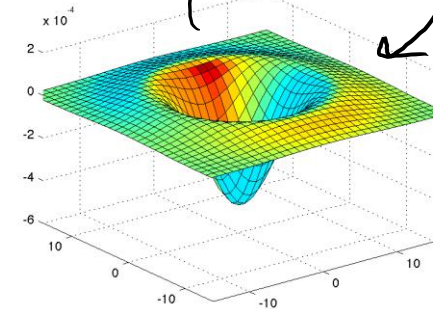


Derivative of Gaussian
(DoG)

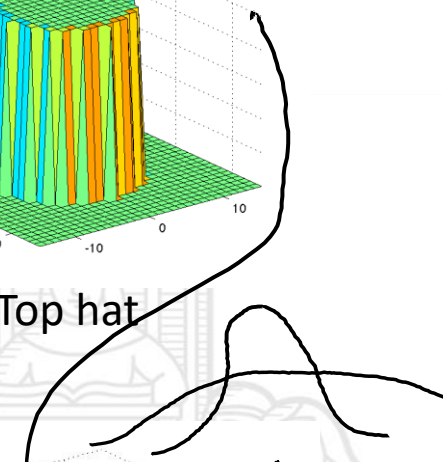
Edge detection



Laplacian of Gaussian
(LoG)



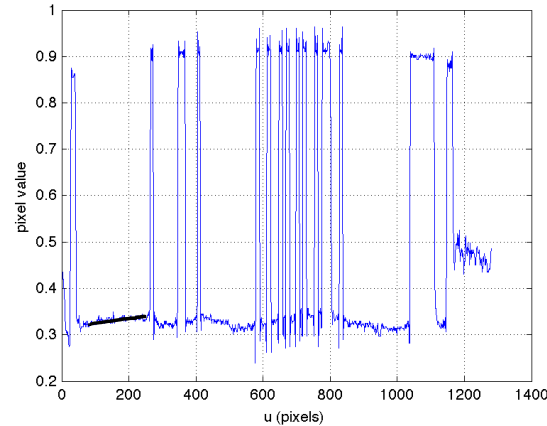
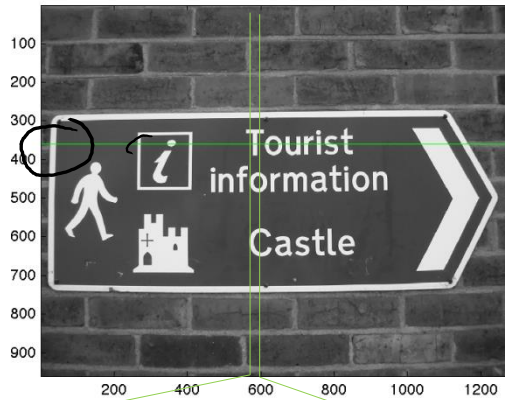
Difference of Gaussian
(DiffG)



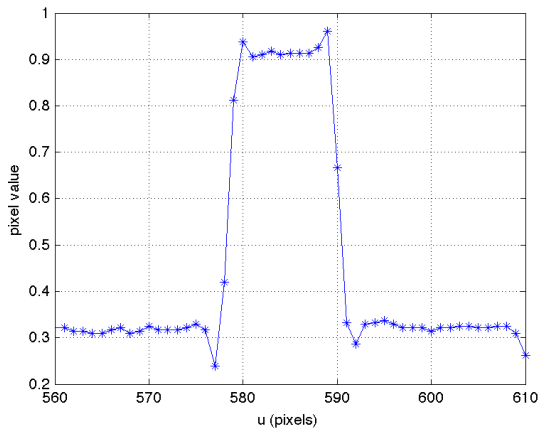
$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} \rightarrow 2$$

Edge detection



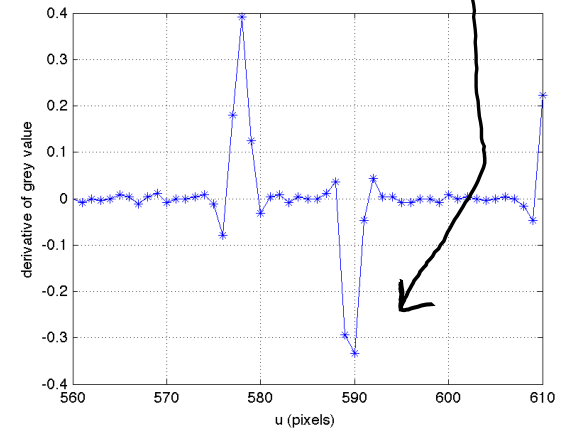
Horizontal profile of the image at $v=360$



$$p'[u] = p[u] - p[u - 1]$$

$$p'[u] = \frac{1}{2}(p[u + 1] - p[u - 1])$$

$$K = \begin{bmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

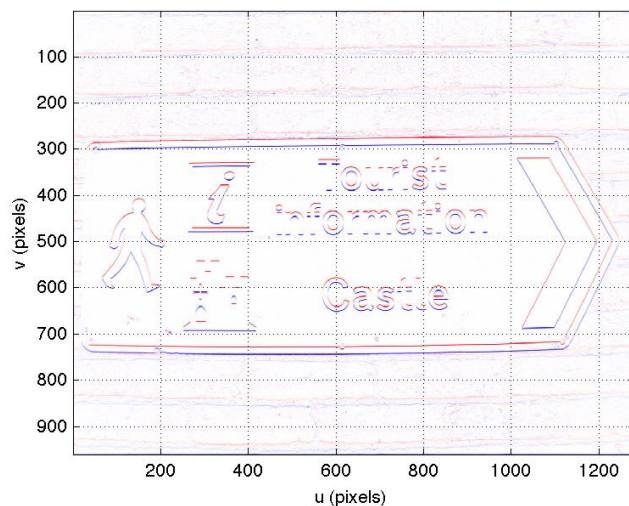


Gradient computation

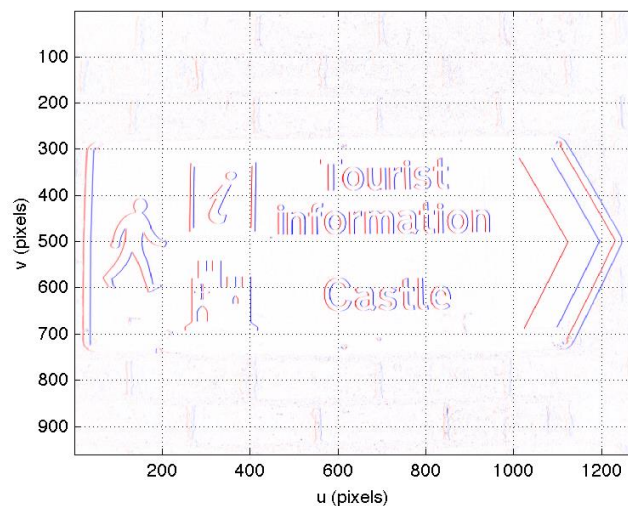
Common convolution kernel: Sobel, Prewitt, Roberts, ...

