DATA MINING 2
Time Series - Similarities & Distances

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Slides edited from Keogh Eamonn's tutorial

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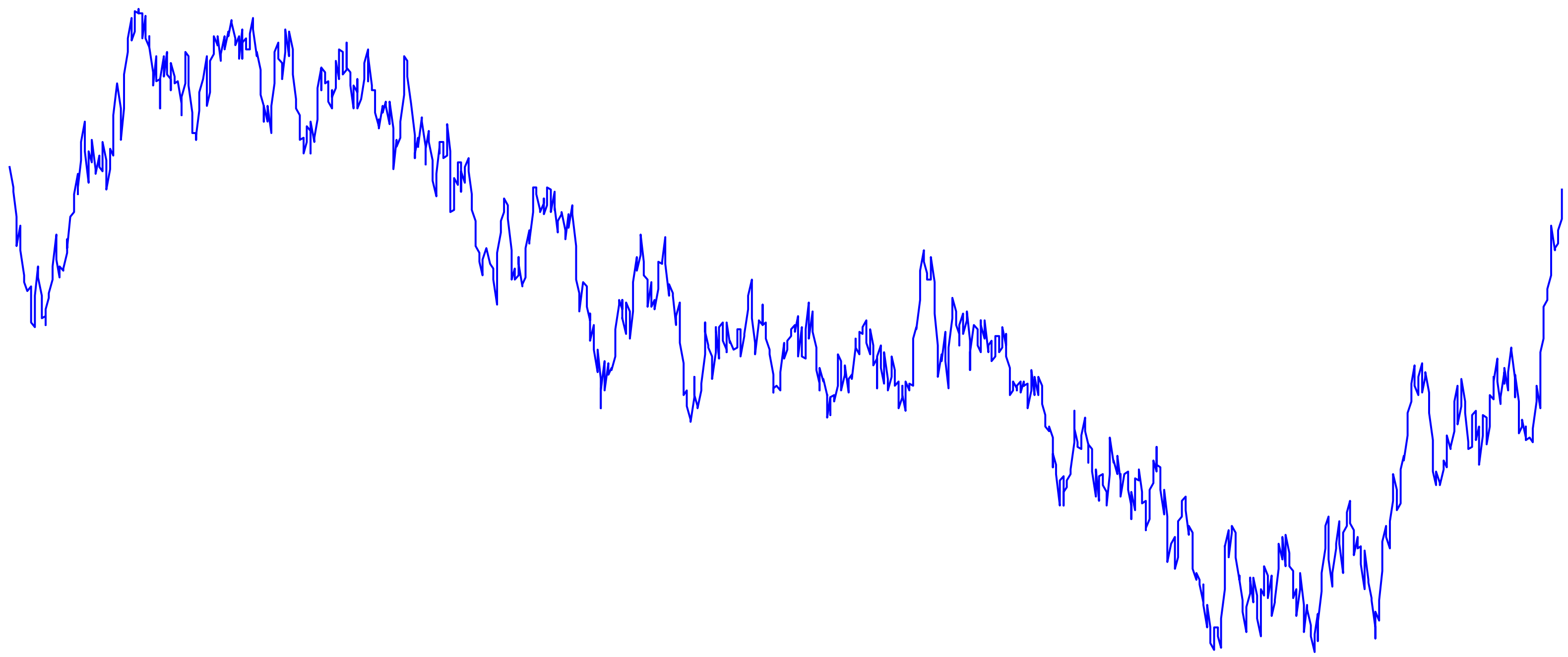
What is a Time Series?

- A time series is a collection of observations made sequentially in time, generally at constant time intervals.



25.1750
25.2250
25.2500
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25.3250
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25.3500
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25.4000
25.3250
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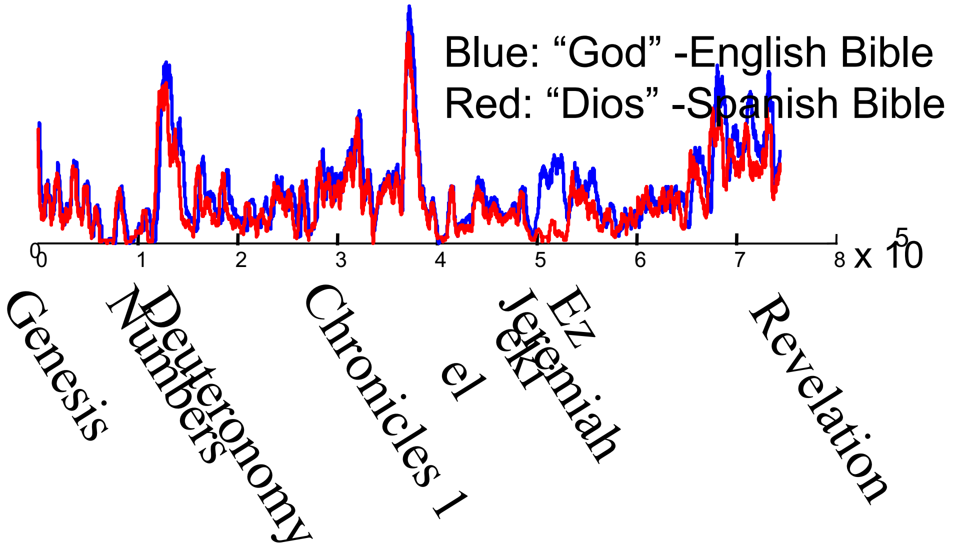
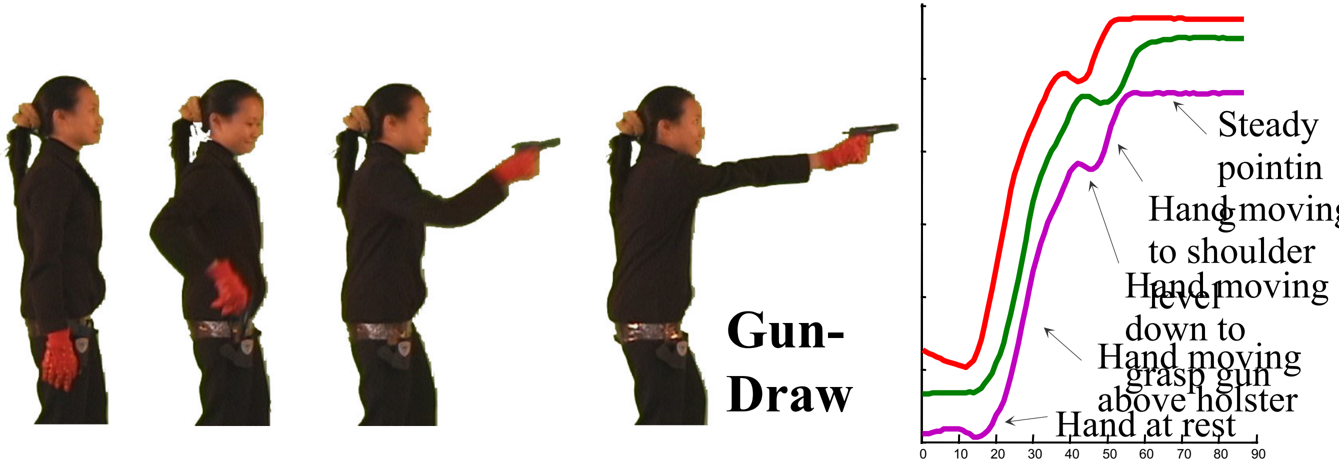
25.2000
25.1750
...
24.6250
24.6750
24.6750
24.6250
24.6250
24.6250
24.6750
24.7500

Time Series are Ubiquitous

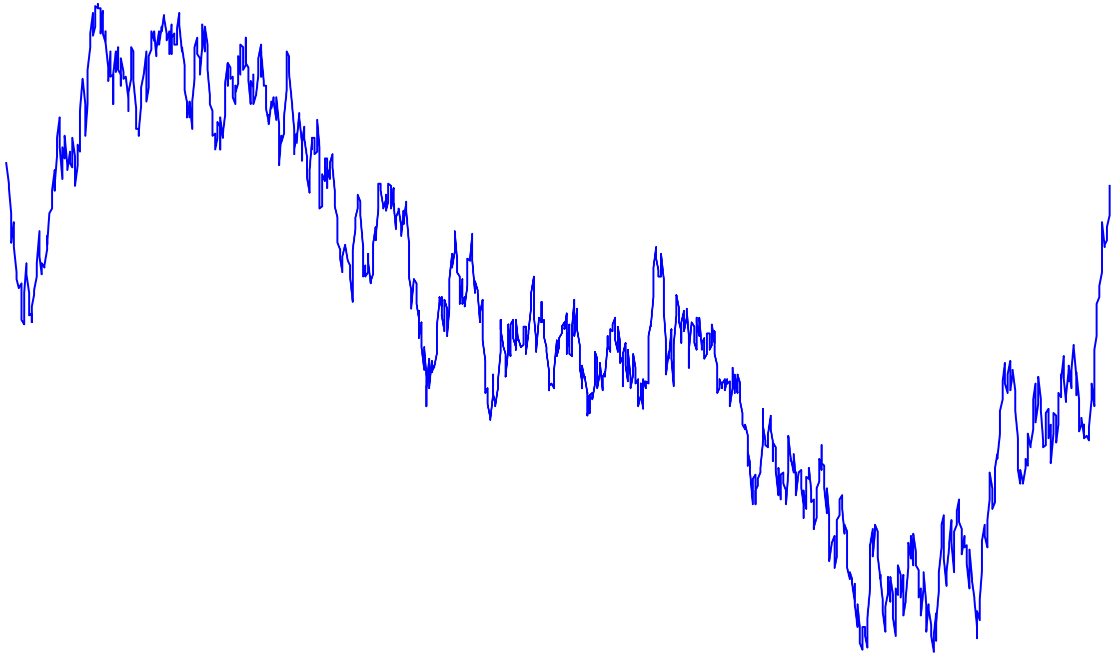
- You can measure many things … and things change over time.
  - Blood pressure
  - Donald Trump’s popularity rating
  - The annual rainfall in Pisa
  - The value of your stocks
- In addition other data type can thought of as time series
  - Text data: words count
  - Images: edges displacement
  - Videos: object positioning





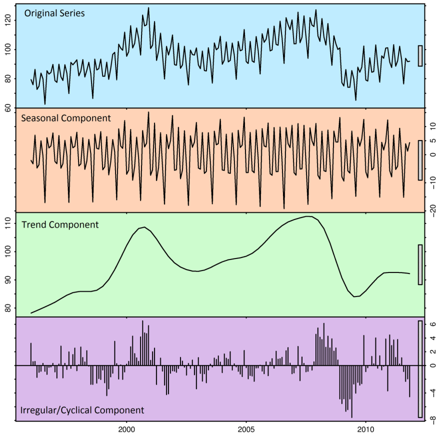
Problems in Working with Time Series

- Large amount of data.
- Similarity is not easy to estimate.
- Different data formats.
- Different sampling rates.
- Noise, missing values, etc.

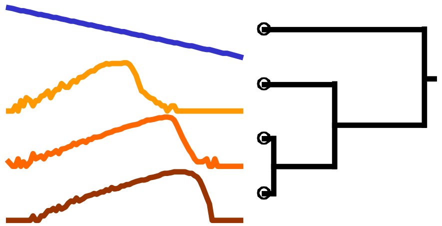


What We Can Do With Time Series?

