Time Series - Shapelet/Motif Discovery



Shapelet

Time Series Classification

- Given a set X of n time series, $X = \{x_1, x_2, ..., x_n\}$, each time series has m ordered values $x_i = \langle x_{t1}, x_{t2}, ..., x_{tm} \rangle$ and a class value c_i .
- The objective is to find a function *f* that maps from the space of possible time series to the space of possible class values.
- Generally, it is assumed that all the TS have the same length *m*.

Shapelet-based Classification

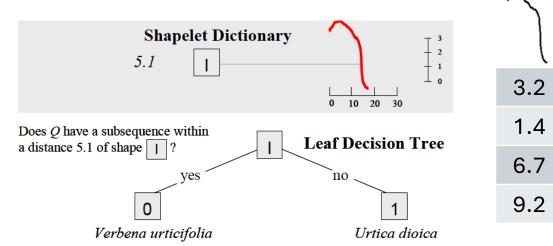
8.7

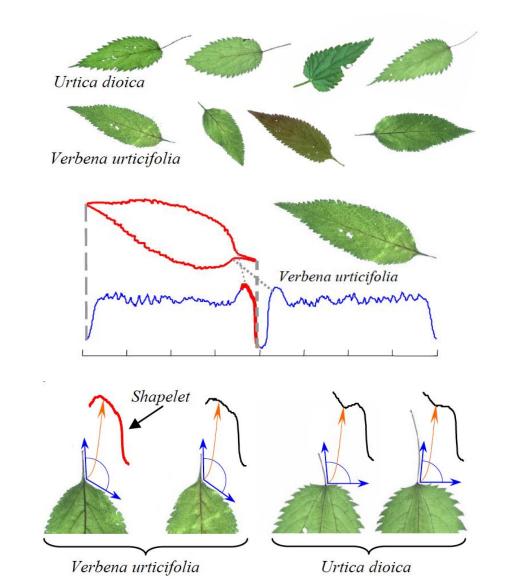
7.9

4.2

3.4

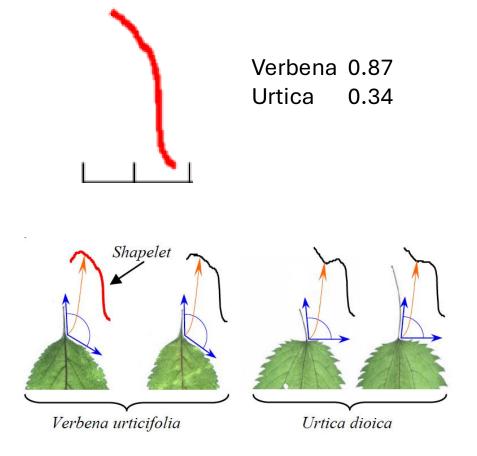
- Represent a TS as a vector of distances with representative subsequences, namely shapelets.
- 2. Use it as input for machine learning classifiers.





Time Series Shapelets

- Shapelets are TS subsequences which are maximally representative of a class.
- Shapelets can provide interpretable results, which may help domain practitioners better understand their data.
- Shapelets can be significantly more accurate/robust because they are *local features*, whereas most other state-of-the-art TS classifiers consider *global features*.



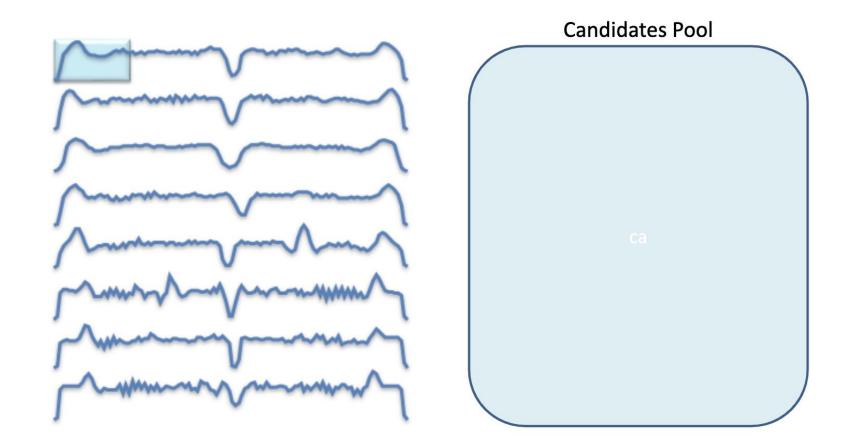
Finding Shapelets

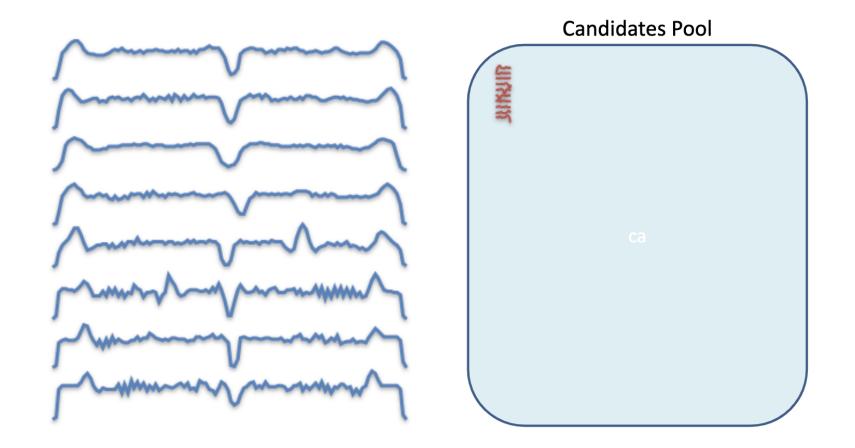
| Findi | FindingShapeletBF (dataset D, MAXLEN, MINLEN) | | | | | | | | | | |
|-------|--|--|--|--|--|--|--|--|--|--|--|
| 1 | candidates GenerateCandidates(D, MAXLEN, MINLEN) | | | | | | | | | | |
| 2 | $bsf_gain \leftarrow 0$ | | | | | | | | | | |
| 3 | For each S in candidates | | | | | | | | | | |
| 4 | $gain \leftarrow CheckCandidate(\mathbf{D}, S)$ | | | | | | | | | | |
| 5 | If gain > bsf_gain | | | | | | | | | | |
| 6 | bsf_gain 🗲 gain | | | | | | | | | | |
| 7 | $bsf_shapelet \leftarrow S$ | | | | | | | | | | |
| 8 | EndIf | | | | | | | | | | |
| 9 | EndFor | | | | | | | | | | |
| 10 | Return bsf_shapelet | | | | | | | | | | |