Sobel $D_v = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

$$D_u = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$



$$I_v = D_v \otimes I$$

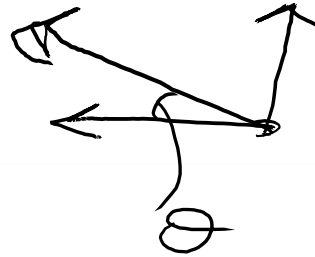


$$I_u = D_u \otimes I$$

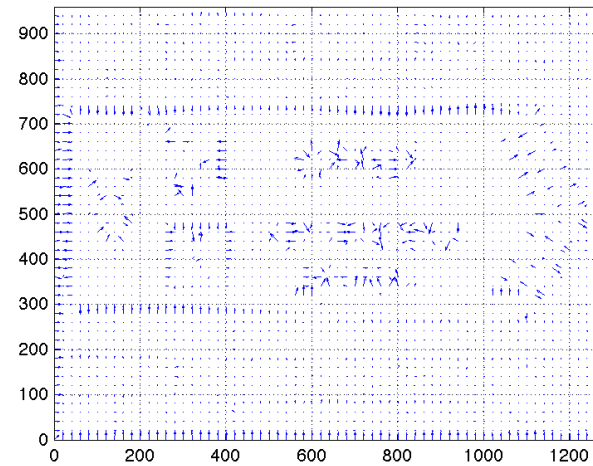
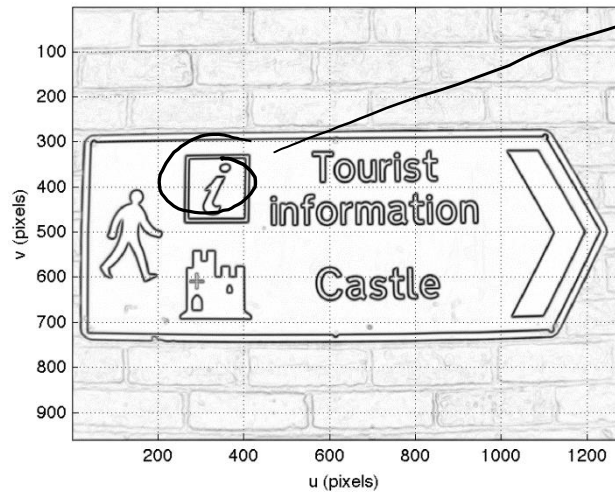


Direction and magnitude

$$m = \sqrt{I_v^2 + I_u^2}$$



$$\theta = \text{atan}(I_v, I_u)$$



Noise amplification

Derivative amplifies high-frequency noise. So, firstly we can smooth the image, after that we can take the derivative:

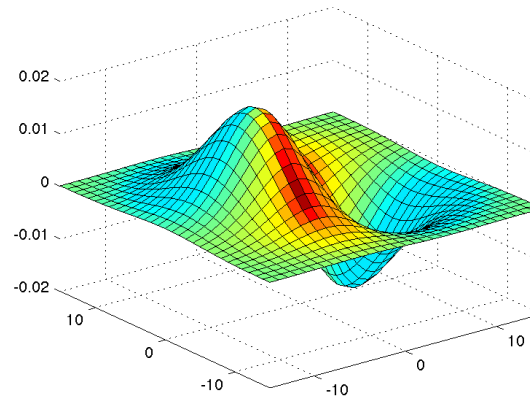
$$I_u = D_u \otimes (G \otimes I)$$

Associative property:

$$I_u = \underbrace{(D_u \otimes G)} \otimes I$$

Derivative of Gaussian
(DoG)

$$G_u = -\frac{u}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



Derivative of Gaussian
(DoG)

<<DoG acts as a bandpass filter!>>



Canny edge detection

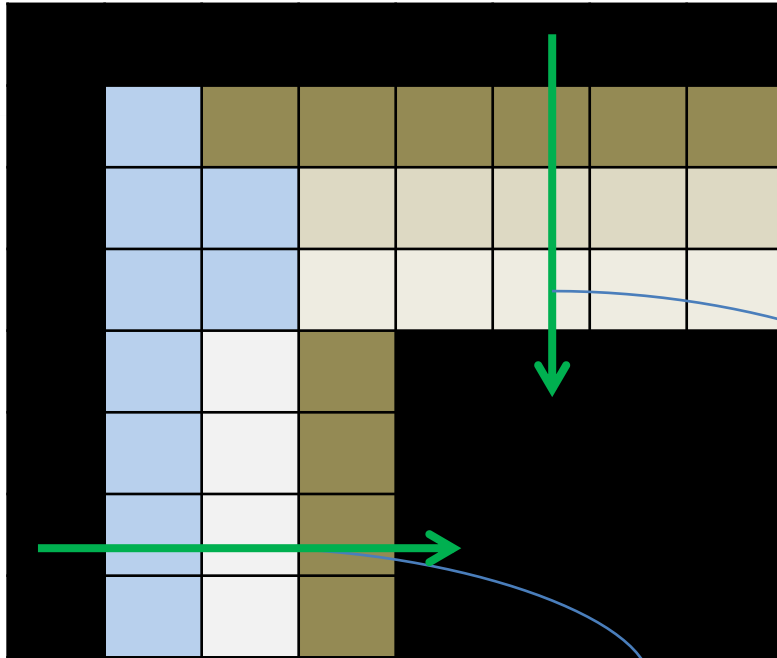
The algorithm is based on a few steps:

1. Gaussian filtering
2. Gradient intensity and direction
3. non-maxima suppression (edge thinning)
4. hysteresis threshold