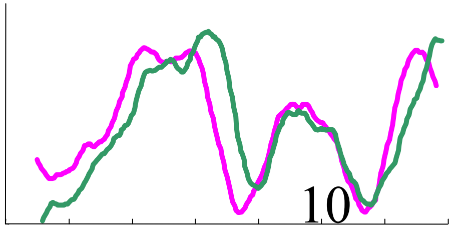
- Trends, Seasonality



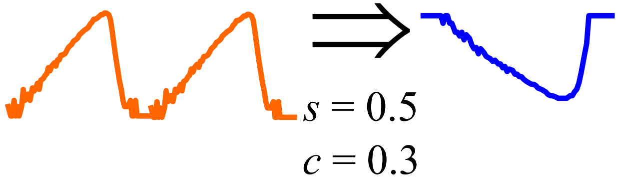
- **Clustering**



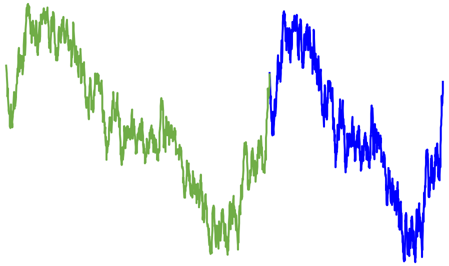
- **Motif Discovery**



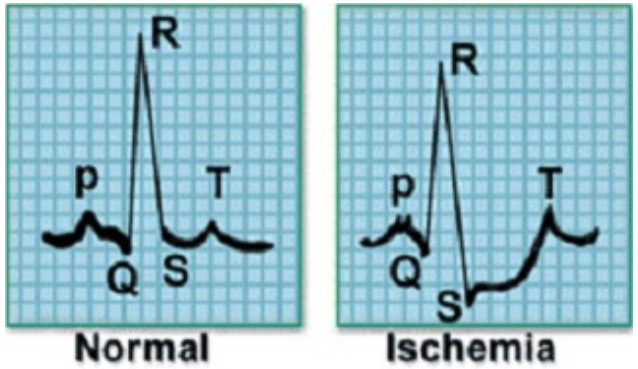
- **Rule Discovery**



- Forecasting



- **Classification**

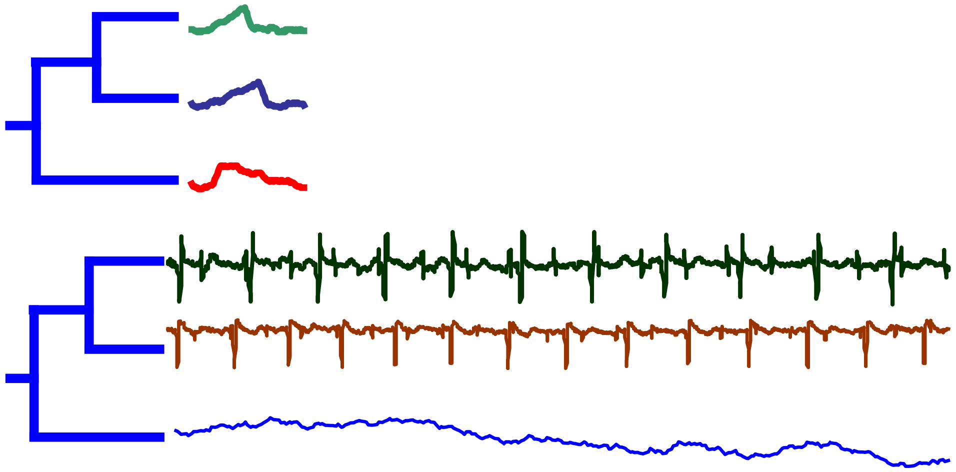


Similarity

- All these problems require similarity matching.

- What is Similarity?
  - It is the quality or state of being similar, likeness, resemblance, as a similarity of features.

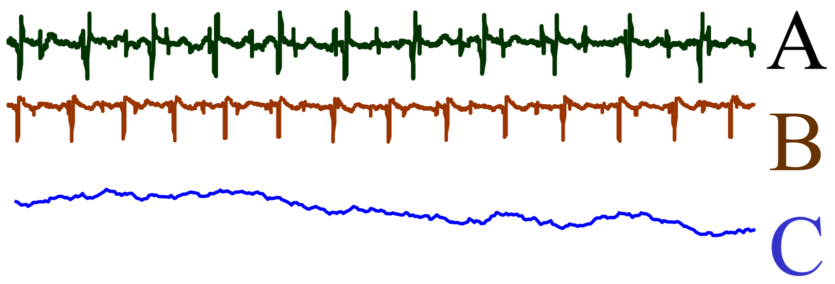
- In time series analysis we recognize two kinds of similarity:
  - Similarity at the level of ***shape***
  - Similarity at the ***structural level***



Structural-based Similarities

Structure or Model Based Similarity

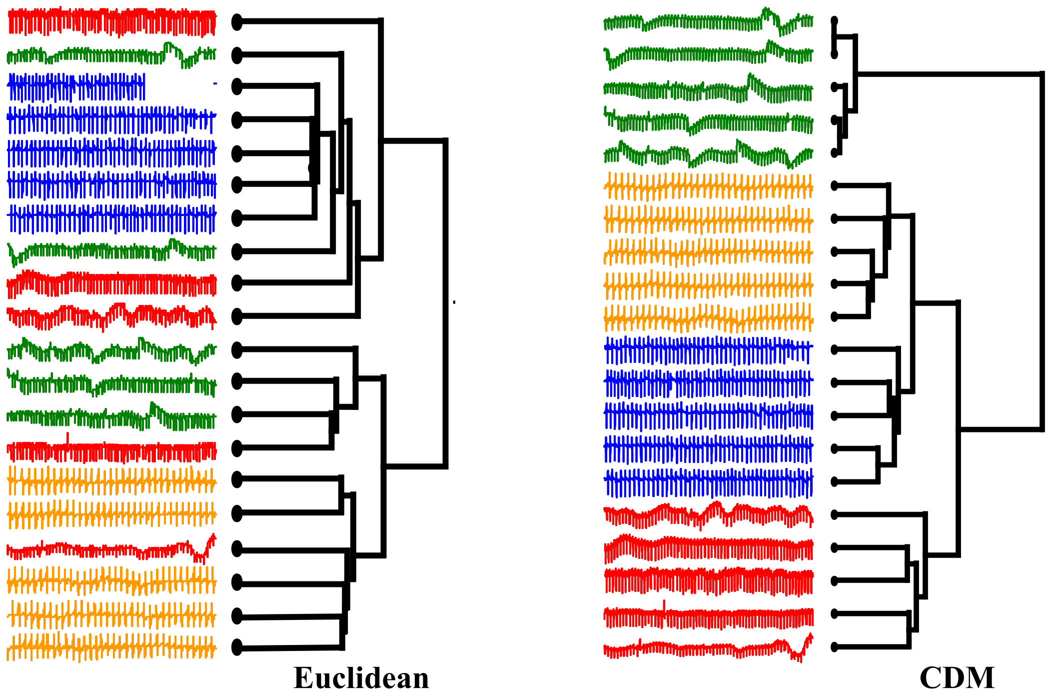
- For long time series, shape based similarity give very poor results.
- We need to measure similarly based on high level structure.
- The basic idea is to:
  1. extract *global* features from the time series,
  2. create a feature vector, and
  3. use it to measure similarity and/or classify
- Example of features:
  - mean, variance, skewness, kurtosis,
  - 1st derivative mean, 1st derivative variance, …
  - parameters of regression, forecasting, Markov model



Feature\Time Series	A	B	C
Max Value	11	12	19
Mean	5.3	6.4	4.8
Min Value	3	2	5
Autocorrelation	0.2	0.3	0.5
…	…	…	…

Compression Based Dissimilarity

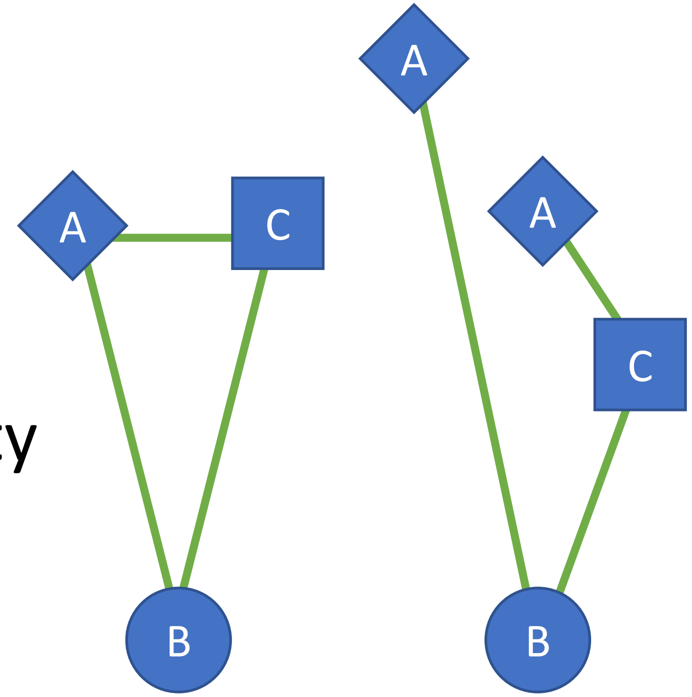
- Use as features whatever structure a given compression algorithm finds.
- $d(x, y) = CDM(x, y) = \frac{C(x,y)}{C(x)+C(y)}$



Shape-based Similarities

Defining Distance Measures

- Let A and B be two objects from the universe of possible objects. The distance (dissimilarity) is denoted by D(A,B).
- Properties in a distance measure.
  - D(A,B) = D(B,A) Symmetry
  - D(A,A) = 0 Constancy
  - D(A,B) = 0 IIf A = B Positivity
  - D(A,B) ≤ D(A,C) + D(B,C) Triangular Inequality