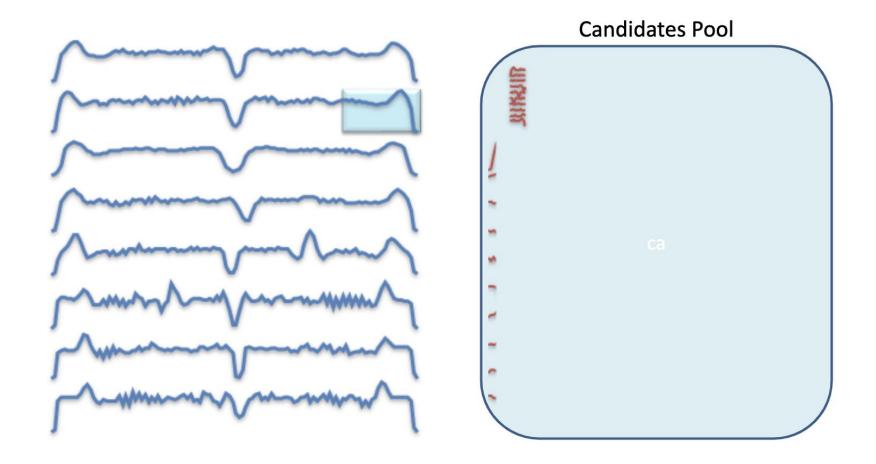
Generate Candidate

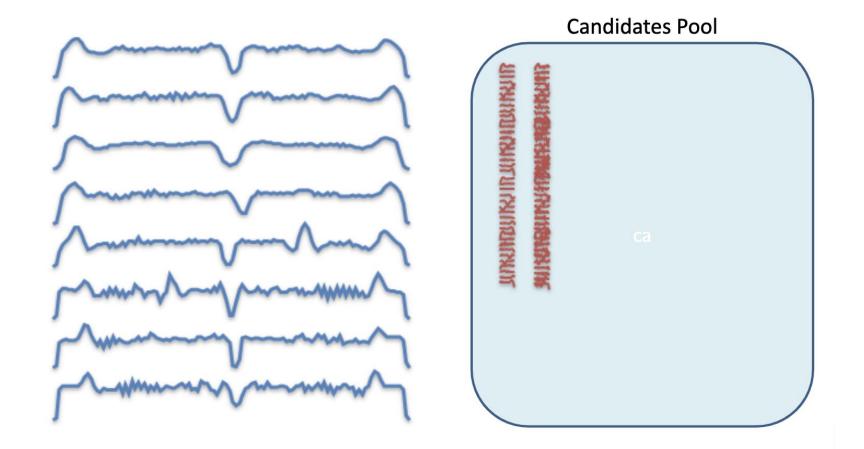
Sliding a **window of size** *l* across all of the time series objects in the dataset D, extracts all of the possible candidates and adds them to the pool

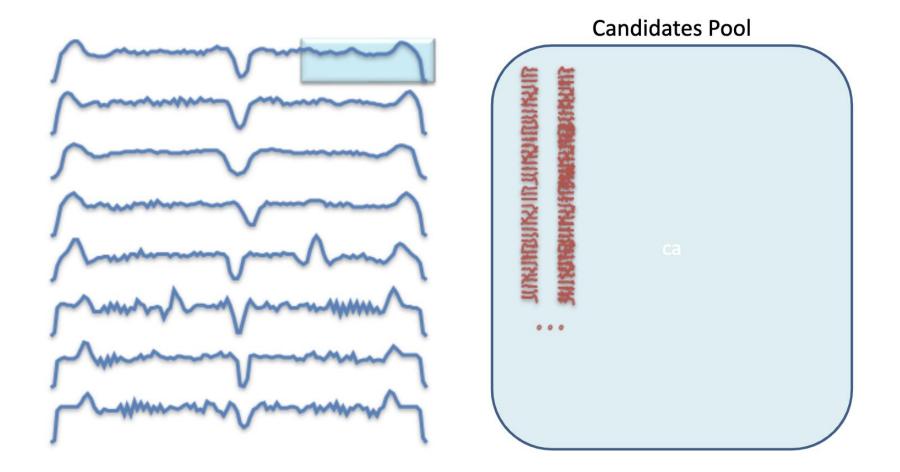
| Gene | erateCandidates (dataset D , MAXLEN, MINLEN) |
|------|---|
| 1 | $pool \leftarrow \emptyset$ |
| 2 | $l \leftarrow MAXLEN$ |
| 3 | While $l \ge MINLEN$ |
| 4 | For T in D |
| 5 | $pool \leftarrow pool \cup \mathbf{S}_T^{\ l}$ |
| 6 | EndFor |
| 7 | $l \leftarrow l - 1$ |
| 8 | EndWhile |
| 9 | Return pool |











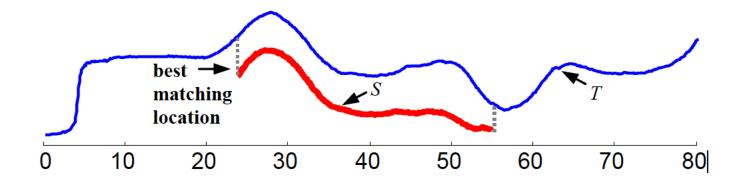
Check Candidates

| Chec | ckCandidate (dataset D , shapelet candidate <i>S</i>) |
|------|--|
| 1 | objects_histogram ← Ø |
| 2 | For each T in D |
| 3 | dist \leftarrow SubsequenceDist(T, S) |
| 4 | insert T into objects_histogram by the key dist |
| 5 | EndFor |
| 6 | Return CalculateInformationGain(<i>objects_histogram</i>) |

- Inserts all of the time series objects into the histogram objects_histogram according to the distance from the time series object to the candidate
- Calculate Information Gain

Distance with a Subsequence

- Distance from the TS to the subsequence *SubsequenceDist(T, S)* is a distance function that takes time series *T* and subsequence *S* as inputs and returns a non-negative value *d*, which is the distance from *T* to *S*.
- SubsequenceDist(T, S) = min(Dist(S, S')), for $S' \in S_T^{/S/}$
 - where $S_T^{/S/}$ is the set of all possible subsequences of T
- Intuitively, it is the distance between *S* and its best matching location in *T*.

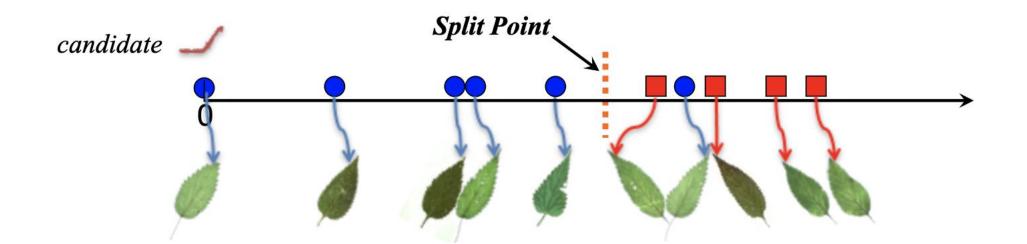


Check Candidates with IG

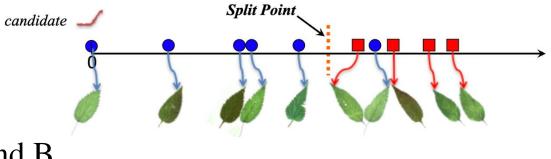
| Calcı | ulateInformationGain (distance histogram <i>obj_hist</i>) |
|-------|--|
| 1 | <pre>split_dist</pre> |
| 2 | $\tilde{\mathbf{D}}_1 \leftarrow \emptyset, \mathbf{D}_2 \leftarrow \emptyset$ |
| 3 | For d in obj_hist |
| 4 | If $d.dist < split_dist$ |
| 5 | $\mathbf{D}_1 \leftarrow \overline{\mathbf{D}}_1 \cup \overline{d}.objects$ |
| 6 | Else |
| 7 | $\mathbf{D}_2 \leftarrow \mathbf{D}_2 \cup d.objects$ |
| 8 | EndIf |
| 9 | EndFor |
| 10 | Return $I(\mathbf{D}) - \hat{I}(\mathbf{D})$ |