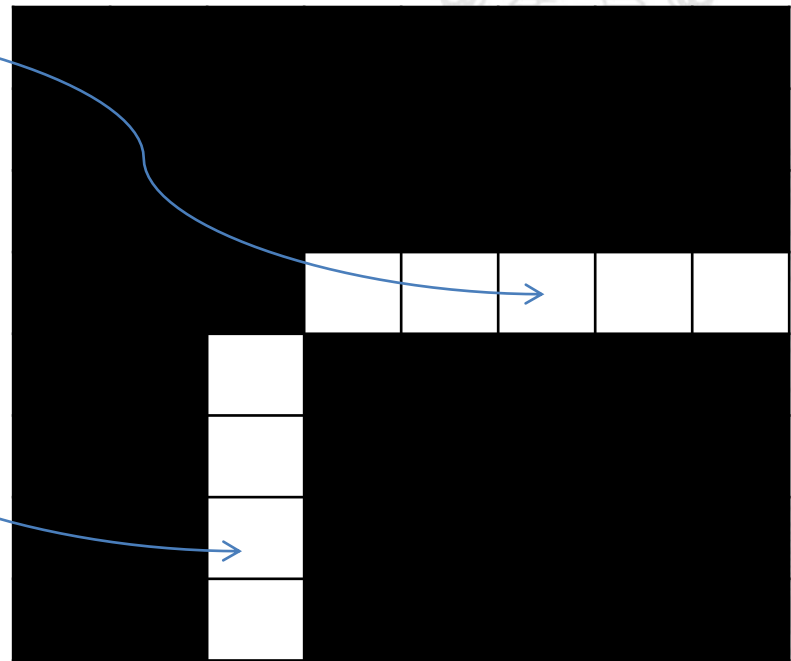


Canny edge detection

3. Non local maxima suppression



Evaluation along gradient direction

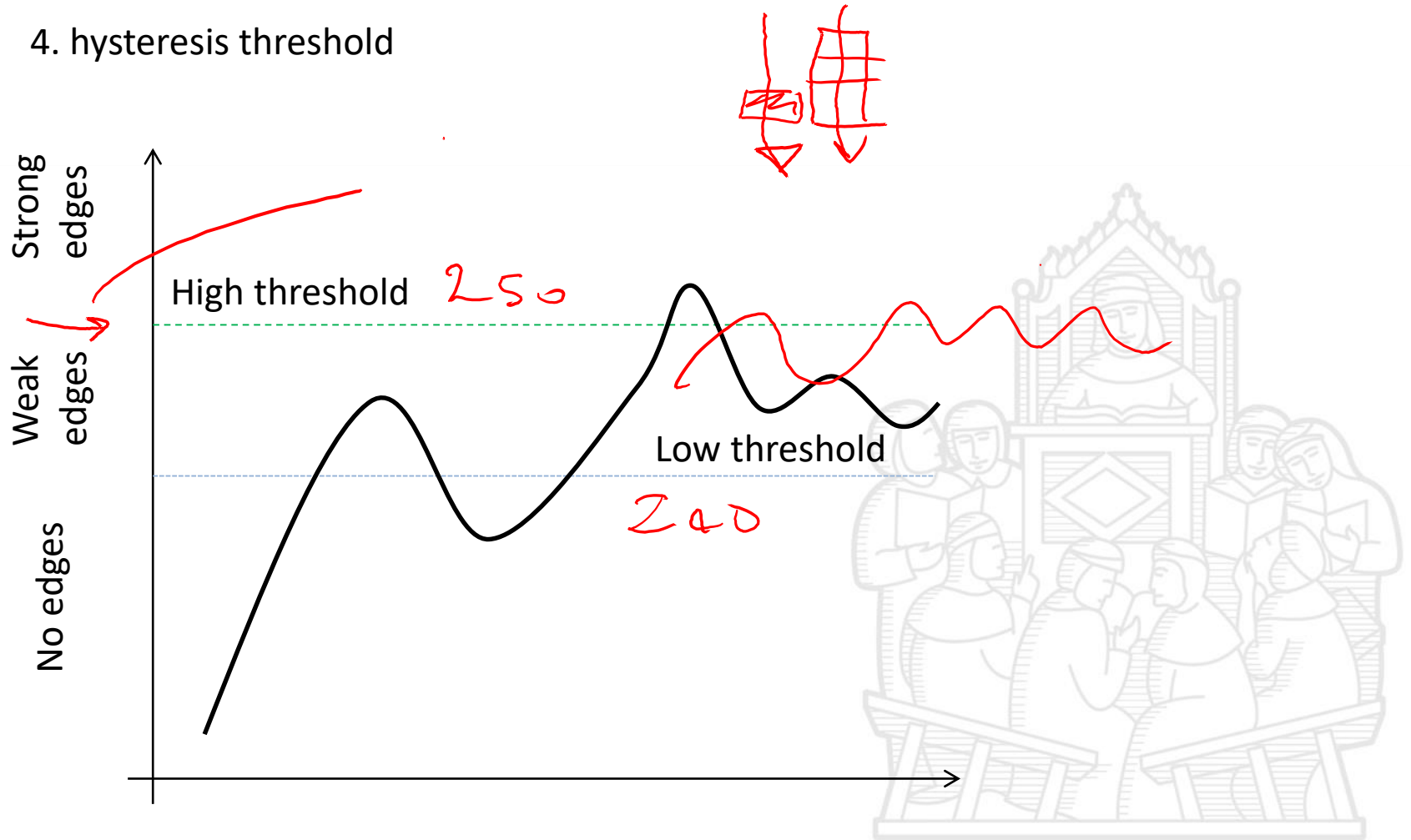


Maxima detection



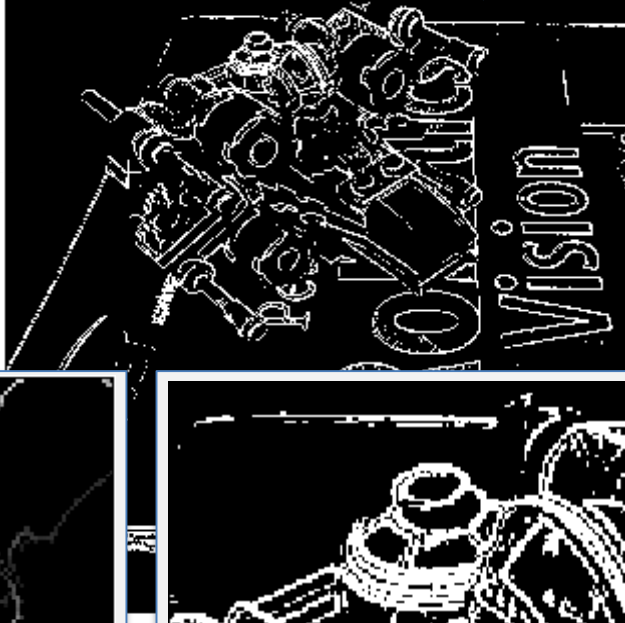
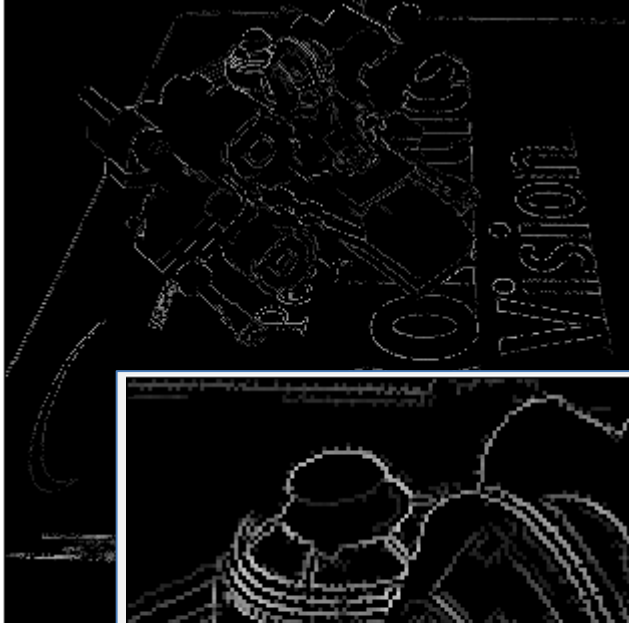
Canny edge detection