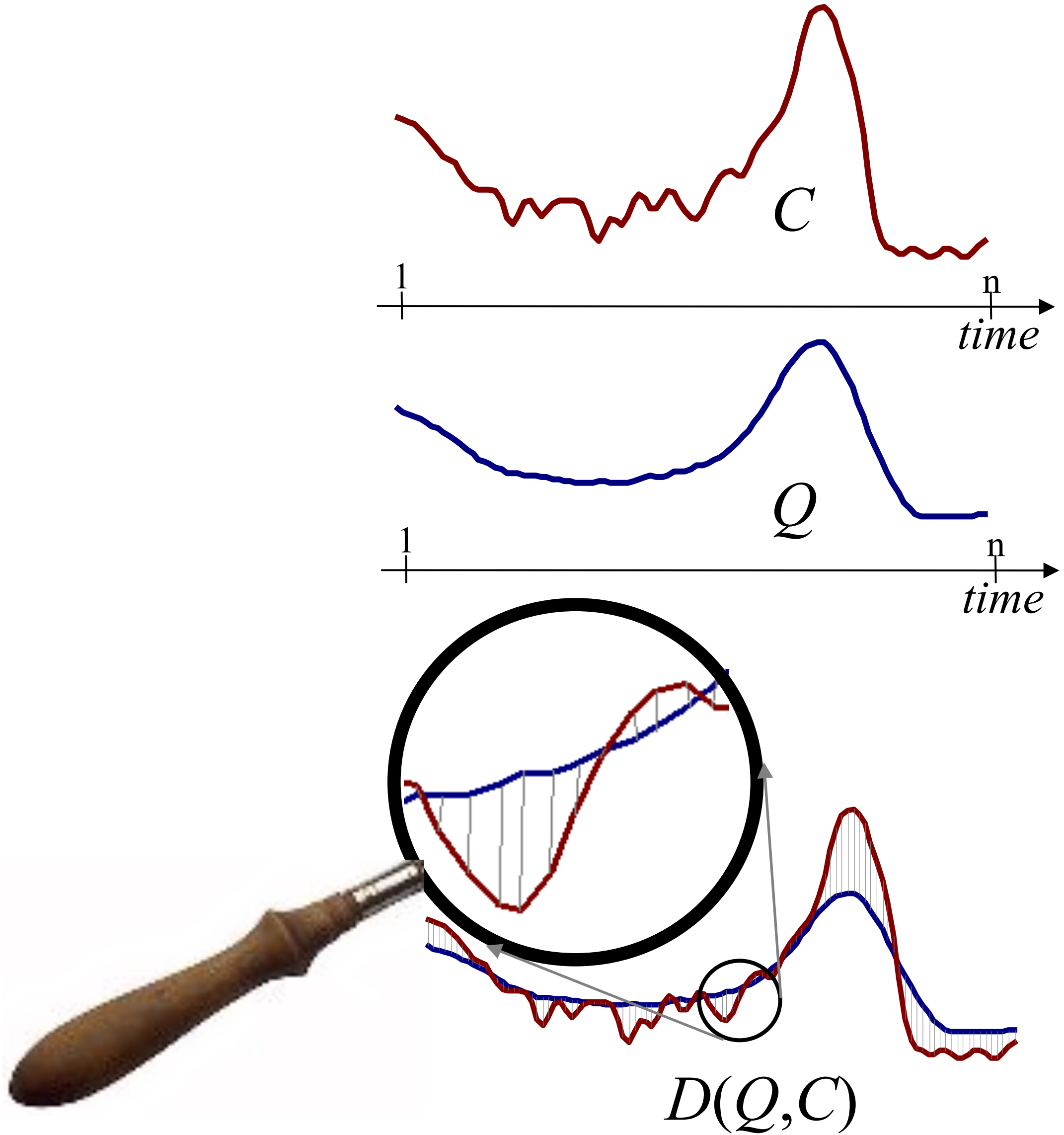
Euclidean Distance

- Given two time series:
  - *Q* = $q_1 \dots q_n$
  - *C* = $c_1 \dots c_n$

$$D(Q,C) \equiv \sqrt{\sum_{i=1}^{n}(q_i - c_i)^2}$$

- *T1 = < 56, 176, 110, 95 >*
- *T2 = < 36, 126, 180, 80 >*

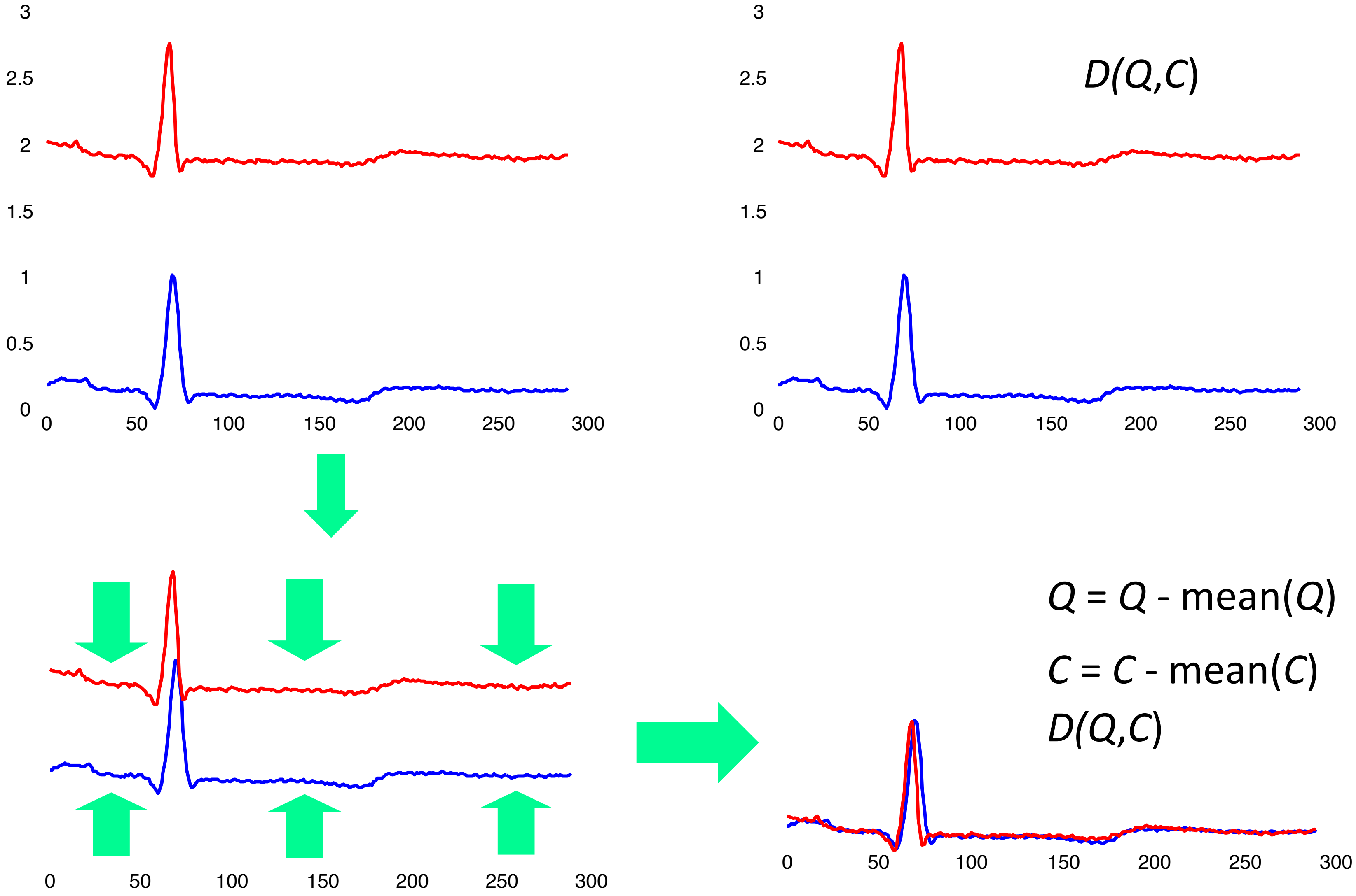
$D(T1,T2) = sqrt\ [\ (56-36)^2 + (176-126)^2 + (110-180)^2 + (95-80)^2\ ]$



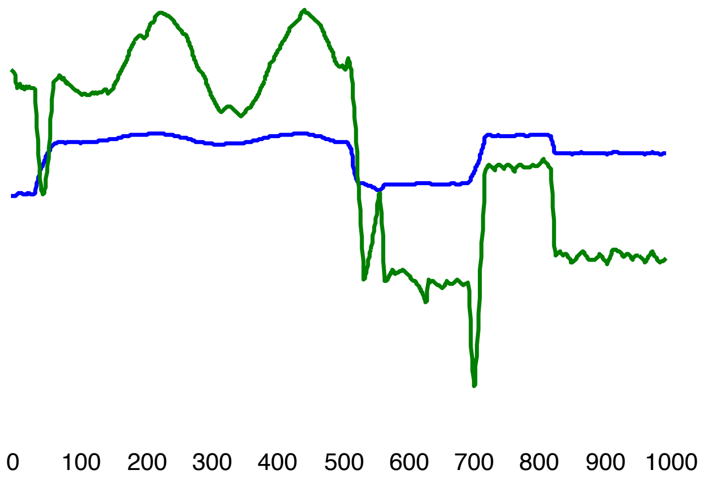
Problems with Euclidean Distance

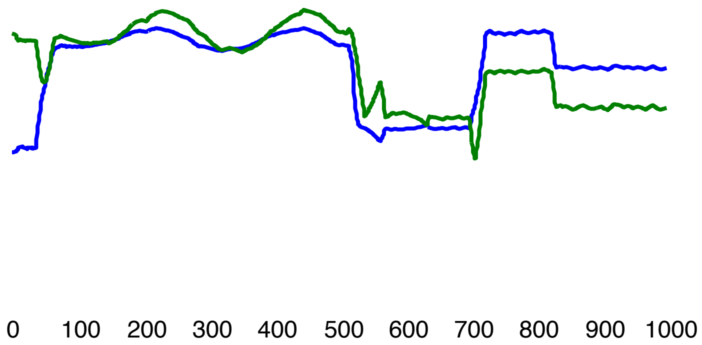
- Euclidean distance is very sensitive to “distortions” in the data.
- These distortions are dangerous and should be removed.
- Most common distortions:
  - Offset Translation
  - Amplitude Scaling
  - Linear Trend
  - Noise
- They can be removed by using the appropriate transformations.

Transformation I: Offset Translation



Transformation II: Amplitude Scaling





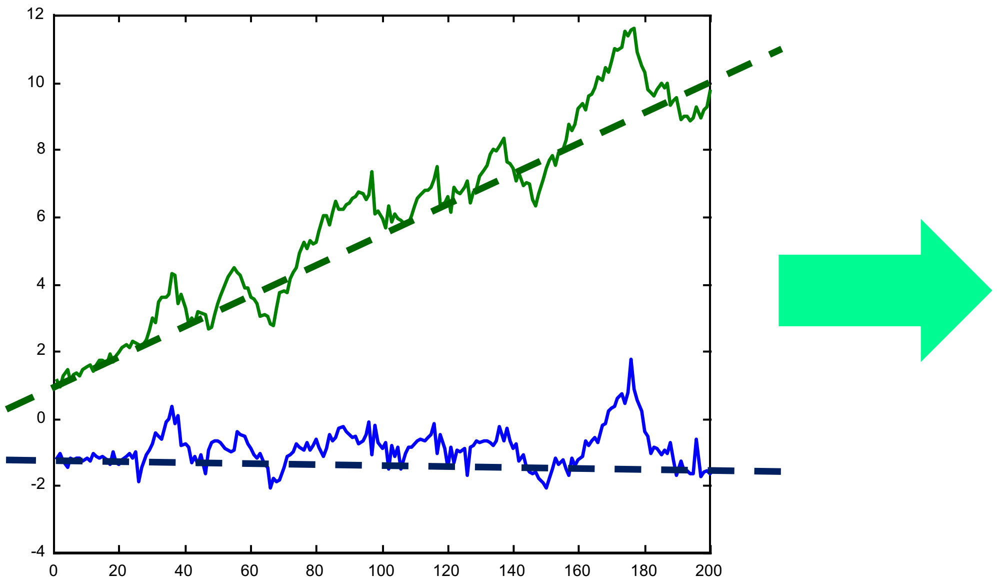
Q = (*Q* - mean(*Q*)) / std(*Q*)

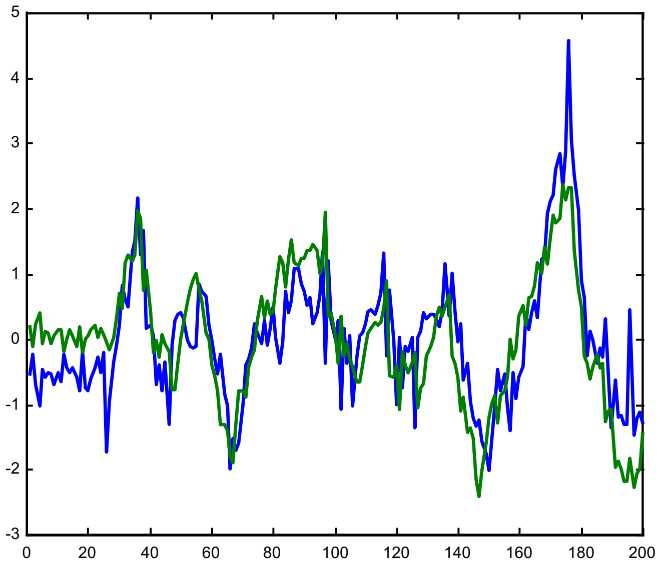
C = (*C* - mean(*C*)) / std(*C*)

D(*Q*,*C*)

Transformation III: Linear Trend

- Removing linear trend: fit the best fitting straight line to the time series, then subtract that line from the time series.

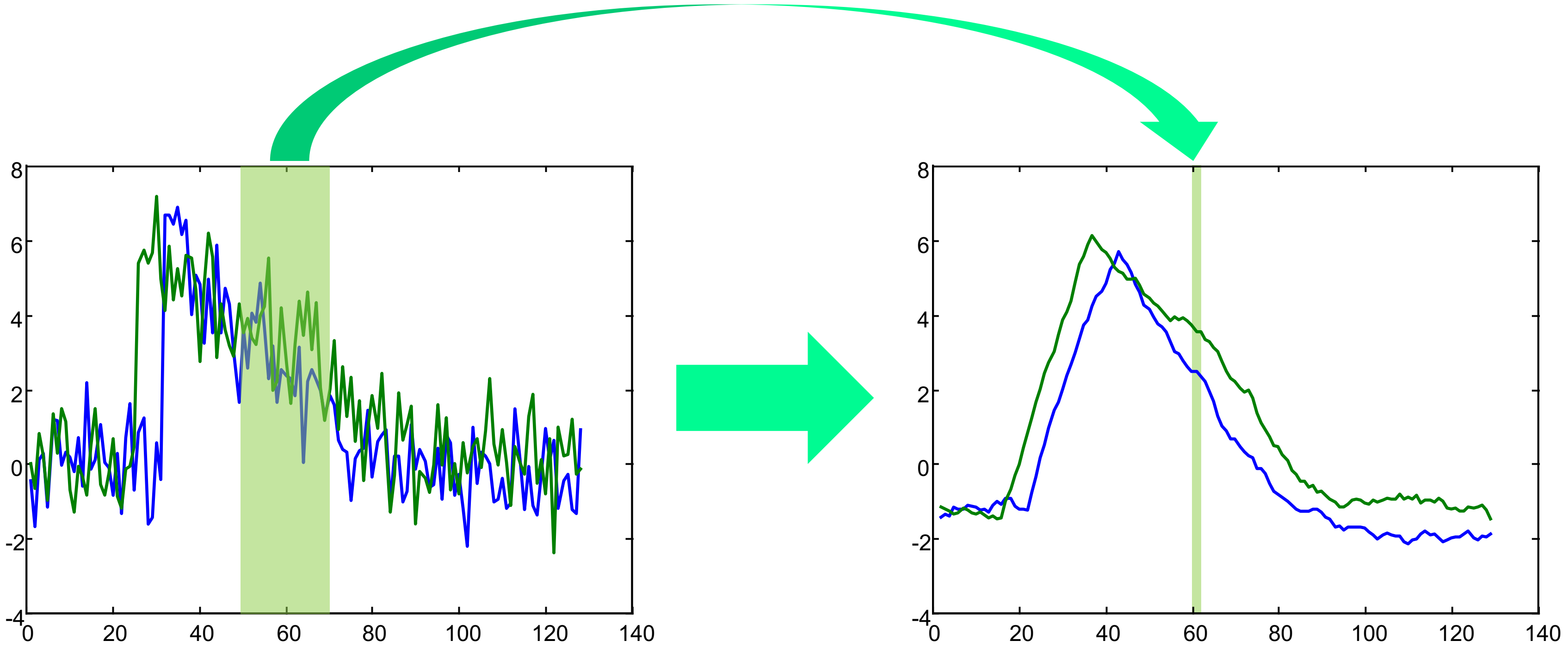




Removed linear trend,
offset translation,
amplitude scaling

Transformation IV: Noise

- The intuition behind removing noise is to average each datapoints value with its neighbors.