Testing The Utility of a Candidate Shapelet

- Arrange the TSs in the dataset *D* based on the distance from the candidate.
- Find the optimal split point that maximizes the information gain (same as for Decision Tree classifiers)
- Pick the candidate achieving best utility as the shapelet

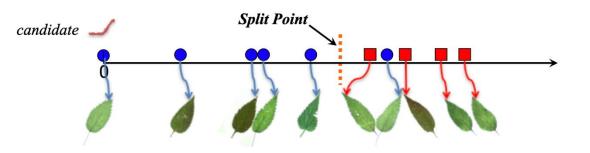


Entropy



- A TS dataset *D* consists of two classes, A and B.
- Given that the proportion of objects in class A is p(A) and the proportion of objects in class B is p(B),
- The **Entropy** of D is: I(D) = -p(A)log(p(A)) p(B)log(p(B)).
- Given a strategy that divides D into two subsets D_1 and D_2 , the information remaining in the dataset after splitting is defined by the weighted average entropy of each subset.
- If the fraction of objects in D_1 is $f(D_1)$ and in D_2 is $f(D_2)$, the total entropy of D after splitting is $\hat{I}(D) = f(D_1)I(D_1) + f(D_2)I(D_2)$.

Information Gain



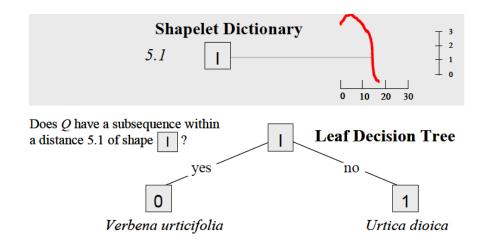
- Given a certain split strategy *sp* which divides D into two subsets D_1 and D_2 , the entropy before and after splitting is I(D) and $\hat{I}(D)$.
- The **information gain** for this splitting rule is:

•
$$Gain(sp) = I(D) - \hat{I}(D) =$$

= $I(D) - f(D_1)I(D_1) + f(D_2)I(D_2).$

• We use the distance from *T* to a shapelet *S* as the splitting rule *sp*.

Split point distance from shapelet = 5.1



Problem

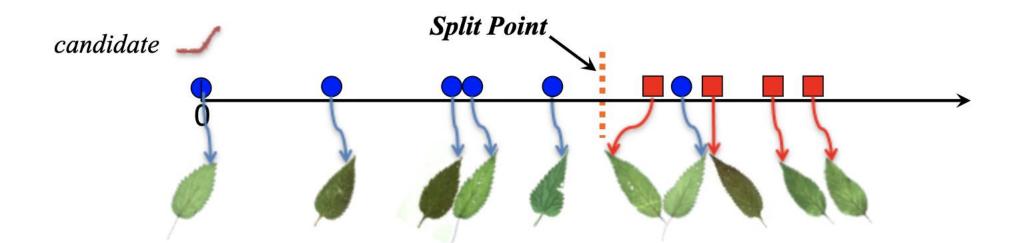
• The total number of candidate is

 $\sum_{l=MINLEN}^{MAXLEN} \sum_{T_i \in D} (|T_i| - l + 1)$

- For each candidate you have to compute the distance between this candidate and each training sample (space inefficiency)
- For instance
 - 200 instances with length 275
 - 7,480,200 shapelet candidates

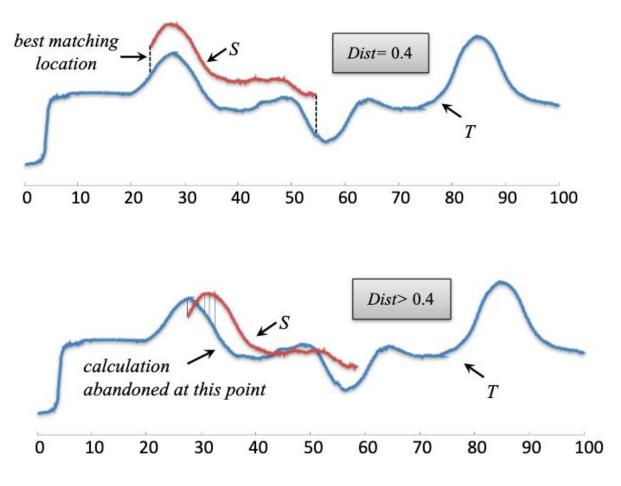
Speedup

- Distance calculations form TSs to shapelet candidates is expensive.
- Reduce the time in two ways
 - Distance Early Abandon: reducing the distance computation time between two TS
 - Admissible Entropy Pruning: reducing the number of distance calculations



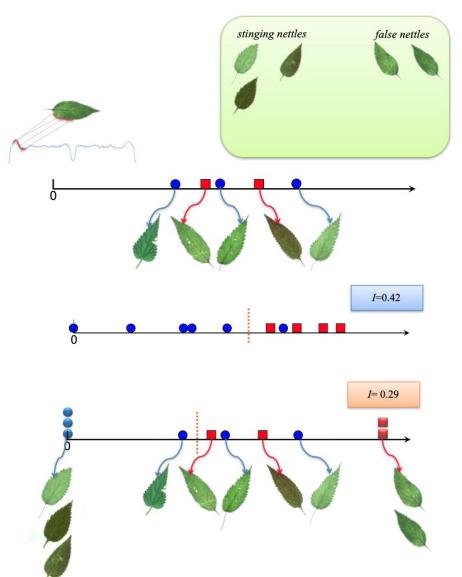
Distance Early Abandon

- We only need the minimum distance.
- Method:
 - Keep the best-so-far distance
 - Abandon the calculation if the partial current distance is larger than best-so-far.
 - We can avoid to compute the full distance for S if the partial one is greater than the best so far



Admissible Entropy Pruning

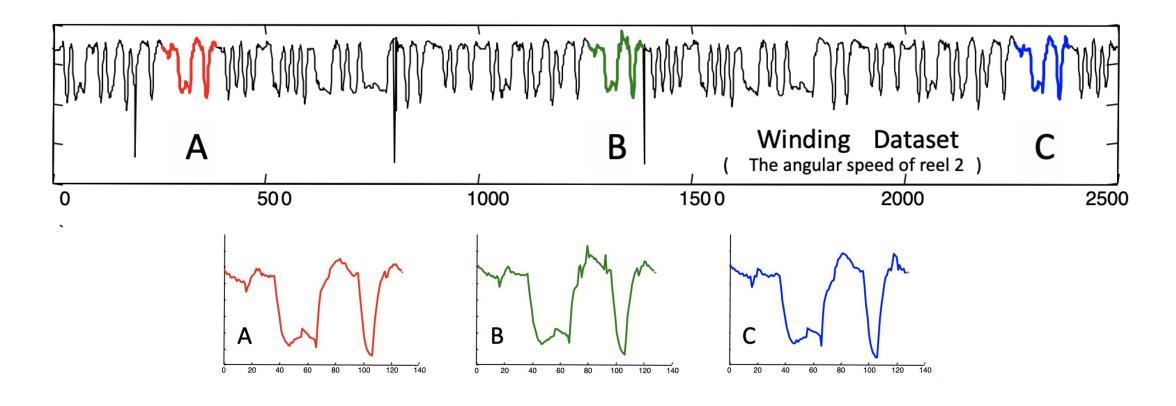
- We only need the best shapelet for each class
- For a candidate shapelet
 - We do not need to calculate the distance for each training sample
 - After calculating some training samples, if the **upper bound** of information gain (corresponding to the optimistic scenario) < best candidate shapelet
 - Stop calculation for that candidate and try next candidate



Motif

Time Series Motif Discovery

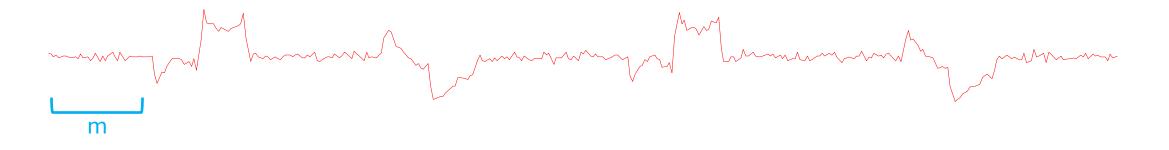
- Finding repeated patterns, i.e., pattern mining.
- Are there any repeated patterns, of length *m* in the TS?