4. hysteresis threshold

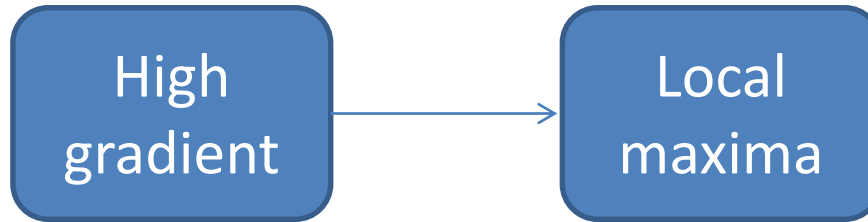


Thresholding

canny



Edge detection

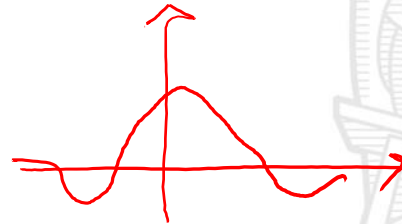


Alternative approach is to use second derivative and to find where there is a zero

Laplacian operator

$$\nabla I^2 = \frac{\partial^2 I}{\partial u^2} + \frac{\partial^2 I}{\partial v^2} = I_{uu} + I_{vv} = L \otimes I$$

$$L = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$



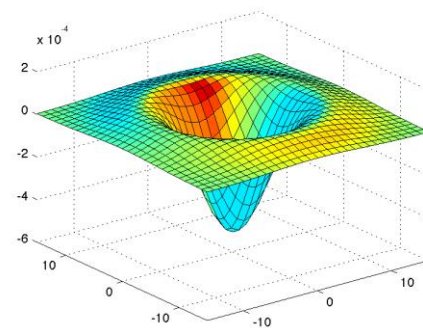
Noise sensitivity

Again, derivative amplifies high-frequency noise. So firstly we can smooth the image, after that we take the derivative:

$$L \otimes (G \otimes I) = \underbrace{(L \otimes G)}_{\text{Laplacian of Gaussian (LoG)}} \otimes I$$

Laplacian of Gaussian
(LoG)

$$\text{LoG}(u, v) = \frac{1}{\pi\sigma^4} \left(\frac{u^2 + v^2}{2\sigma^2} - 1 \right) e^{-\frac{u^2 + v^2}{2\sigma^2}}$$

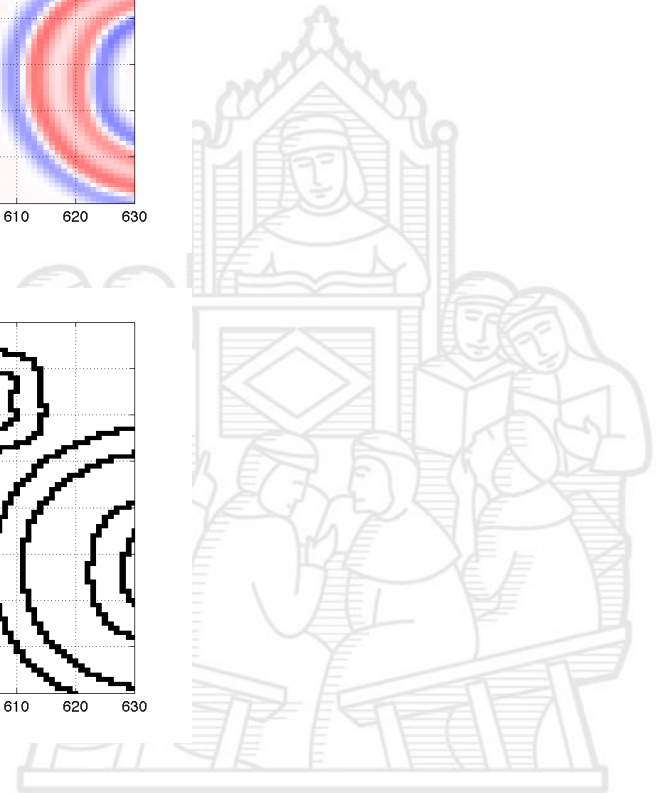
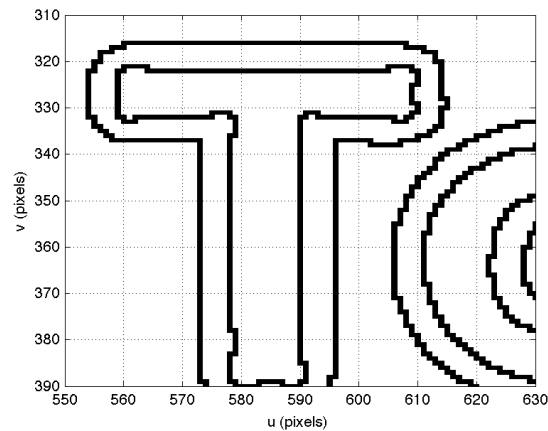
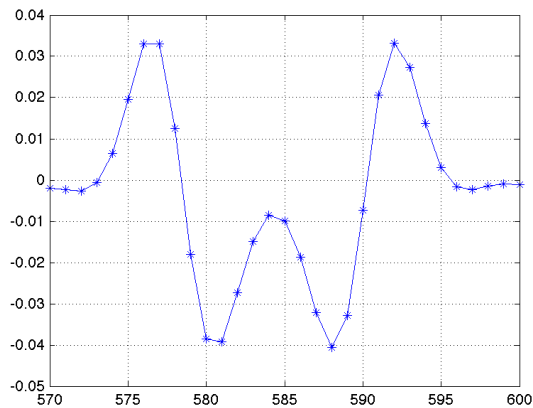
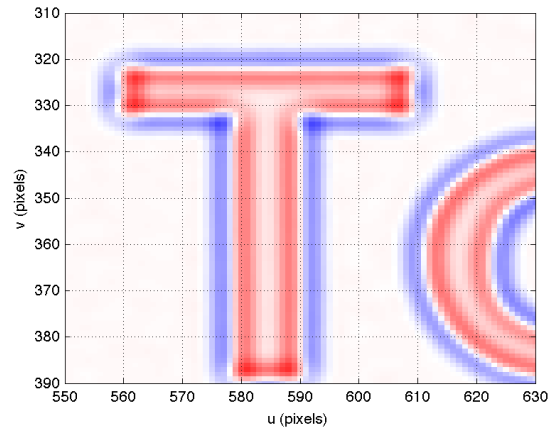
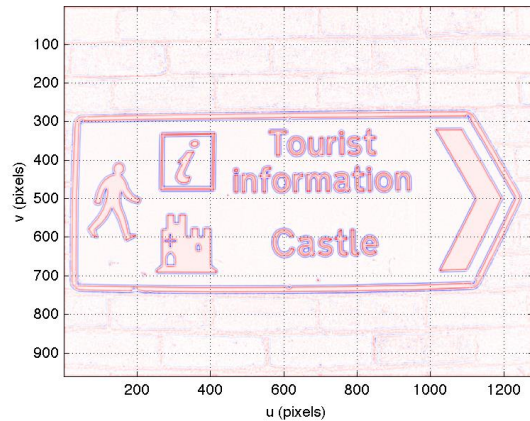