Q = smooth(Q)

C = smooth(C)

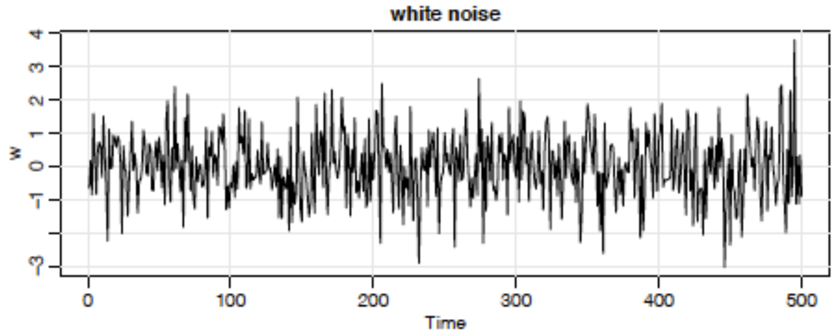
$D(Q,C)$

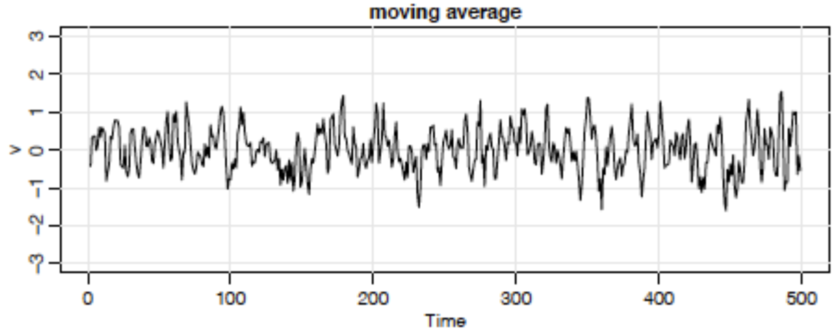
Moving Average

- Noise can be removed by a **moving average** (MA) that smooths the TS.
- Given a window of length *w* and a TS *t*, the MA is applied as follows
- $t_i = \frac{1}{w} \sum_{j=i-w/2}^{w/2} t_j$ for $i = 1, \dots, n$
- For example, if w=3 we have
- $t_i = \frac{1}{3} \left(t_{i-1} + t_i + t_{i+1}\right)$

w=3

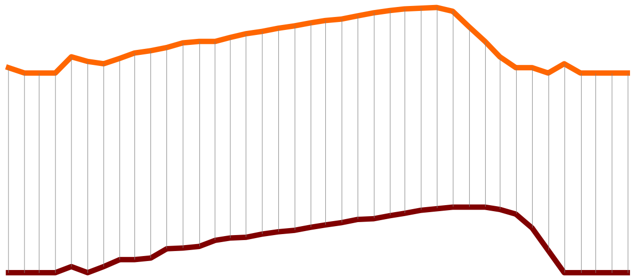
time	value	ma
t1	20	-
t2	24	22.0
t3	22	24.0
t4	26	24.3
t5	25	-



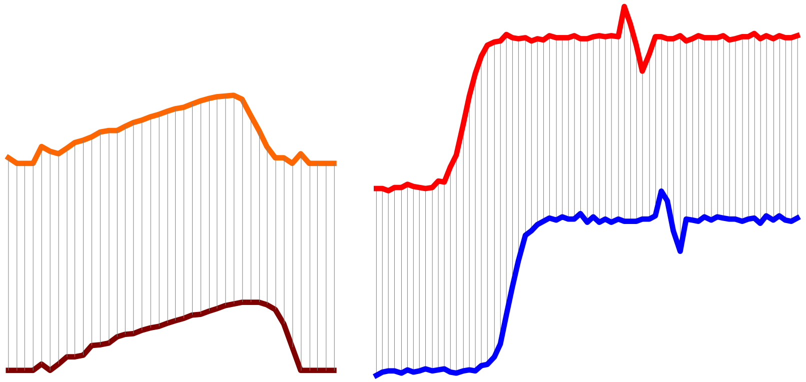


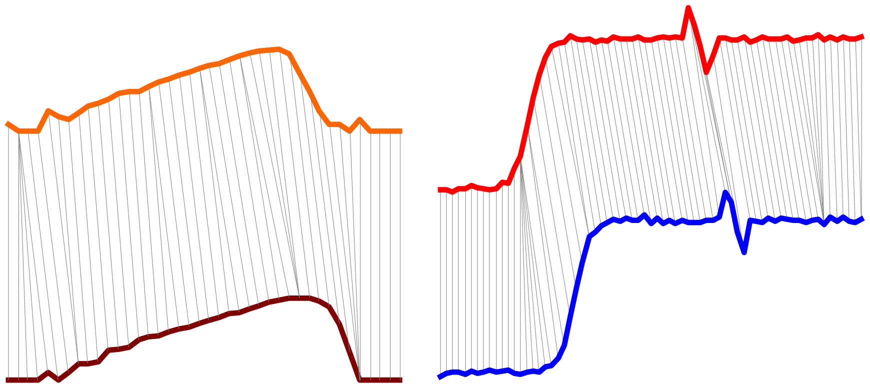
Dynamic Time Warping

- Sometimes two time series that are conceptually equivalent evolve at different speeds, at least in some moments.



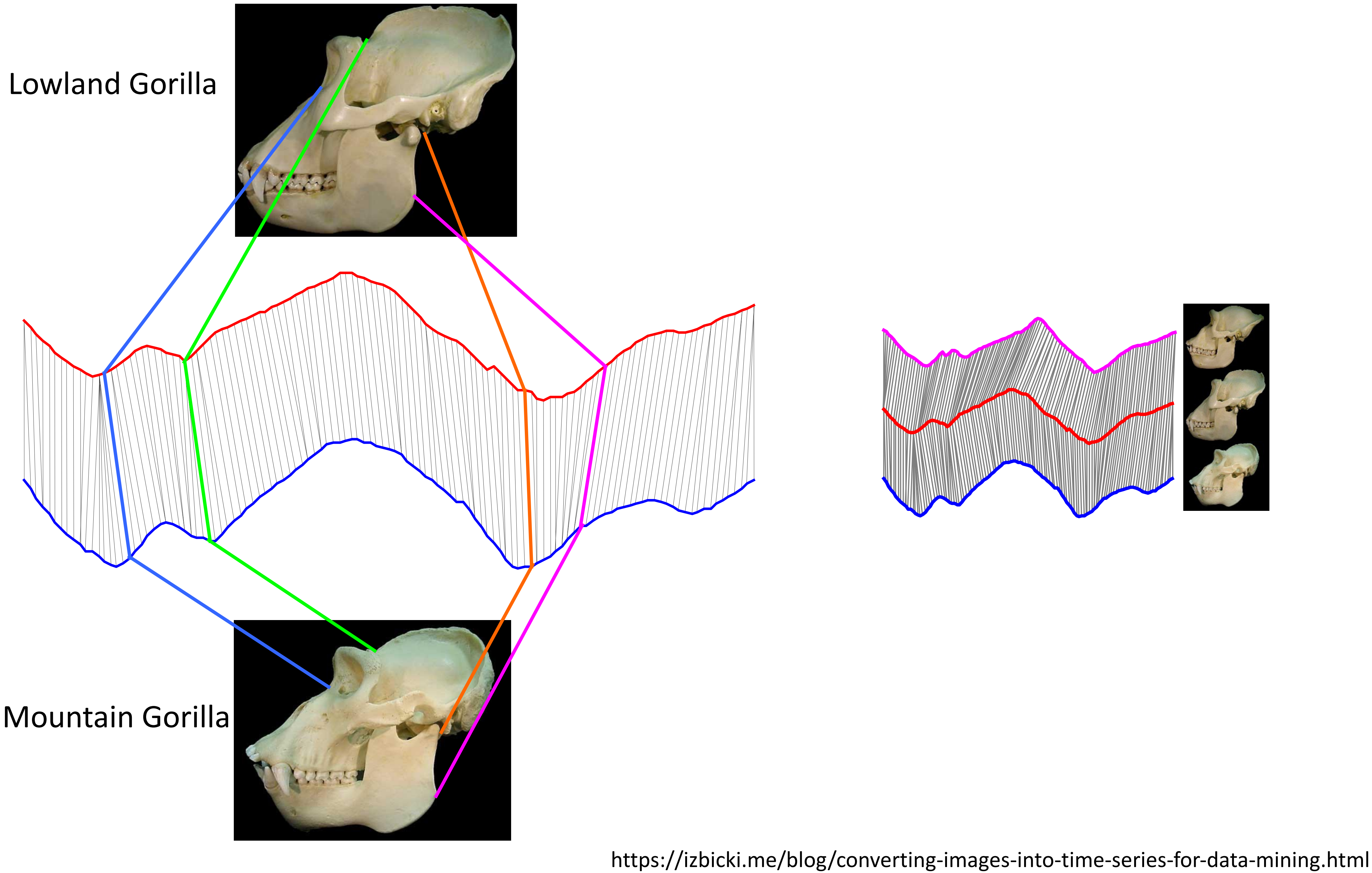
E.g. correspondence of peaks in two similar time series





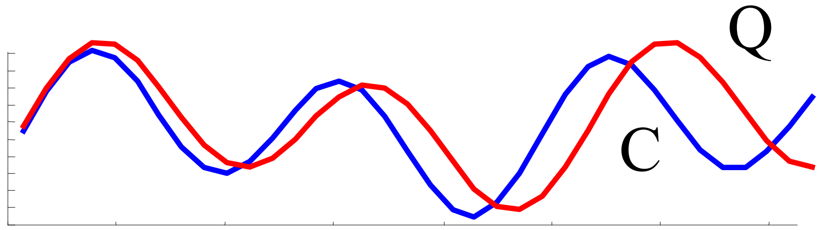
Fixed Time Axis. Sequences are aligned “one to one”. Greatly suffers from the misalignment in data. Euclidean.

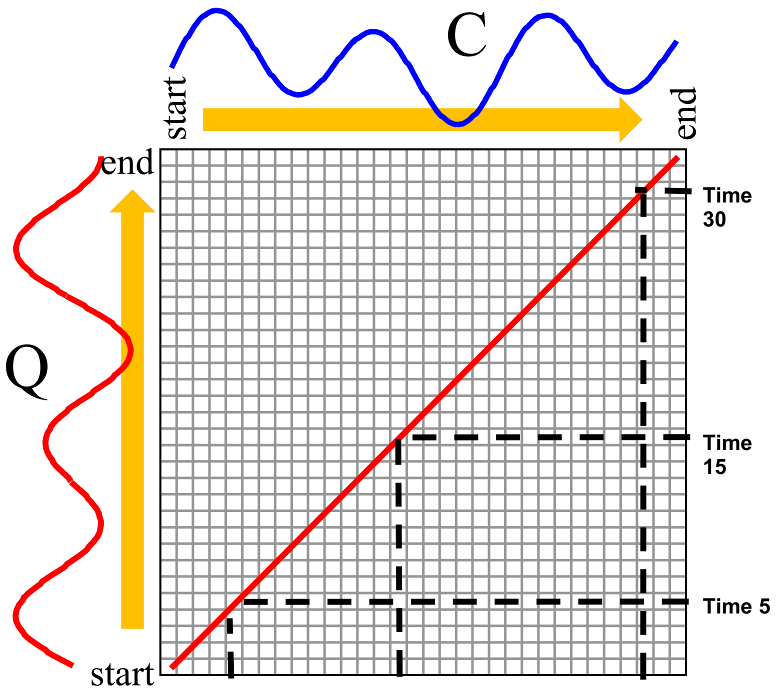
Warped Time Axis. Nonlinear alignments are possible. Can correct misalignments in data. Dynamic Time Warping.


Lowland Gorilla
Mountain Gorilla
https://izbicki.me/blog/converting-images-into-time-series-for-data-mining.html

How is DTW Calculated?

- We create a matrix the size of |Q| by |C|, then fill it in with the distance between every pair of point in our two time series.