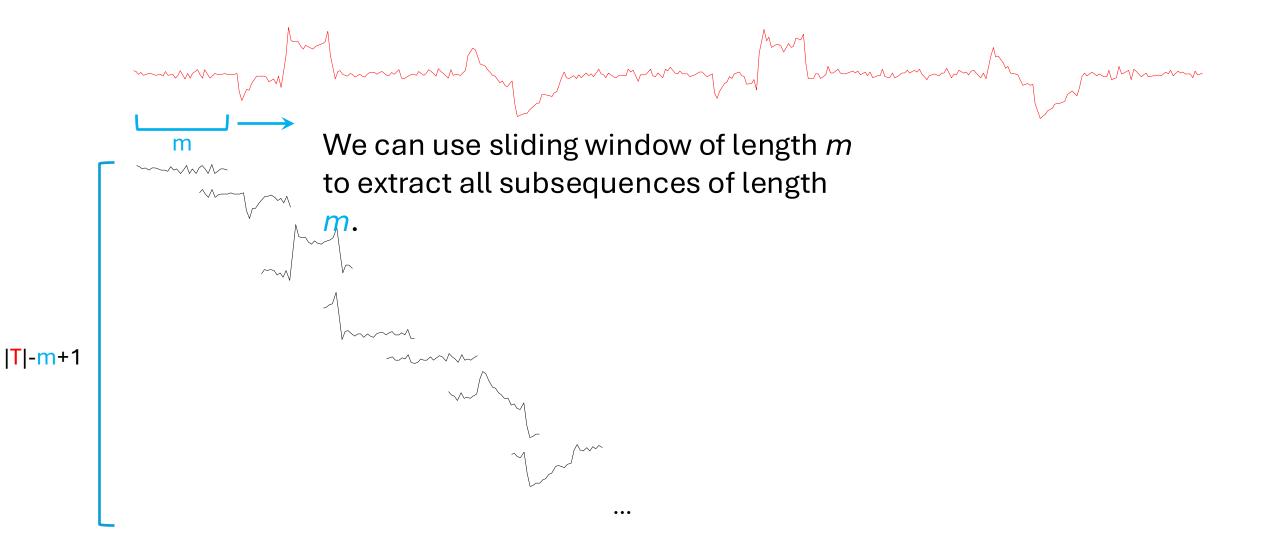


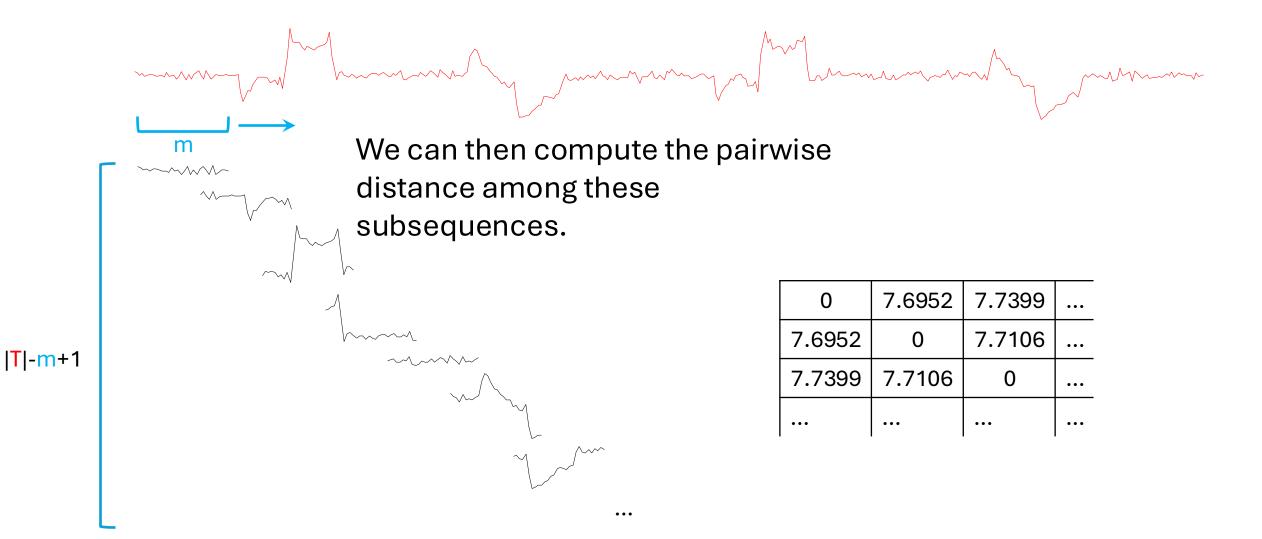
Why Find Motifs?

- Mining association rules in TS requires the discovery of motifs. These are referred to as primitive shapes and frequent patterns.
- Several TS classifiers work by constructing typical prototypes of each class. These prototypes may be considered motifs.
- Many TS anomaly detection algorithms consist of modeling normal behavior with a set of typical shapes (which we see as motifs), and detecting future patterns that are dissimilar to all typical shapes.

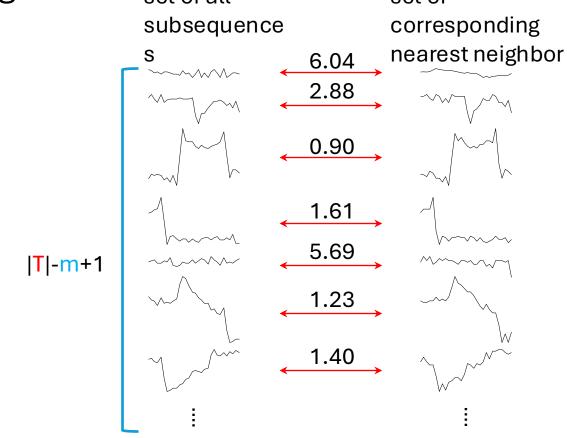
- The Matrix Profile (MP) is a data structure that annotates a TS and can be exploited for many purposed: e.g. efficient Motif Discovery.
- Given a time series, T and a desired subsequence length, m.