Laplacian of Gaussian
(LoG)

Marr-Hildreth operator or the Mexican hat kernel



Edge detection



Gradient and Laplacian

Example

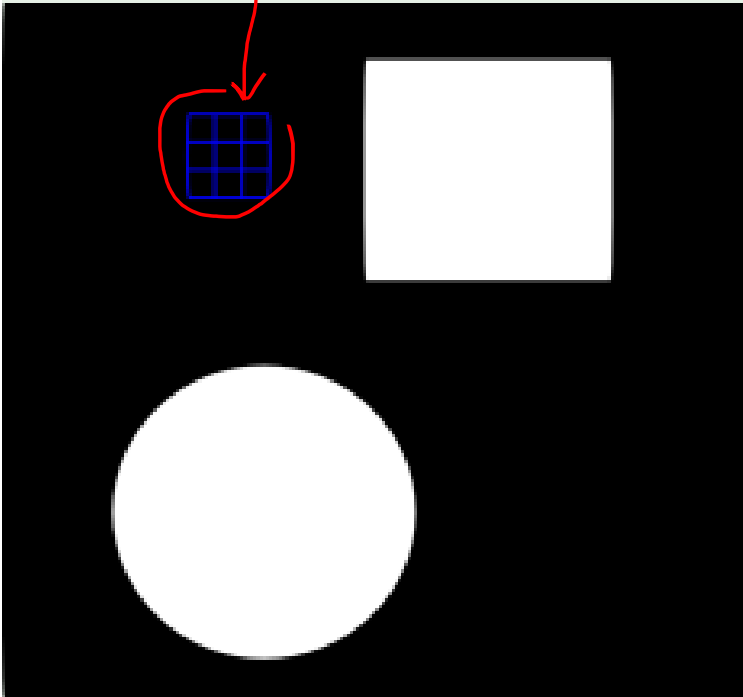


Image window:

$$f(x, y) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\nabla^2 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

Products are:

$$\nabla^2 \otimes f(x, y) = 0$$

$$G_x \otimes f(x, y) = 0$$

Gradient and Laplacian

Example

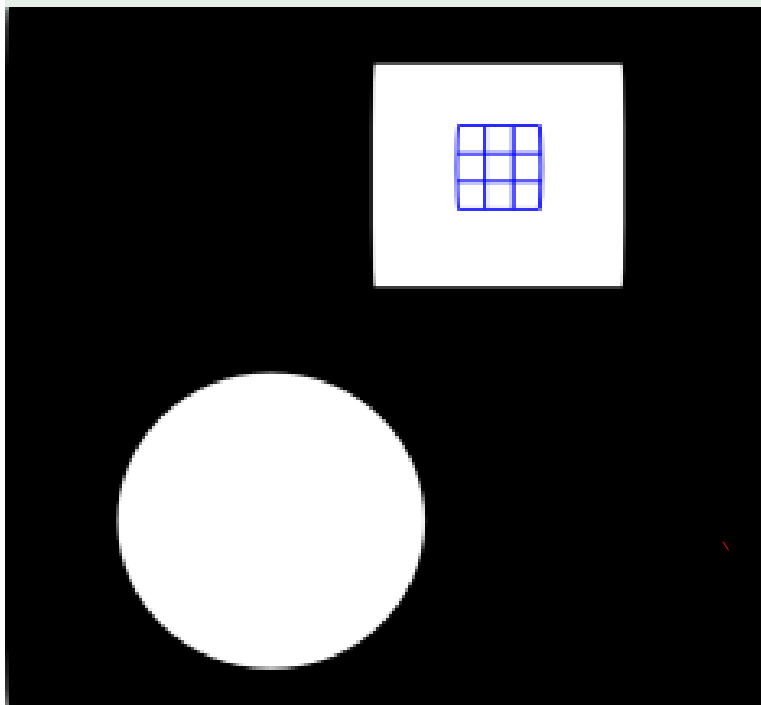


Image window:

$$f(x, y) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\nabla^2 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

Products are:

$$\nabla^2 \otimes f(x, y) = 0$$

$$G_x \otimes f(x, y) = 0$$

Gradient and Laplacian

Example

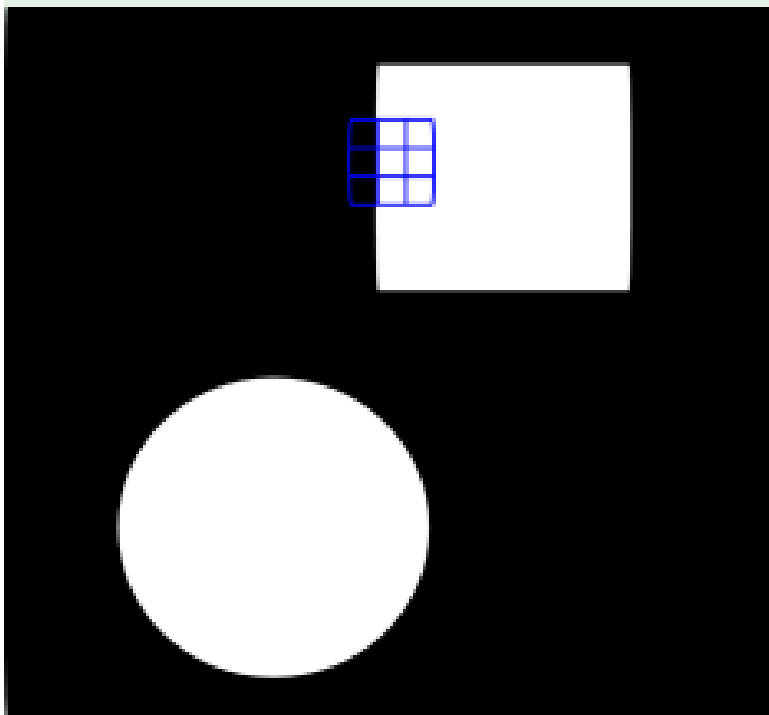


Image window:

$$f(x, y) = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\nabla^2 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