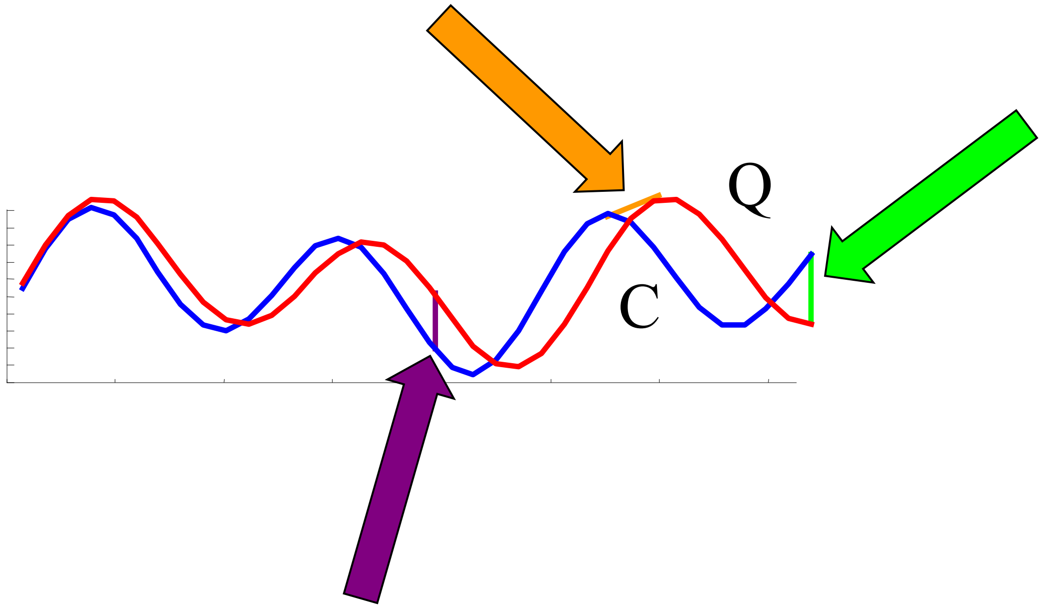


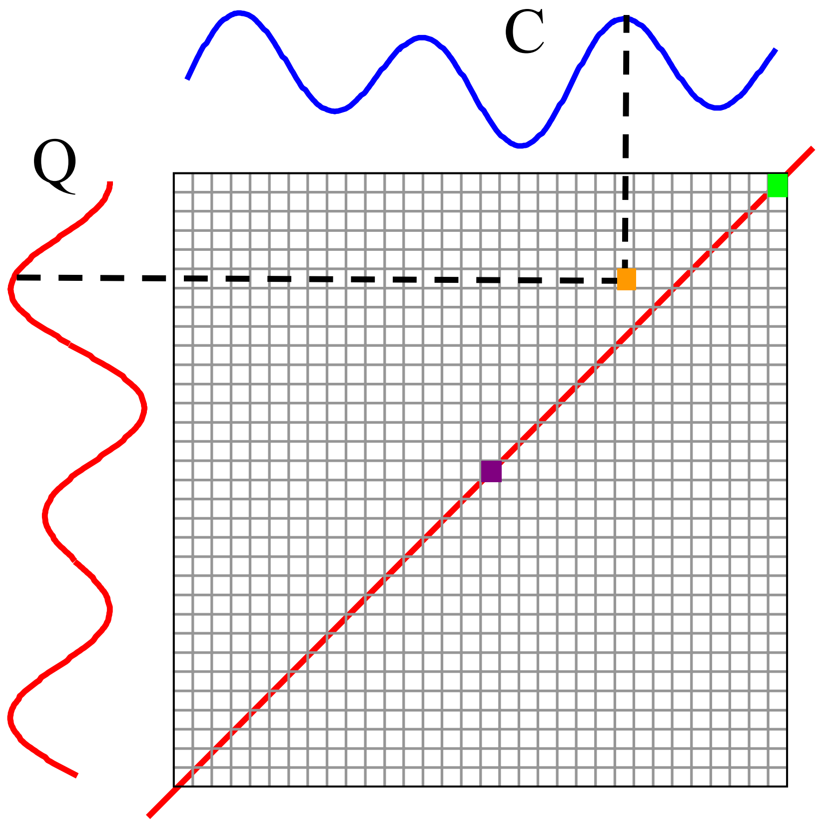
The Euclidean distance works only on the diagonal of the matrix. The sequence of comparisons performed:

- Start from pair of points *(0,0)*
- After point *(i,i)* move to *(i+1,i+1)*
- End the process on *(n,n)*

How is DTW Calculated?

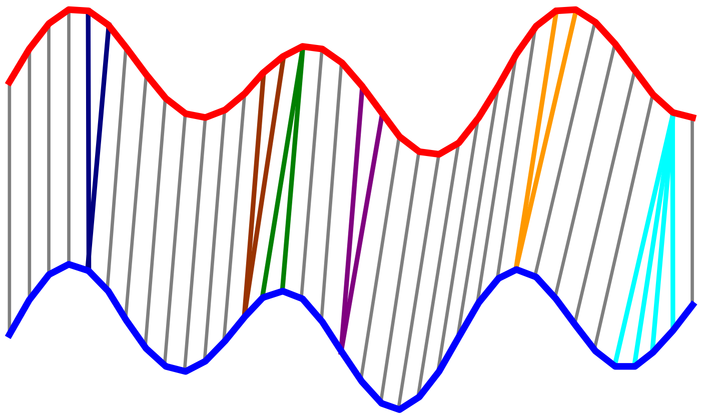
- The DTW distance can “freely” move outside the diagonal of the matrix
- Such cells correspond to temporally shifted points in the two time series

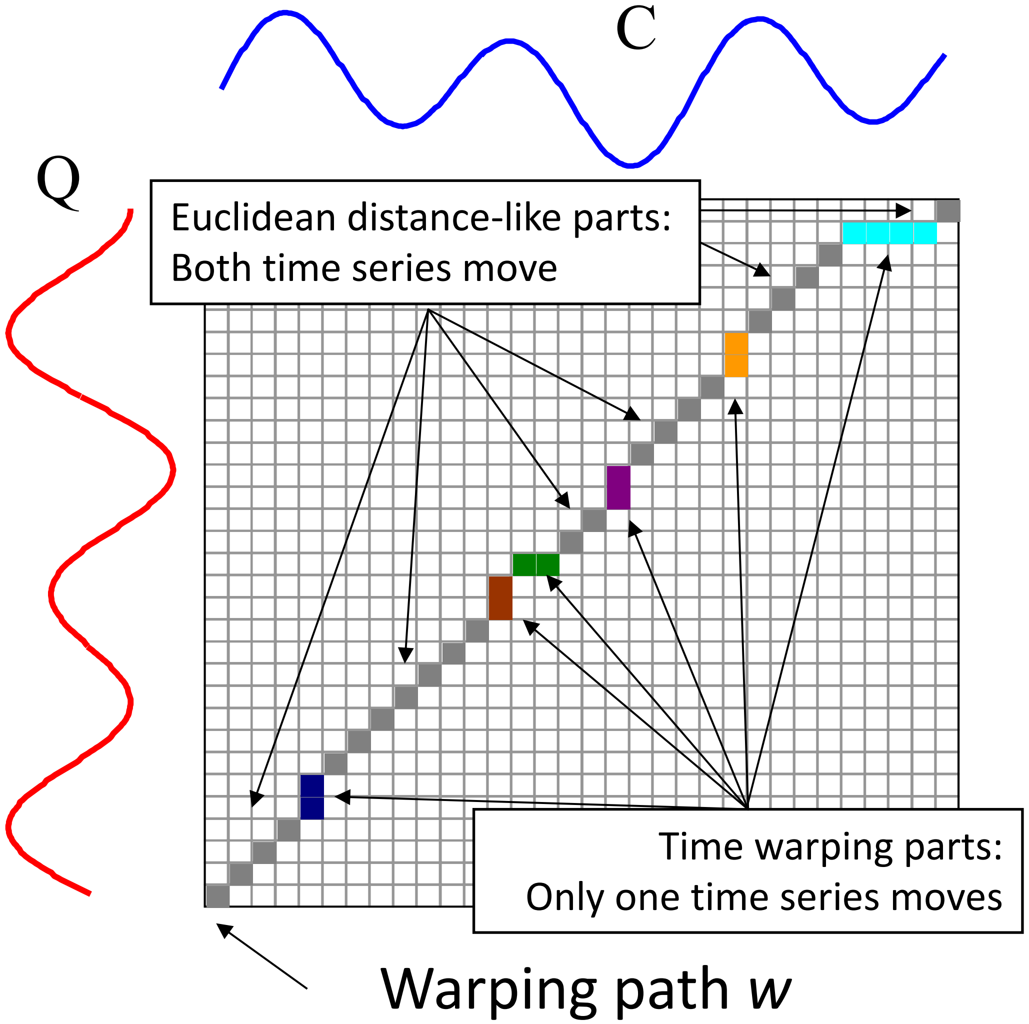




How is DTW Calculated?

- Every possible warping between two time series, is a path through the matrix.
- The constrained sequence of comparisons performed:
  - Start from pair of points *(0,0)*
  - After point *(i,j)*, either *i* or *j* increase by one, or both of them
  - End the process on *(n,n)*





How is DTW Calculated?

- Every possible warping between two time series, is a path through the matrix.
- We find the best one using a recursive definition of the DTW:

$\gamma(i,j)$ = cost of best path reaching cell (i,j)

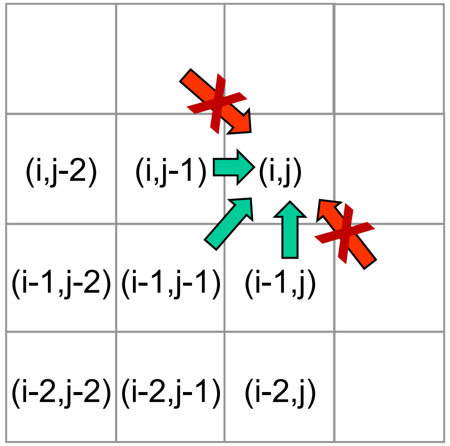
$= d(q_i,c_j) + \min\{ \gamma(i-1,j-1), \gamma(i-1,j), \gamma(i,j-1) \}$

- Idea: best path must pass through (i-1,j), (i-1,j-1) or (i,j-1)

$$DTW(Q,C) = \min\left\{ \sqrt{\sum_{k=1}^{K} w_k} \Big/ K \right.$$

w_k = cost of the k-th points comparison

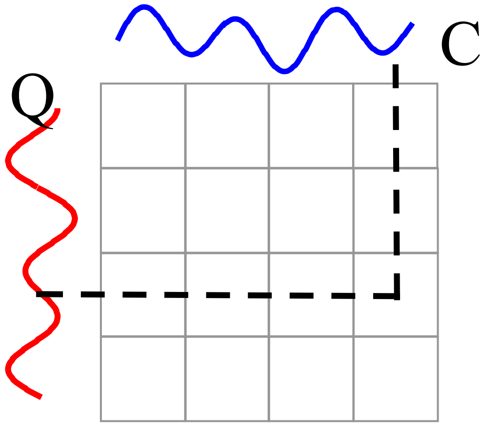
- $w_k = | Q_i - C_j |$
- $w_k = (Q_i - C_j) \wedge 2$



Dynamic Programming Approach

Step 1: compute the matrix of all $d(q_i,c_j)$

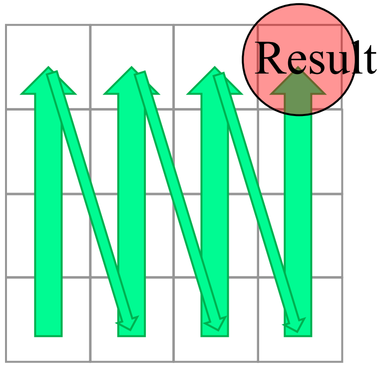
- Point-to-point distances $D(i,j) = | Q_i - C_j |$

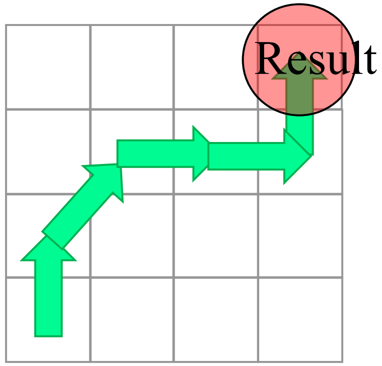


$$\gamma(i,j) = d(q_i,c_j) + \min\{ \gamma(i-1,j-1), \gamma(i-1,j), \gamma(i,j-1) \}$$

Step 2: compute the matrix of all path costs $\gamma(i,j)$

- Start from cell *(1,1)*
- Compute *(2,1), (3,1), …, (n,1)*
- Repeat for columns *2, 3, …, n*
- Final result in last cell computed





Step 3: find the path with the lowest value (best alignment)

Dynamic Programming Approach

$\gamma(i,j) \; = \; d(q_i,c_j) + \min\{\, \gamma(i\text{-}1,j\text{-}1),\ \gamma(i\text{-}1,j\,),\ \gamma(i,j\text{-}1)\,\}$

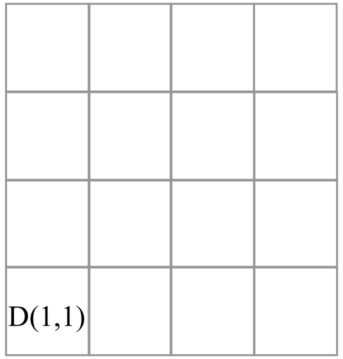
Step 2: compute the matrix of all path costs $\gamma(i,j)$

- Start from cell *(1,1)*

$\gamma(1,1) = d(q_1,c_1) + \min\{\gamma(0,0), \gamma(0,1), \gamma(1,0)\}$

$= d(q_1,c_1)$

$= D(1,1)$

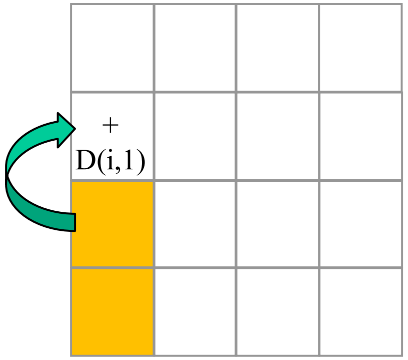


- Compute (2,1), (3,1), …, (n,1)