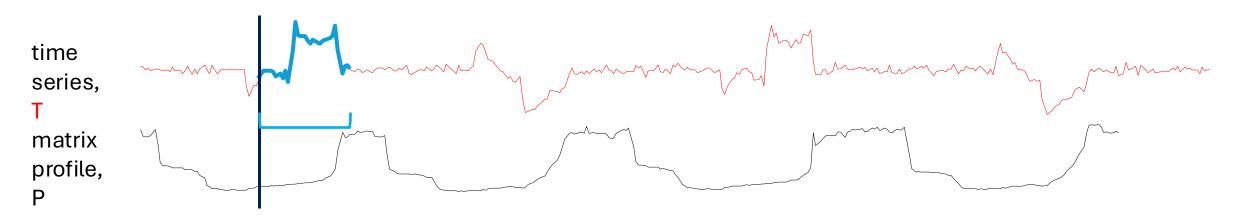




• For each subsequence we keep only the distance with the closest nearest neighbor. set of all set of



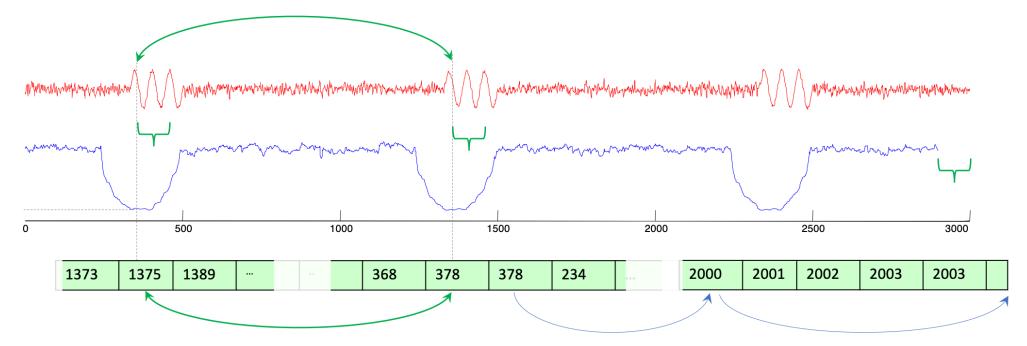
• The **distance** to the corresponding **nearest neighbor** of each subsequence can be stored in a vector called **matrix profile P**.



The matrix profile value at location i is the distance between T_i and its nearest neighbor

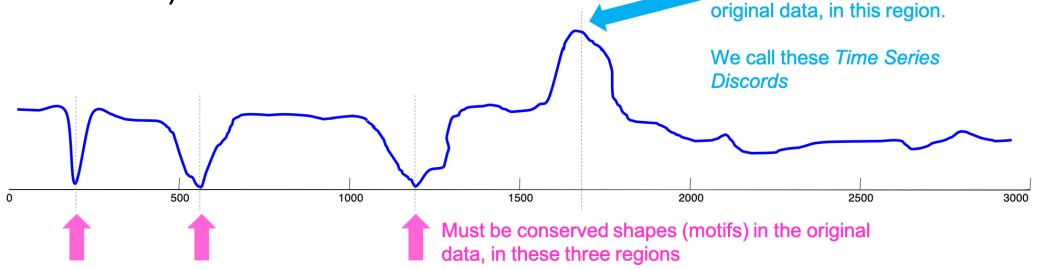
- The index of corresponding nearest neighbor of each subsequence is also stored in a vector called matrix profile index. 194 time series. matrix m profile index, I It turns out that T_i 's nearest neighbor is T_{194} 192 193 195 194 196 The matrix profile value at location *i* is the
 - distance between T_i and its nearest neighbor

- The MP index allows to find the nearest neighbor to any subsequence in constant time.
- Note that the pointers in the matrix profile index are not necessarily symmetric.
- If A points to B, then B may or may not point to A
- The classic TS motif: the two smallest values in the MP must have the same value, and their pointers must be mutual.



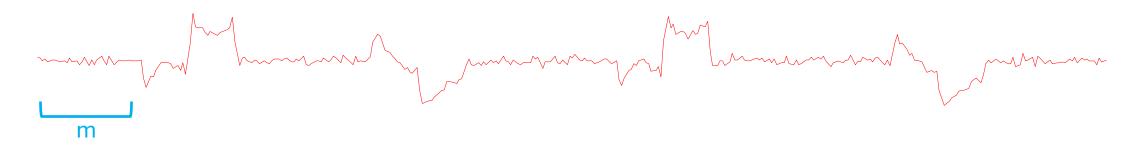
How to "read" a Matrix Profile

- For relatively low values, you know that the subsequence in the original TS must have (at least one) relatively similar subsequence elsewhere in the data (such regions are "motifs")
- For relatively high values, you know that the subsequence in the original TS must be unique in its shape (such areas are anomalies).



How to Compute Matrix Profile?

• Given a time series, T and a desired subsequence length, m.



| inf |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| | | | | | | | | | | | | | | | | | | |

Matrix profile is initialized as inf vector

This is just a toy example, so the values and the vector length does not fit the time series shown above

How to Compute Matrix Profile?

• Given a time series, T and a desired subsequence length, m.

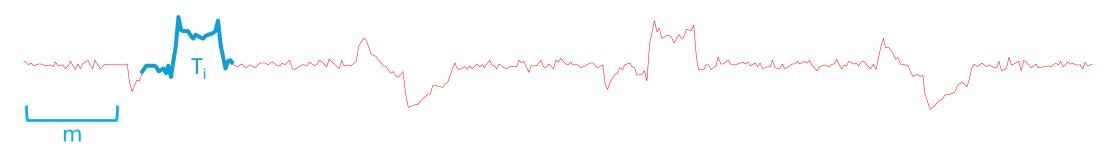


| | | inf |
|--|--|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|--|--|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|

At the first iteration, a subsequence T_i is randomly selected from T

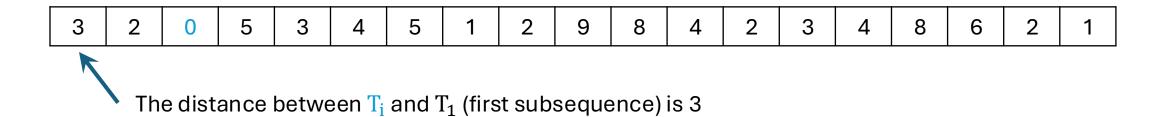
How to Compute Matrix Profile?

• Given a time series, T and a desired subsequence length, m.



| inf |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| | | | | | | | | | | | | | | | | | | |

We compute the distances between T_i and every subsequences from T (time complexity = O(|T|log(|T|))) We then put the distances in a vector based on the position of the subsequences



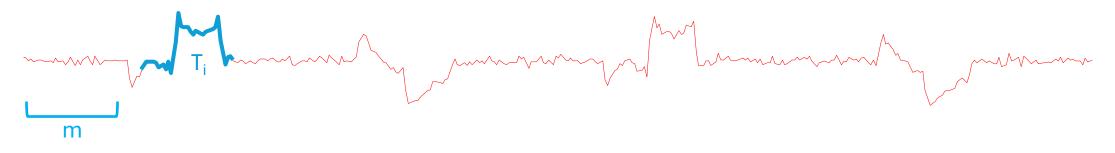
m

• Given a time series, T and a desired subsequence length, m. \mathcal{T}_{i}

inf inf

We compute the distances between T_i and every subsequences from T (time complexity = O(|T|log(|T|))) We them put the distances in a vector based on the position of the subsequences