Products are:

$$\begin{aligned} \nabla^2 \otimes f(x, y) &= 1 \\ G_x \otimes f(x, y) &= 4 \end{aligned}$$

Gradient and Laplacian

Example

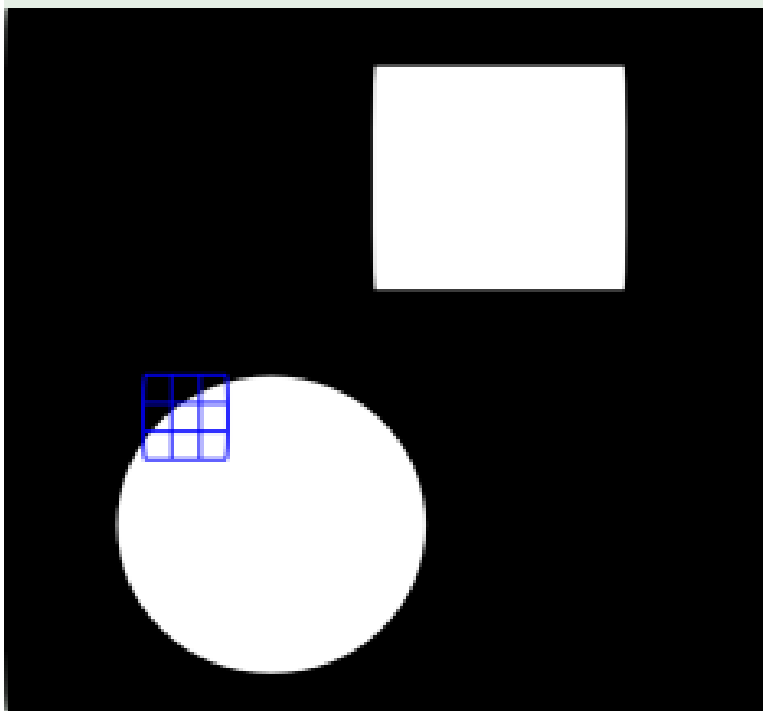


Image window:

$$f(x, y) = \begin{array}{|c|c|c|} \hline 0 & 0 & 1 \\ \hline 0 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

$$\nabla^2 = \begin{array}{|c|c|c|} \hline 0 & -1 & 0 \\ \hline -1 & 4 & -1 \\ \hline 0 & -1 & 0 \\ \hline \end{array}$$

$$G_x = \begin{array}{|c|c|c|} \hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$$

Products are:

$$\nabla^2 \otimes f(x, y) = 2$$

$$G_x \otimes f(x, y) = 3$$



Gradient and Laplacian

Example

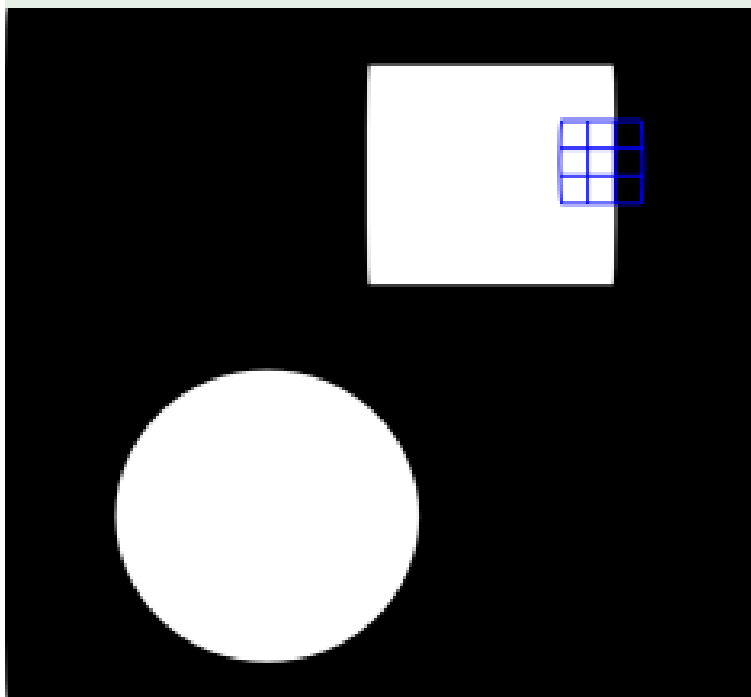


Image window:

$$f(x, y) = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\nabla^2 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

Products are:

$$\nabla^2 \otimes f(x, y) = 1$$

$$G_x \otimes f(x, y) = -4$$



Code sample >

```
% denoising/edge detection
dx=[-1 0 1;-2 0 1; -1 0 1];
dy=[-1 -2 -1;0 0 0;1 2 1];
K=kgauss(3);
K1=ones(19,19).*1/(19*19);
xwingDenoisMean=iconv(K1,xwing_grey);
idisp(xwingDenoisMean)
xwingDenoisGaus=iconv(K,xwing_grey);
idisp(xwingDenois)
xwinglx=iconv(dx,xwing_grey);
idisp(xwinglx)
xwingly=iconv(dy,xwing_grey);
idisp(xwingly)
magnGrad=sqrt(xwinglx.^2+xwingly.^2);
idisp(magnGrad)
edgeGrad=magnGrad>250;

edgeLapl=iconv(klog(2),xwing_grey);
idisp(iint(edgeLapl)>250);

edgeLapl=iconv(klog(1),xwing_grey);
idisp(iint(edgeLapl)>250);

edgeLapl=iconv(klog(3),xwing_grey);
idisp(iint(edgeLapl)>250);
```