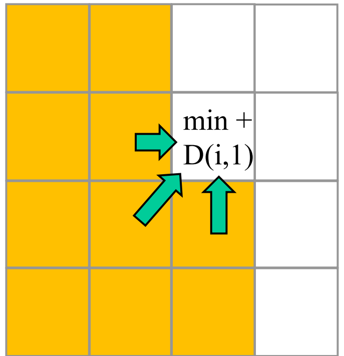
$\gamma(i,1) = d(q_i,c_1) + \min\{\gamma(i\text{-}1,0), \gamma(i\text{-}1,1), \gamma(i,0)\}$

$= d(q_i,c_1) + \gamma(i\text{-}1,1)$

$= D(i,1) + \gamma(i\text{-}1,1)$



- Repeat for columns *2, 3, …, n*
  - The general formula applies

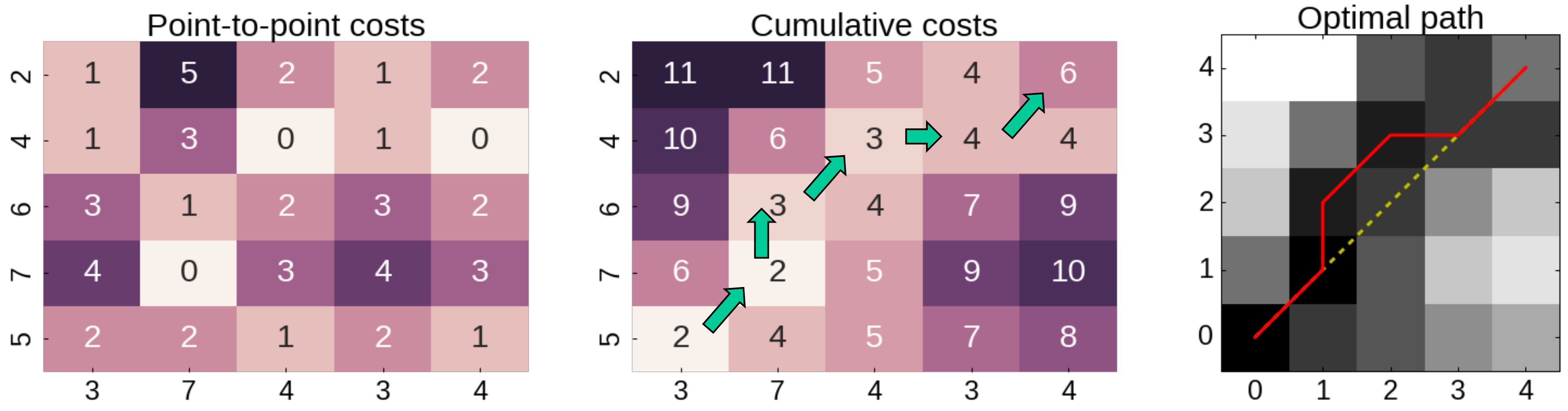


Dynamic Programming Approach

Example

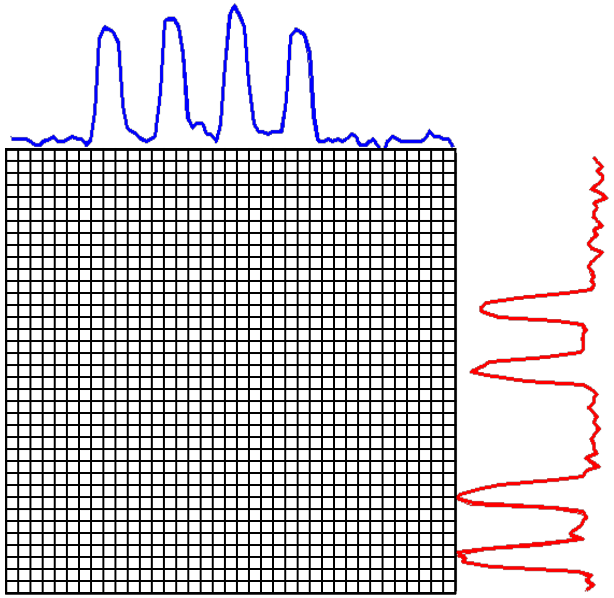
- c = < 3, 7, 4, 3, 4 >
- q = < 5, 7, 6, 4, 2 >

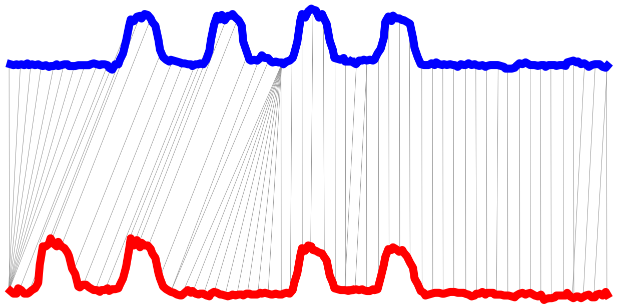
$$\gamma(i,j) \;=\; d(q_i,c_j) + \min\{\, \gamma(i\text{-}1,j\text{-}1),\, \gamma(i\text{-}1,j\,),\, \gamma(i,j\text{-}1) \,\}$$

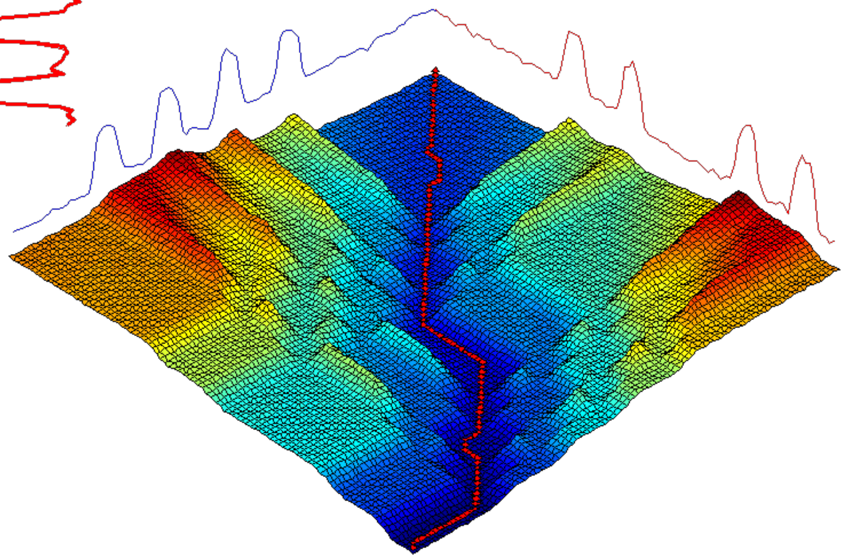


Dynamic Time Warping – A Real Example

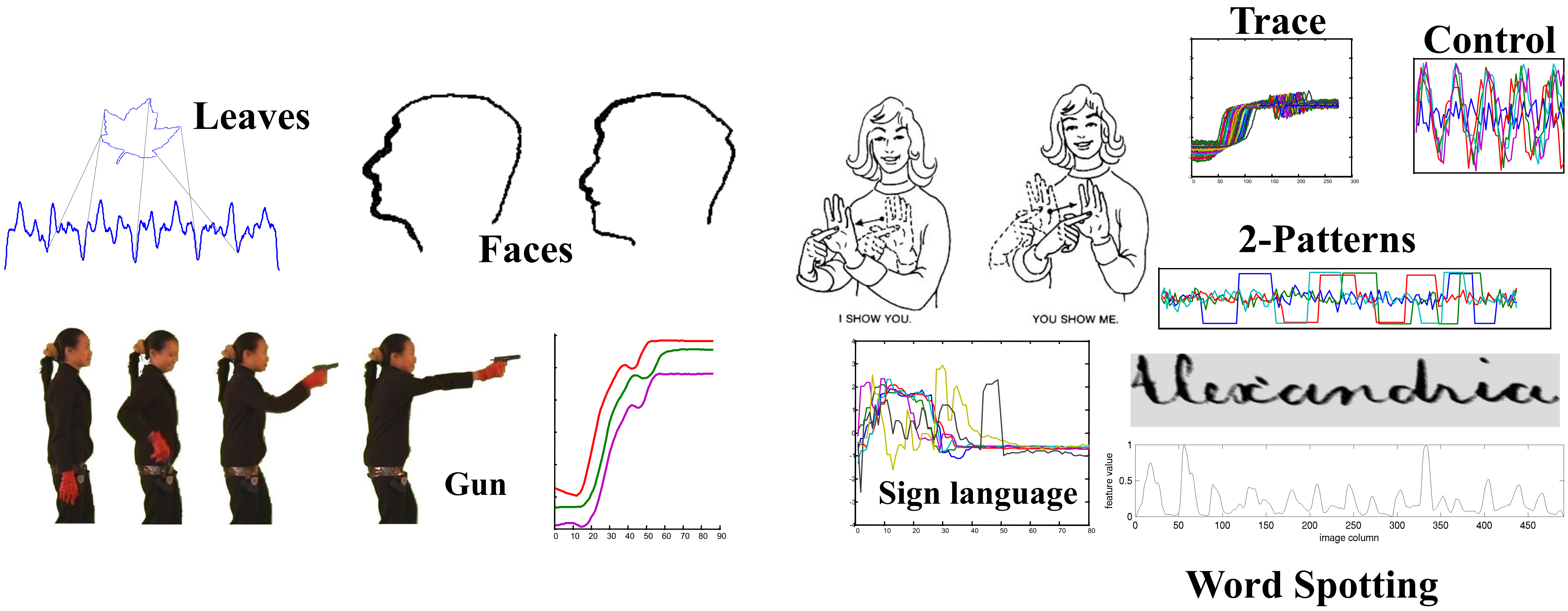
- A Real Example
- This example shows 2 one-week periods from the power demand time series.
- Note that although they both describe 4-day work weeks, the blue sequence had Monday as a holiday, and the red sequence had Wednesday as a holiday.







Comparison of Euclidean Distance and DTW



Comparison of Euclidean Distance and DTW

- Classification using 1-NN
- Class(x) = class of most similar training object
- Leaving-one-out evaluation
- For each object: use it as test set, return overall average

Error Rate

Dataset	Euclidean	DTW
Word Spotting	4.78	1.10
Sign language	28.70	25.93
GUN	5.50	1.00
Nuclear Trace	11.00	0.00
Leaves#	33.26	4.07
(4) Faces	6.25	2.68
Control Chart*	7.5	0.33
2-Patterns	1.04	0.00

Comparison of Euclidean Distance and DTW

- Classification using 1-NN
- Class(x) = class of most similar training object
- Leaving-one-out evaluation
- For each object: use it as test set, return overall average
- DTW is two to three orders of magnitude slower than Euclidean distance.