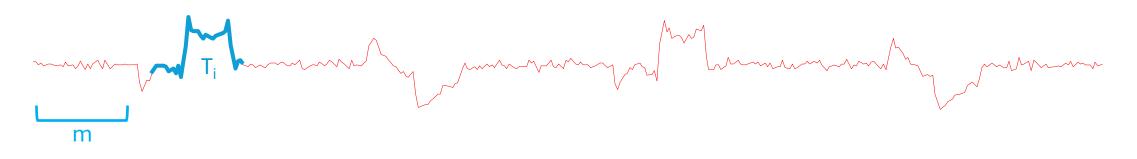
3 5 2 5 3 2 9 8 2 3 8 6 2 $\mathbf{0}$ 4 4 4 Let say T_i happen to be the third subsequences, therefore the third value in the distance vector is 0



| | inf | inf | inf | inf | inf | inf | inf | inf | inf | inf | inf |
|----|-----|-----|-----|-----|-----|-----|-----|-----|------------------|-----|-------|-------|-------|-------|--------|--------|-----|-----|-----|
| mi | n 🚺 | | | | | | | • | rofile /o vec | • | dated | by ap | ply e | lemer | ntwise | e mini | mum | to | |
| | 3 | 2 | 0 | 5 | 3 | 4 | 5 | 1 | 2 | 9 | 8 | 4 | 2 | 3 | 4 | 8 | 6 | 2 | 1 |



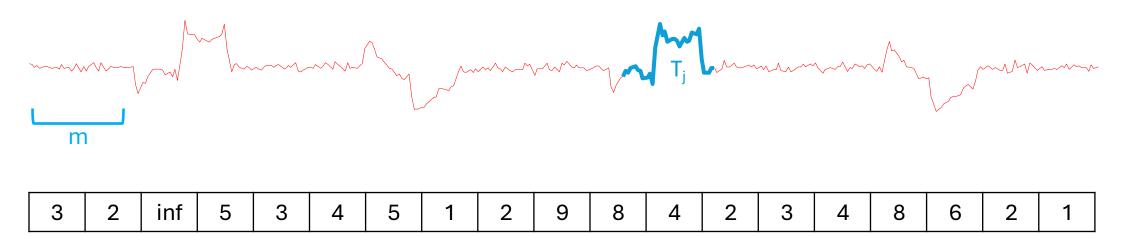
| 3 | inf | inf | inf | inf | inf | inf | inf | inf | inf | inf | inf | inf |
|----|-----|-----|-----|-----|-----|-----|-------------------|-----|-----|-------|-------|-------|-------|--------|--------|-----|-----|-----|
| mi | n 🚺 | | | | | | atrix p ese tw | | • | dated | by ap | ply e | lemer | ntwise | e mini | mum | to | |
| 3 | 2 | 0 | 5 | 3 | 4 | 5 | 1 | 2 | 9 | 8 | 4 | 2 | 3 | 4 | 8 | 6 | 2 | 1 |



| 3 | 2 | inf | 5 | 3 | 4 | 5 | 1 | 2 | 9 | 8 | 4 | 2 | 3 | 4 | 8 | 6 | 2 | 1 |
|---|---|-----|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | |

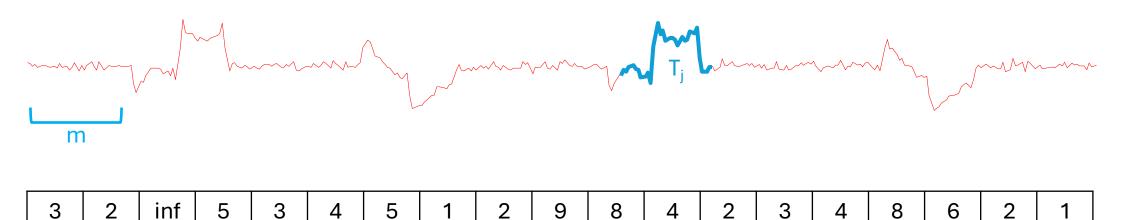
After we finish to update matrix profile for the first iteration

• Given a time series, T and a desired subsequence length, m.



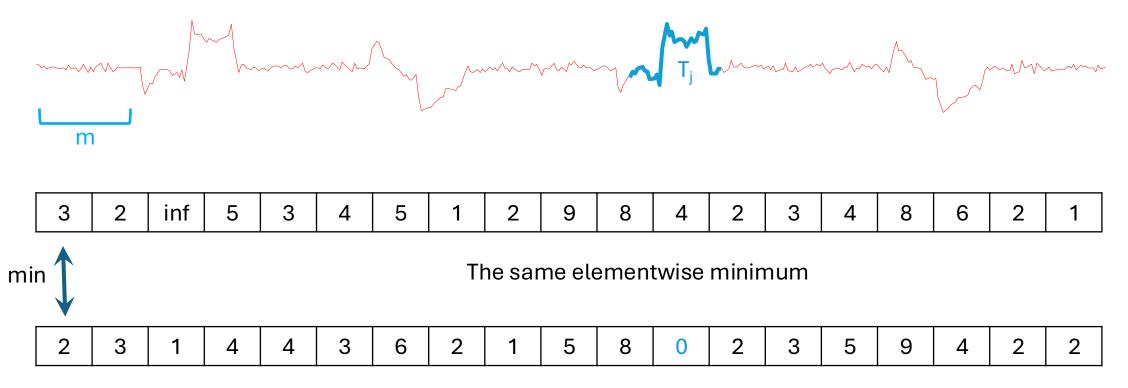
In the second iteration, we randomly select another subsequence T_j and it happens to be the 12^{th} subsequences

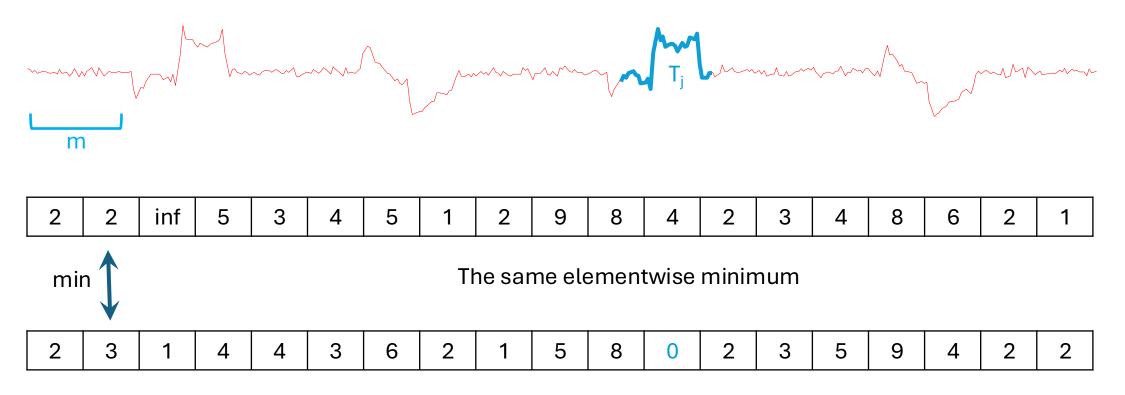
• Given a time series, T and a desired subsequence length, m.