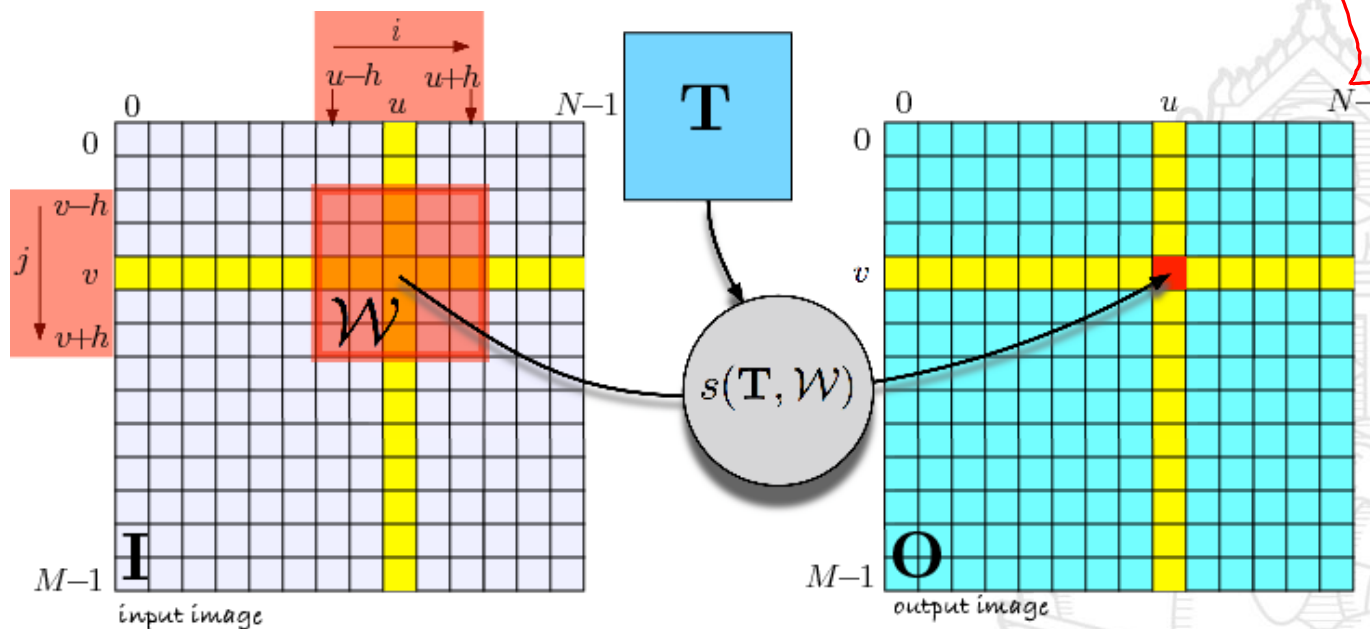


Template matching

$$O[u, v] = s(\mathbf{T}, \underline{\mathbf{W}}), \quad \forall (u, v) \in I$$

$u, v \rightarrow I$ E

DEAR
MARCO
X
JOHN



Template matching

Similarity measures

Sum of absolute differences	
SAD	$s = \sum_{(u,v) \in I} I_1[u, v] - I_2[u, v] $
ZSAD	$s = \sum_{(u,v) \in I} (I_1[u, v] - \bar{I}_1) - (I_2[u, v] - \bar{I}_2) $
Sum of squared differences	
SSD	$s = \sum_{(u,v) \in I} (I_1[u, v] - I_2[u, v])^2$
ZSSD	$s = \sum_{(u,v) \in I} ((I_1[u, v] - \bar{I}_1) - (I_2[u, v] - \bar{I}_2))^2$
Cross correlation	
NCC	$s = \frac{\sum_{(u,v) \in I} I_1[u, v] \cdot I_2[u, v]}{\sqrt{\sum_{(u,v) \in I} I_1^2[u, v] \cdot \sum_{(u,v) \in I} I_2^2[u, v]}}$
ZNCC	$s = \frac{\sum_{(u,v) \in I} (I_1[u, v] - \bar{I}_1) \cdot (I_2[u, v] - \bar{I}_2)}{\sqrt{\sum_{(u,v) \in I} (I_1[u, v] - \bar{I}_1)^2 \cdot \sum_{(u,v) \in I} (I_2[u, v] - \bar{I}_2)^2}}$



Non-parametric similarity measures

Census

		150	35	85		
		120	80	90		
		70	80	50		

10101101

Rank transform = 5

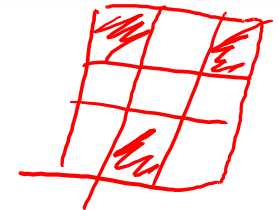
$$s(x) = \begin{cases} 1, & \text{if } x > R \\ 0, & \text{otherwise} \end{cases}$$

11011010

110101101

Census representation
Hamming distance

101101
10



Non-parametric similarity measures

Rank transform is more compact but does not encode position information

50	10	205
1	25	2
102	250	240

10	26	2
101	25	202
1	250	214

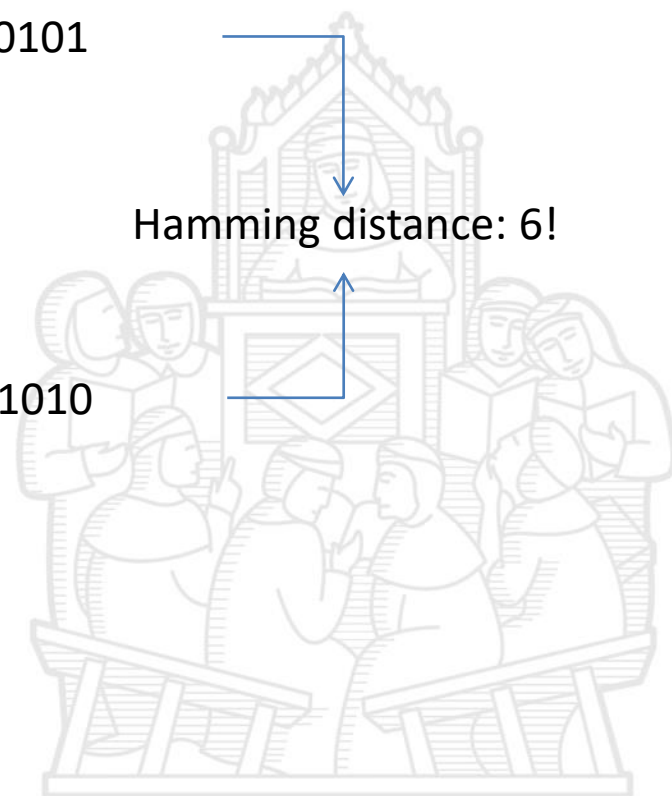
Census: 01110101

Rank: 5

Census: 10111010

Rank: 5

Hamming distance: 6!