Milliseconds

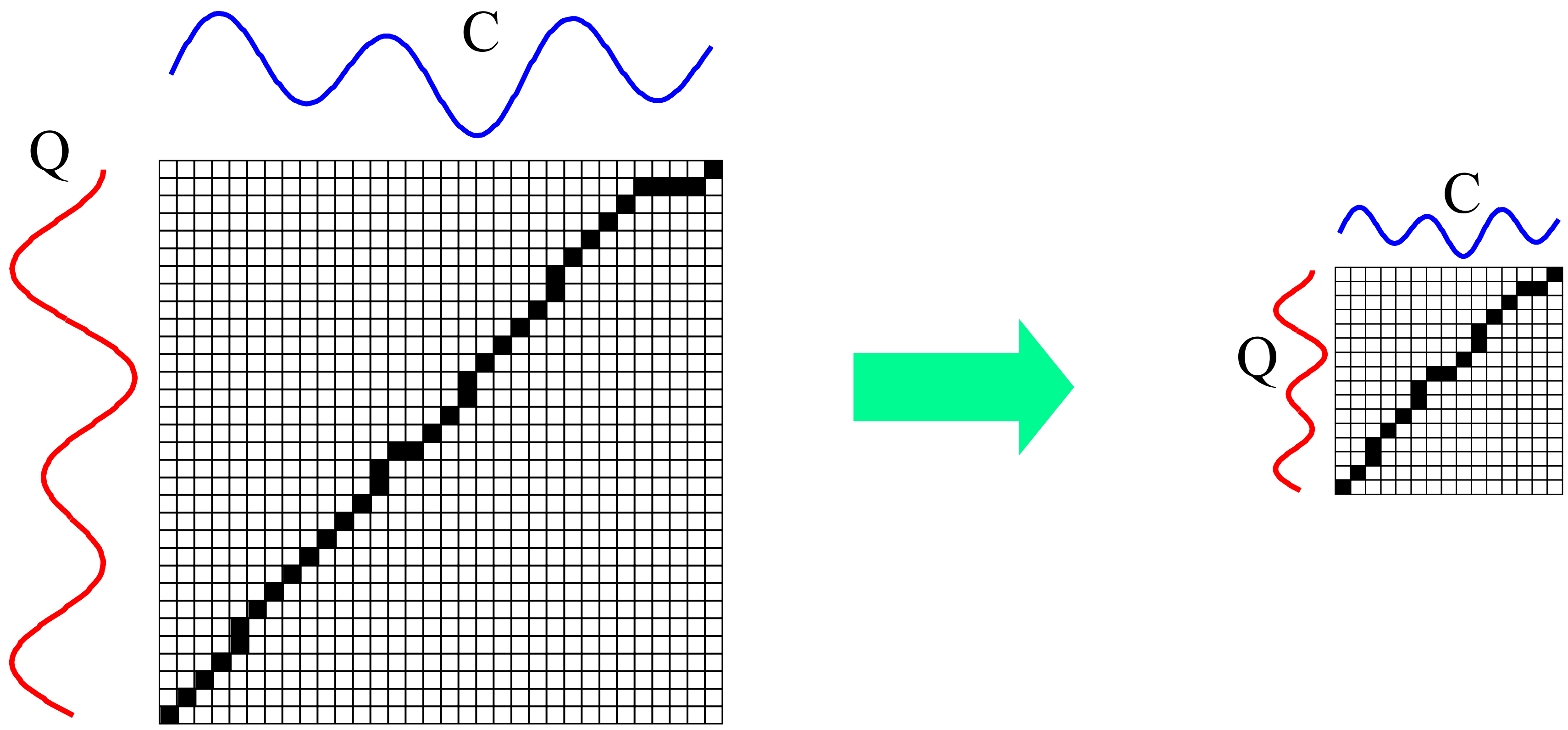
Dataset	Euclidean	DTW
Word Spotting	40	8,600
Sign language	10	1,110
GUN	60	11,820
Nuclear Trace	210	144,470
Leaves	150	51,830
(4) Faces	50	45,080
Control Chart	110	21,900
2-Patterns	16,890	545,123

What we have seen so far…

- Dynamic Time Warping gives much better results than Euclidean distance on many problems.
- Dynamic Time Warping is very very slow to calculate!
- Is there anything we can do to speed up similarity search under DTW?

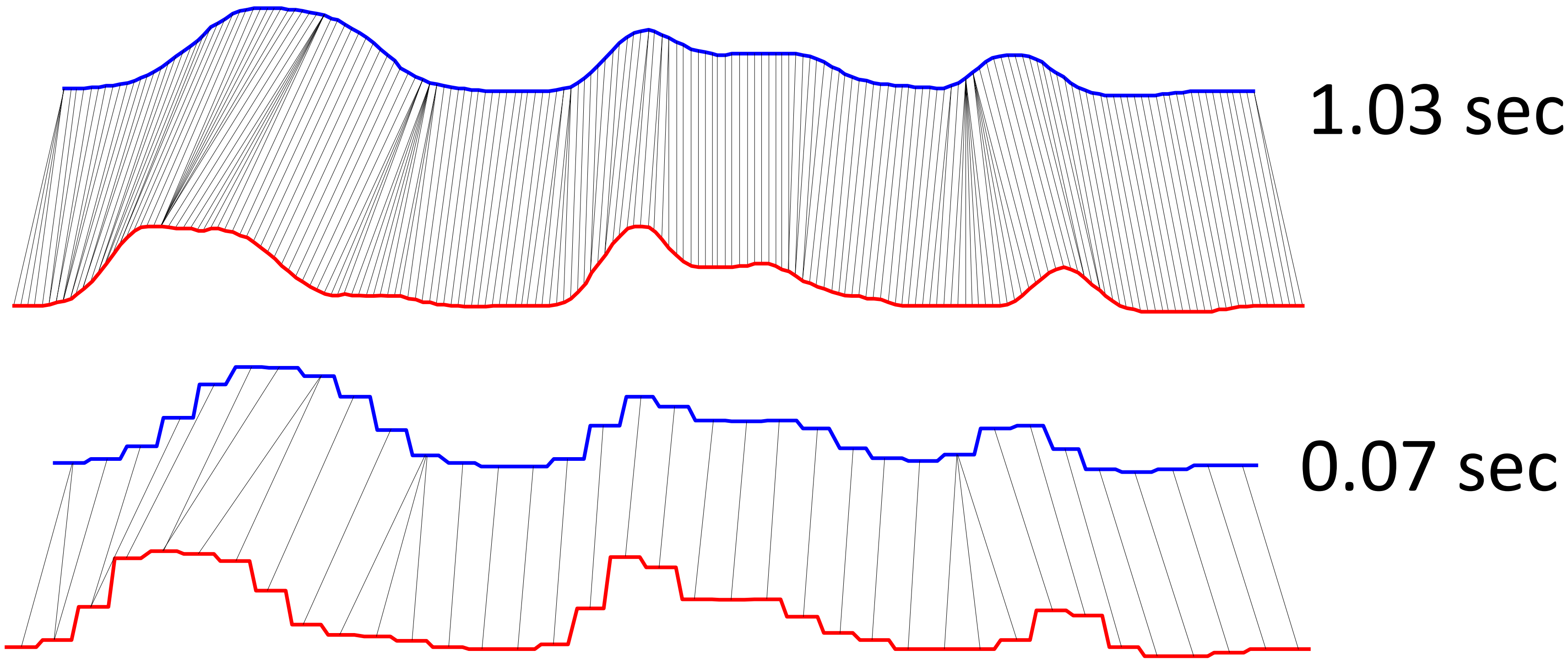
Fast Approximations to DTW

- Approximate the time series with some compressed or downsampled representation, and do DTW on the new representation.



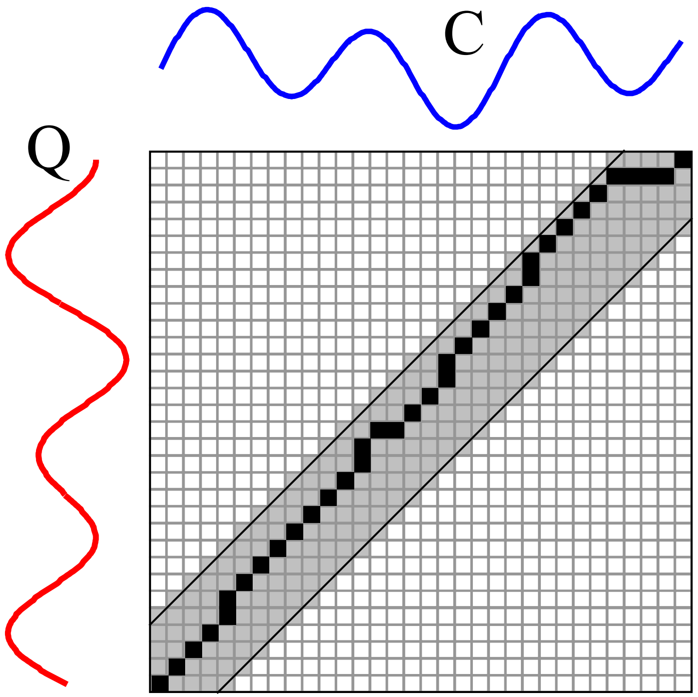
Fast Approximations to DTW

- There is strong visual evidence to suggests it works well
- In the literature there is good experimental evidence for the utility of the approach on clustering, classification, etc.