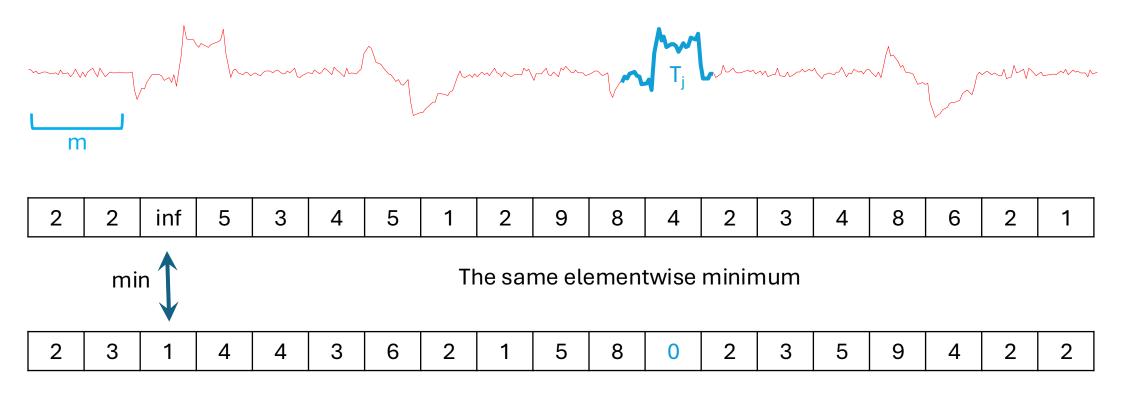


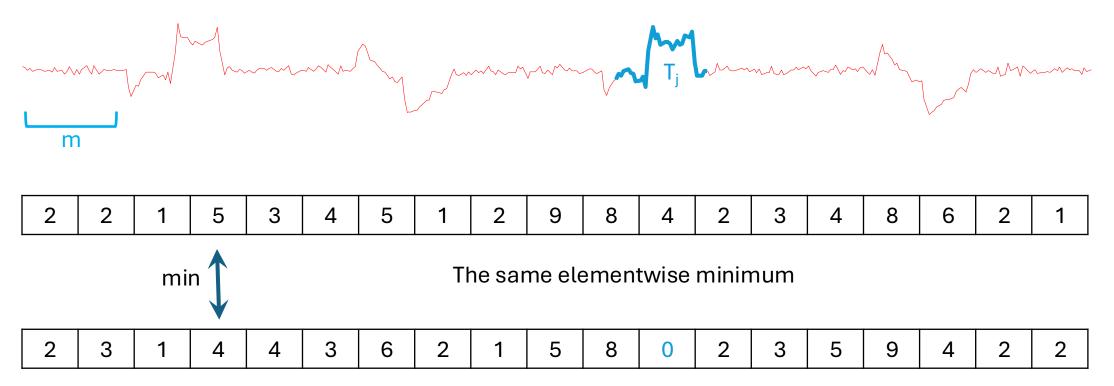
Once again, we compute the distance between T_i and every subsequences of T

| 2 | 3 | 1 | 4 | 4 | 3 | 6 | 2 | 1 | 5 | 8 | 0 | 2 | 3 | 5 | 9 | 4 | 2 | 2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | 1 |

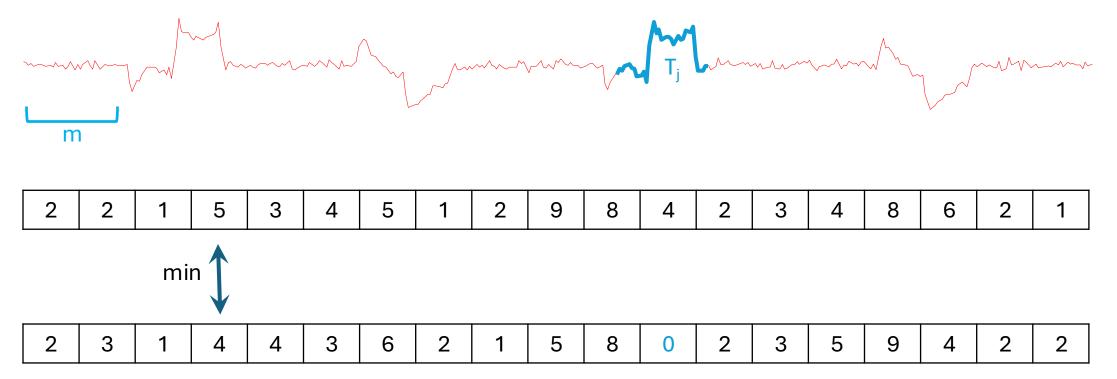






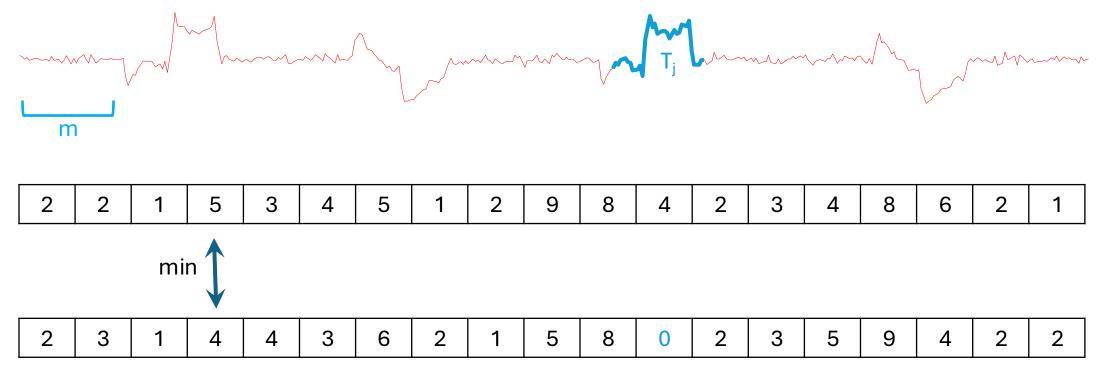


• Given a time series, T and a desired subsequence length, m.



We repeat the two steps (distance computation and update) until we have used every subsequences

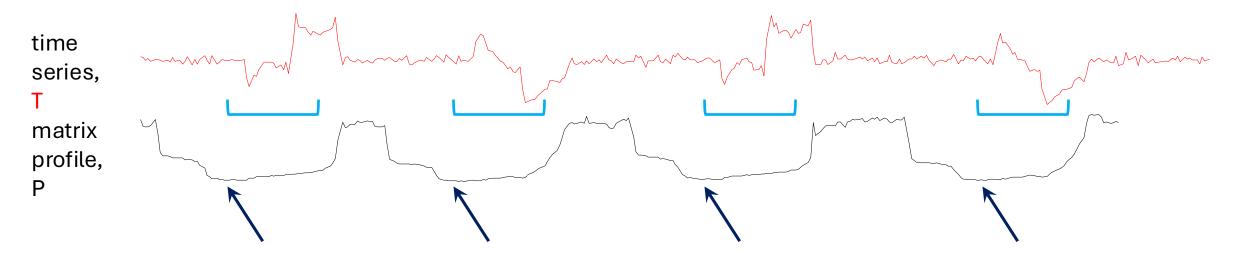
• Given a time series, T and a desired subsequence length, m.



There are |T| subsequences and the distance computation is O(|T|log(|T|))

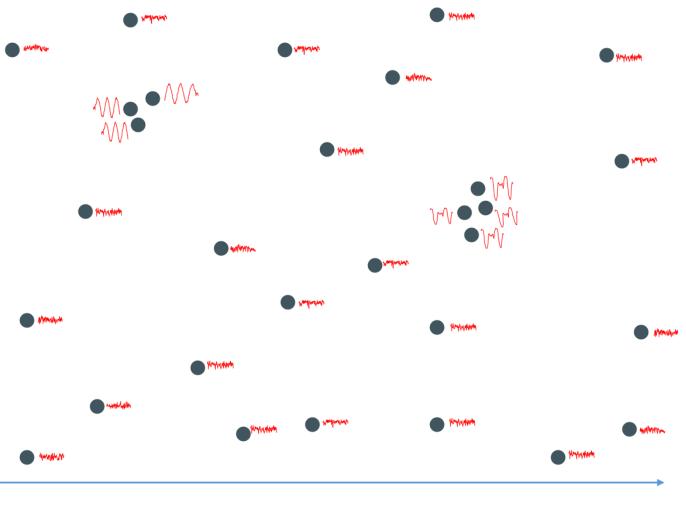
The overall time complexity is $O(|T|^2 \log(|T|))$

Motif Discovery From Matrix Profile



Local minimums are corresponding to motifs

Motif Discovery From Matrix Profile



 It is sometime useful to think of time series subsequences as points in m-dimensional space.