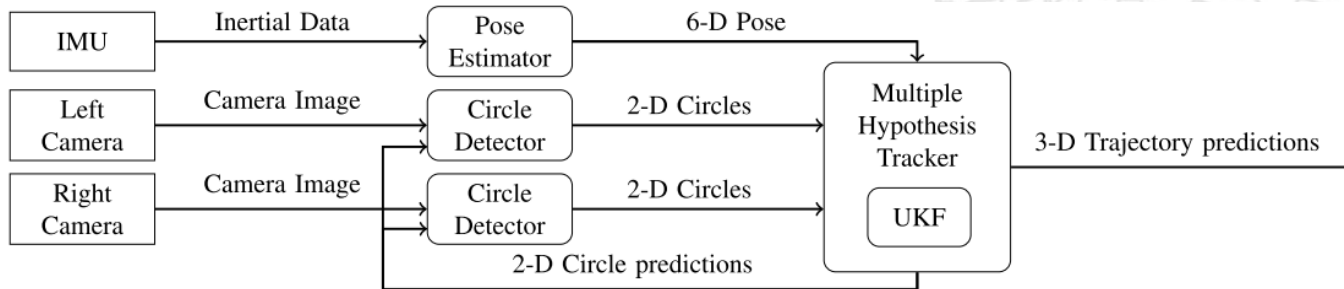
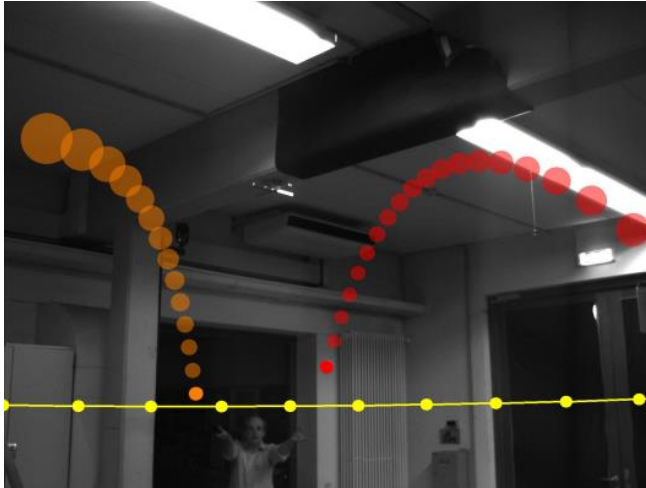


Non-linear operators

- Variance measure (on windows): Edge detection
- Median filter: noise removal
- Rank transform: non-local maxima suppression



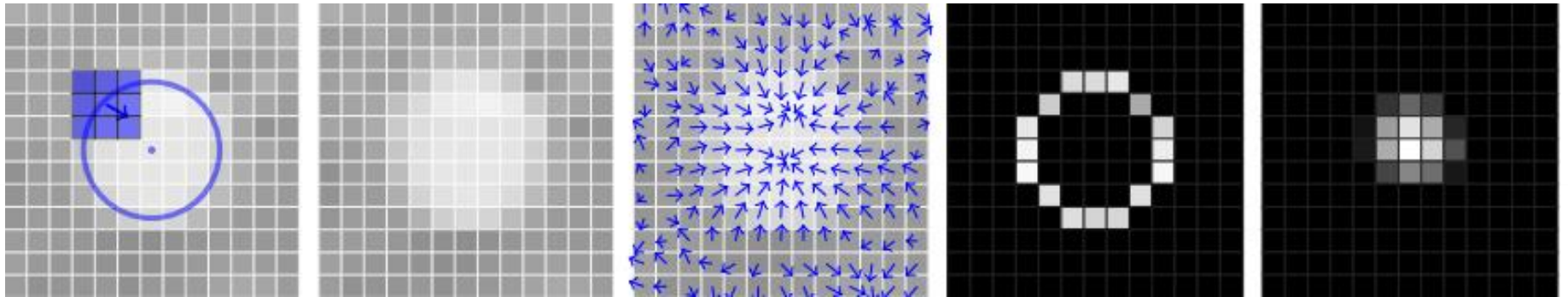
Example DLR



Oliver Birbach, Udo Frese and Berthold Baumli, (2011) 'Realtime Perception for Catching a Flying Ball with a Mobile Humanoid'



Example DLR

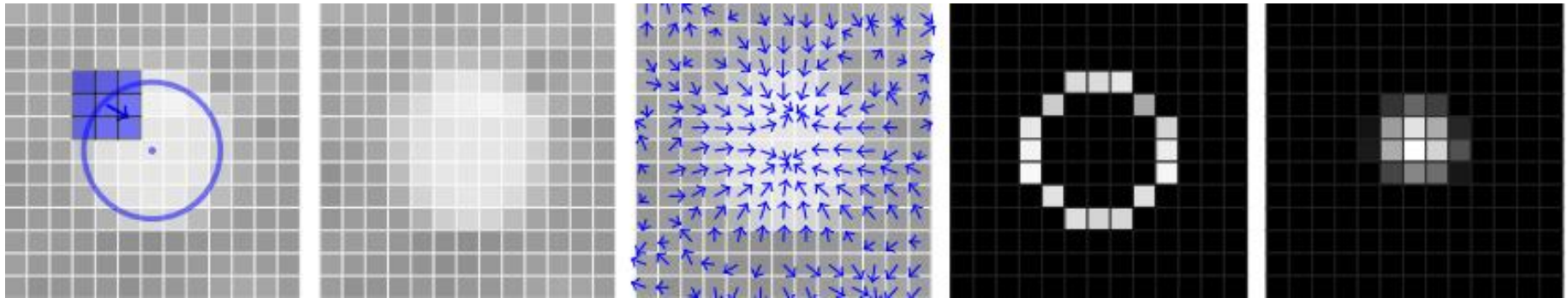


$$C = \frac{\sqrt{2} \left(\begin{pmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{pmatrix} * I, \begin{pmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{pmatrix} * I \right)^T}{\sqrt{16 \left(\begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{pmatrix} * I^2 - \left(\begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{pmatrix} * I \right)^2 + \epsilon^2}}$$

$$R(x, y, \alpha) = \left(\begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix} \cdot C(x, y) \right)^2 = |C(x, y)|^2 \cdot \cos^2 \delta$$

$$CR(x_c, y_c, r) = \frac{1}{2\pi} \int_{\alpha=0}^{2\pi} R(x_c + r \cos \alpha, y_c + r \sin \alpha, \alpha) d\alpha$$

Example DLR



Filtraggio con Sobel

$$C = \frac{\sqrt{2} \left(\begin{pmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{pmatrix} * I, \begin{pmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{pmatrix} * I \right)^T}{\sqrt{16 \left(\begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{pmatrix} * I^2 - \left(\begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{pmatrix} * I \right)^2 + \epsilon^2}}$$

Normalizzazione rispetto alla varianza locale

$$R(x, y, \alpha) = \left(\begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix} \cdot C(x, y) \right)^2 = |C(x, y)|^2 \cdot \cos^2 \delta$$

$$CR(x_c, y_c, r) = \frac{1}{2\pi} \int_{\alpha=0}^{2\pi} R(x_c + r \cos \alpha, y_c + r \sin \alpha, \alpha) d\alpha$$