 In this view, dense regions in the m-dimensional space correspond to regions of the time series that have a low corresponding MP.

1000

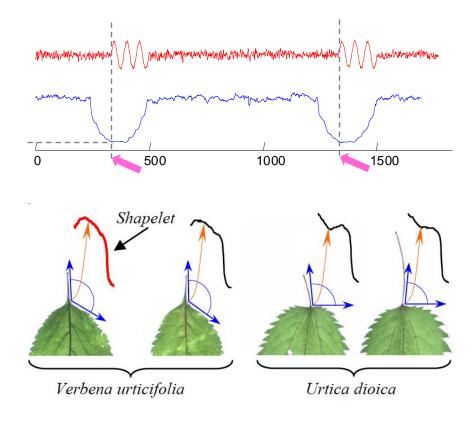
1500

500

Motif/Shapelet Summary

• A **motif** is a repeated pattern/subsequence in a given TS.

• A **shapelet** is a pattern/subsequence which is maximally representative of a class with respect to a given dataset of TSs.



References

- Matrix Profile I: All Pairs Similarity Joins for Time Series: A Unifying View that Includes Motifs, Discords and Shapelets. Chin-Chia Michael Yeh et al. 1997
- Time Series Shapelets: A New Primitive for Data Mining. Lexiang Ye and Eamonn Keogh. 2016.
- Josif Grabocka, Nicolas Schilling, Martin Wistuba, Lars Schmidt-Thieme (2014): Learning Time-Series Shapelets, in Proceedings of the 20th ACM SIGKDD Conference on Knowledge Discovery and Data Mining, KDD 2014

Hoang Anh Dau, 'Diego Furtado Silva, 'Abdullah Mueen, and Eamonn Keogh Iniversity of California, Riverside, 'Universidade de São Panlo, ¹University of New Mexico 5, Inlan001, nbegu001, yding007, hdsu001]@scr.edu, diegofulva@icmc.usp.br, ameen@sum.edu, eamo Advance—The elipsivisaticity oversity for isolativity justs for damper latence in the standard of the start ing) at best p It is exact, providing no false positives or false dismis de speedup. In this work we introduce It is simple and parameter-free. In contrast, the mo general metric space APSS algorithms require building and tuning spatial access methods and/or hash function Our algorithm requires an inconsequential space overher O(n) with a small constant factor While our exact algorithm is extremely, ly large datasets we can compute the results in of our ideas for many time region data mining or nytime fashion, allowing ultra-fast approximate solut Juding matif discovery, negalty discovery, shandlet discover segmentation, density estimation, and contrast se incrementativy update it very streaming and incrementative provides and the streaming data forever. Our method provides *full* joins, eliminating the need to which subjustive threathold, which as we will show, is a Keywords-Time Series; Similarity Joins; Motif Discover 1. INTRODUCTION specify a similarity threshold, which as we will show, is a near impossible task in this domain. ilarity-search (also known as similari omes in several variants. The basic task is this ion of data objects, ratrieve the nearest neighbor et. In the text domain the algorithm has Our algorithm is embarrassingly parallelizable, both on multicour processors and in distributed systems Time Series Shapelets: A New Primitive for Data Mining We believe that this lack of progress stems not from a lack nterest in this useful primitive, but from the daunting nature the problem. Consider the following example that reflects the rds of an industrial collaborator. A boiler at a chemica Lexiang Ye Dept. of Computer Science & Engineering University of California, Riverside, CA 92521 lexiangy@cs.ucr.edu Dept. of Computer Science & Engineering University of California, Riverside, CA 92521 eamonn@cs.ucr.edu ce a minute. After a year, we have series of length 525,600. A plant manager may wish to d rity self-join on this data with week-long subse to discover operating regimes (summer vs. winter o istillate vs. heavy distillate etc.) The obvious nested loo hm requires 132,880,692,960 Euclidean distance ABSTRACT Classification of time series has been attracting great interest over the past decade. Recent empirical evidence has strongly suggested that the simple nearest neighbor algorithm is very difficult to beat for most time series problems. While this may be considered good ome time to consider a detailed mot shows some examples of leaves from two classes, Urtica di (stinging nettles) and Verbana urticifolia. These two plants en the join will take 135.6 says, ork is to show that we can reduce this time to n off-the-shelf desktop computer. Moreover, in one he computed and/or updated increaommonly confused, hence the colloquial name "false nett eray, given the simplicity of imples nenting the nearest neighb algorithm, there are some negative consequen ould maintain this join essentially forever on a standard earest neighbor algorithm requires storing and searching th e dataset, resulting in a time and space complexity that limit pplicability, especially on resource-limited In this work we introduce a new time serie e series shapelet primitives can be interpretable, more accura and significantly faster than state-of-the-art classifier Categories and Subject Descriptors General Terms Algorithms, Experime 1. INTRODUCTION 1. INTRODUCTION While the last decide has seen a large interest in time series classification, to date the most accurate and robust method is the simple enserts meighbor algorithm (4)[21][4]. While the merest neighbor algorithm has the advantages of simplicity and nor requiring extensive parameter training, it does have serveral important disadvantages. Chief among these are its pace and time important disadvantages. Chief among these are its pace and interments, and the fact that it does not tell us anything about a particular object was assigned to a particular class ed time serves shapetett. Informatiy, shapetets are time serves sequences which are in some sense maximally representative class. While we believe shapetets can have many uses in data ning, one obvious implication of them is to mitigate the two aknesses of the nearest neighbor algorithm noted above. on to make digital or hard copies of all or part of this work fo

Matrix Profile I: All Pairs Similarity Joins for Time Series A Unifying View that Includes Motifs, Discords and Shapelets Chin-Chia Michael Yeh, Yan Zhu, Liudmila Ulanova, Nurjahan Begum, Yifei Ding

D 19, June 29-July 1, 2009, Paris, France

Figure 1: Samples of leaves from two species. Note that leaves have the insect-hite damage uppose we wish to build a classifier to dista

Eamonn Keogh

color and size within each class completely dwarfs the rability between classes, our best hope is based on t the leaves. However, as we can see in Figure 1, the d the global shape are very subtle. Furthermore, i common for leaves to have distortions or "occlusions" due to insect damage, and these are likely to confuse any global measures of shape. Instead we attempt the following. We first convert each leaf into a one-dimensional representation as shown

tions have been succe ears [8]. However, here we find that using a near

to be due to the fact that the data is somewhat poicy (i.e. ins bites, and different stem lengths), and this noise is enough t vamp the subtle differences in the shapes

erional or classroom use is granted without tee provided that copies are or made or distributed for profit or commercial advantage and that opies bear this notice and the full citation on the first page. To copy therwise, or republish, to post on servers or to redistribute to lists, service nois exactly neuroistic to the first.

time series will be made a