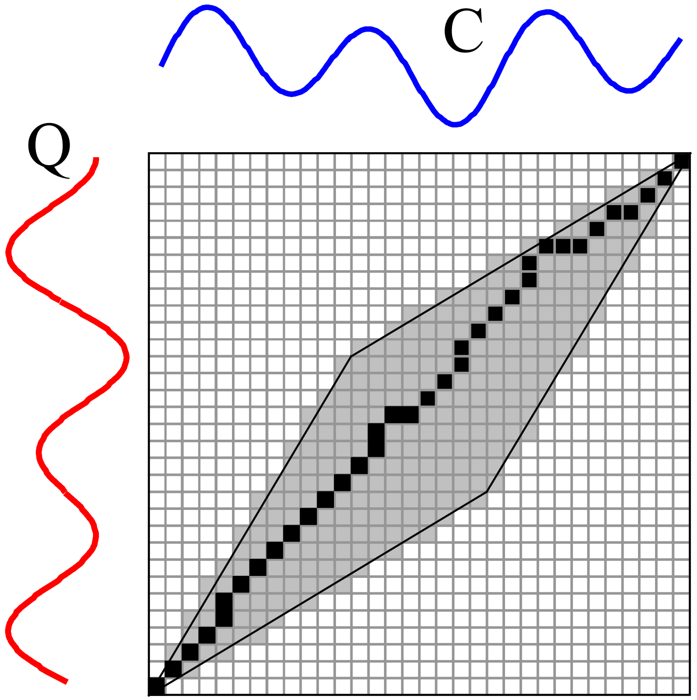


Global Constraints

- Slightly speed up the calculations
- Prevent pathological warpings



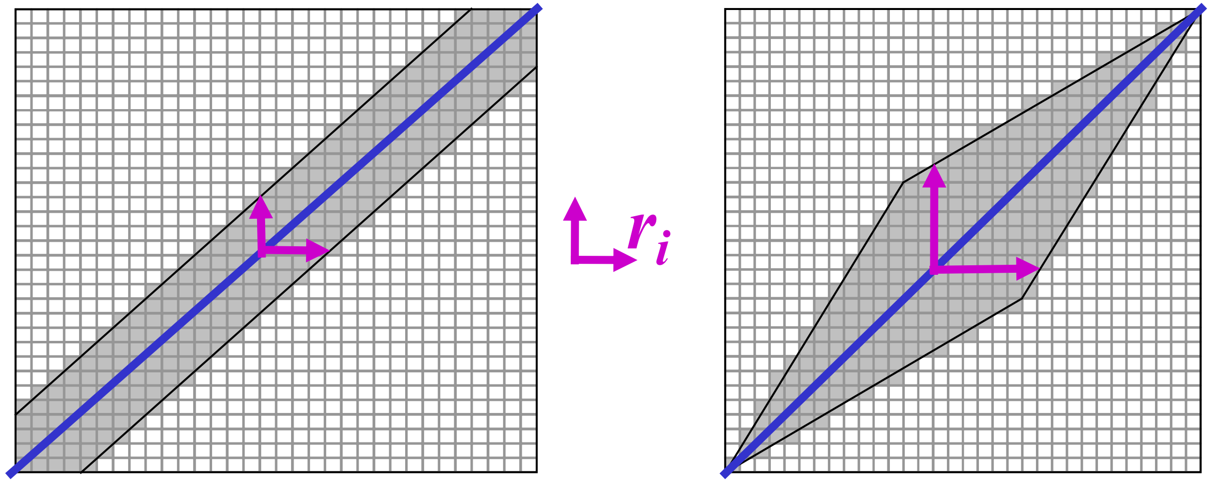
Sakoe-Chiba Band



Itakura Parallelogram

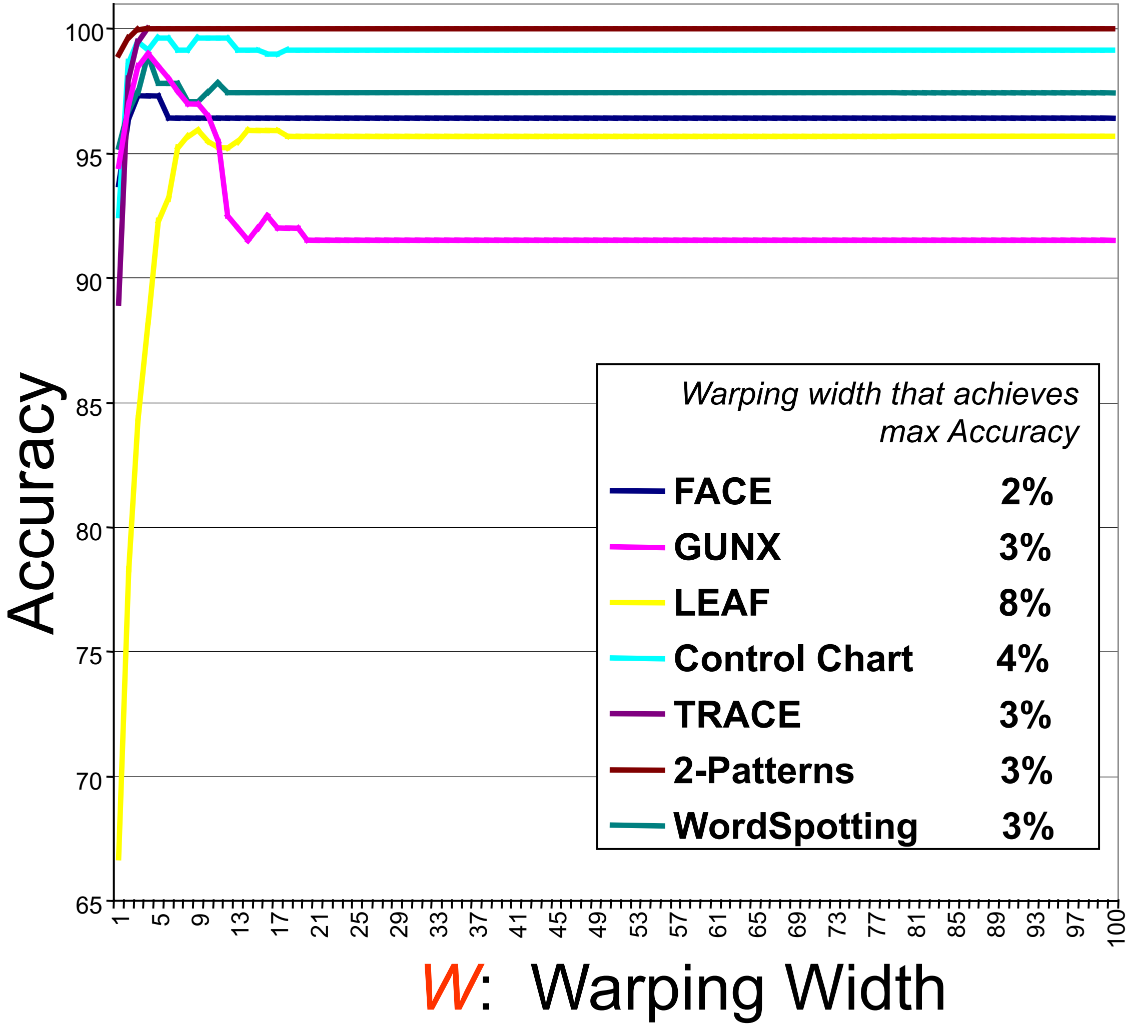
Global Constraints

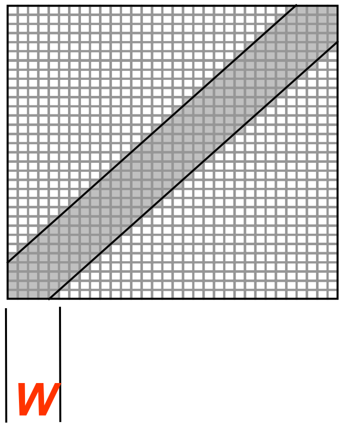
- A global constraint constrains the indices of the warping path w_k = $(i,j)_k$ such that $j\text{-}r \leq i \leq j\text{+}r$, where r is a term defining allowed range of warping for a given point in a sequence.
- r can be considered as a *window* that reduces the number of calculus.



Sakoe-Chiba Band   Itakura Parallelogram

Accuracy vs. Width of Warping Window





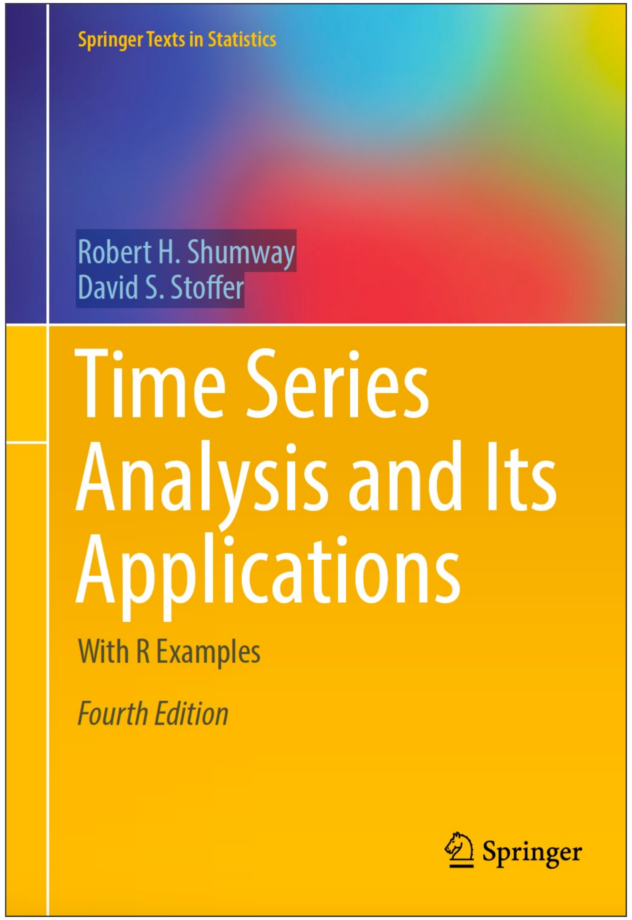
Summary of Time Series Similarity

- If you have short time series
  - use DTW after searching over the warping window size

- If you have long time series
  - if you do know something about your data =>
    extract features
  - and you know nothing about your data =>
    try compression/approximation based dissimilarity

References

- Forecasting: Principles and Practic. Rob J Hyndman and George Athanasaopoulus. (https://otexts.com/fpp2/)
- Time Series Analysis and Its Applications. Robert H. Shumway and David S. Stoffer. 4th edition.(https://www.stat.pitt.edu/stoffer/tsa4/tsa4.pdf)
- Mining Time Series Data. Chotirat Ann Ratanamahatana et al. 2010. (https://www.researchgate.net/publication/227001229_Mining_Time_Series_Data)
- Dynamic Programming Algorithm Optimization for Spoken Word Recognition. Hiroaki Sakode et al. 1978.
- Experiencing SAX: a Novel Symbolic Representation of Time Series. Jessica Line et al. 2009
- Compression-based data mining of sequential data. Eamonn Keogh et al. 2007.



Exercises Dynamic Time Warping

DTW – Exercise 1

- Given the following input time series:

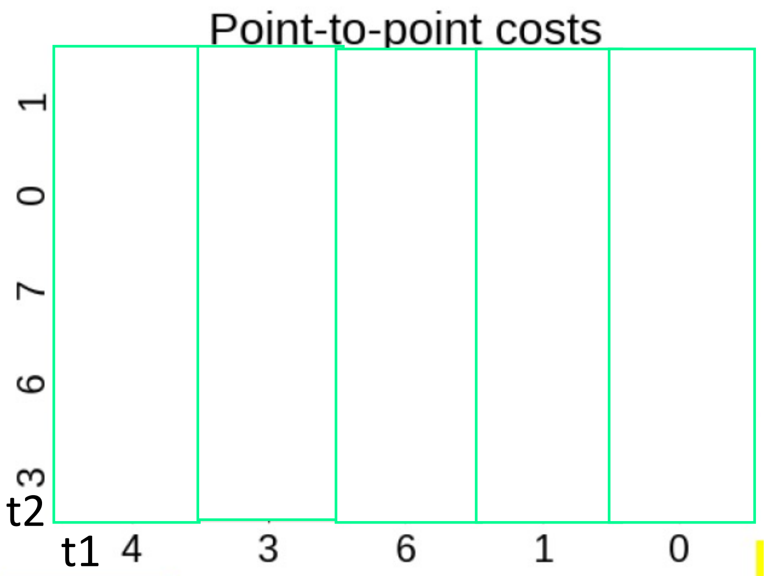
t1	< 4, 3, 6, 1, 0 >
t2	< 3, 6, 7, 0, 1 >

- A) Compute the distance between “t1” and “t2”, using the DTW with distance between points computed as d(x,y) = |x – y|.
- B) If we repeat the computation of point (A) above, this time with a Sakoe-Chiba band of size r=1, does the result change? Why?
- C) If we compute DTW(T1,T2), where T1 is equal to t1 in reverse order (namely T1=<0,1,6,3,4>) and similarly for T2 (namely T2=<1,0,7,6,3>), is it true that DTW(T1,T2) = DTW(t1,t2)? Discuss the problem without providing any computation.

t1	< 4, 3, 6, 1, 0 >
t2	< 3, 6, 7, 0, 1 >

DTW – Exercise 1 - Solution

- A)

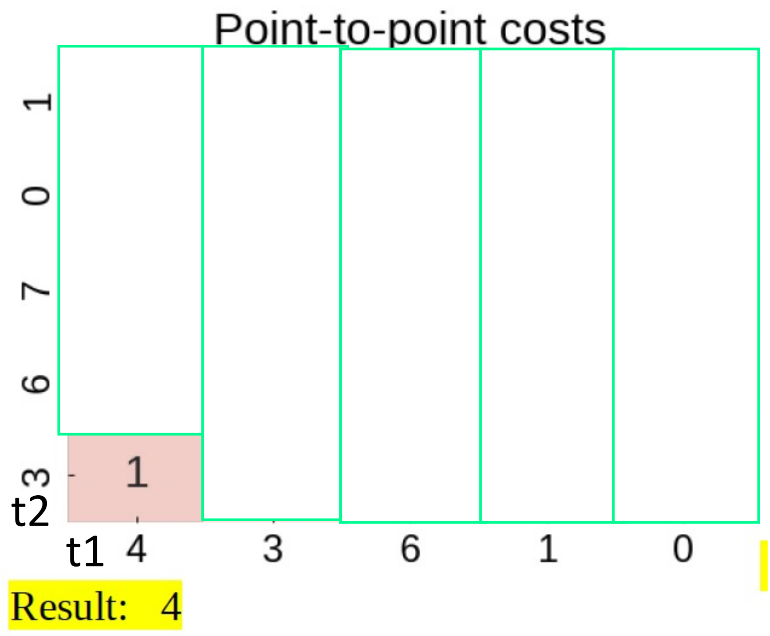


Result: 4

DTW – Exercise 1 - Solution

t1	< 4, 3, 6, 1, 0 >
t2	< 3, 6, 7, 0, 1 >

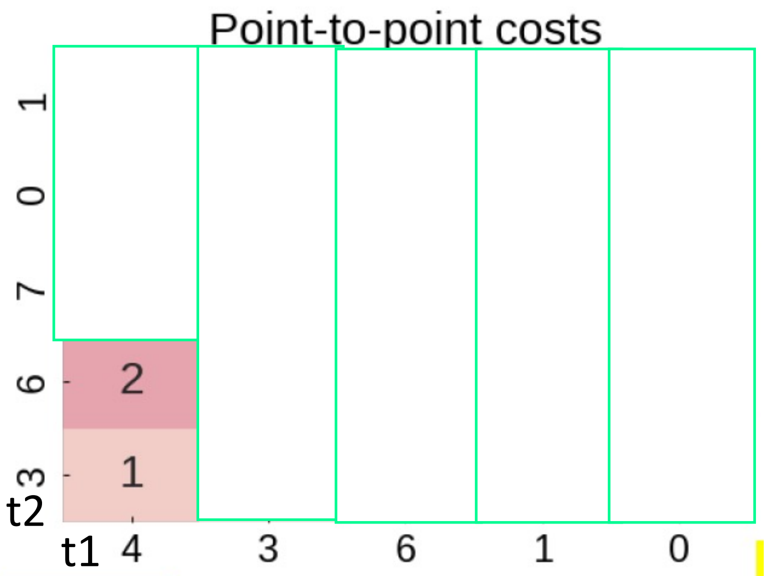
- A)



DTW – Exercise 1 - Solution

t1	< 4, 3, 6, 1, 0 >
t2	< 3, 6, 7, 0, 1 >

- A)

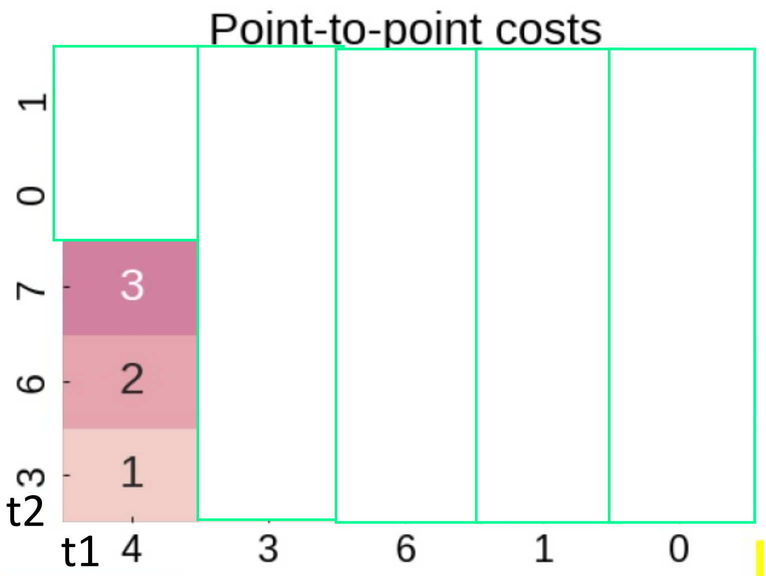


Result: 4

t1	< 4, 3, 6, 1, 0 >
t2	< 3, 6, 7, 0, 1 >

DTW – Exercise 1 - Solution

- A)

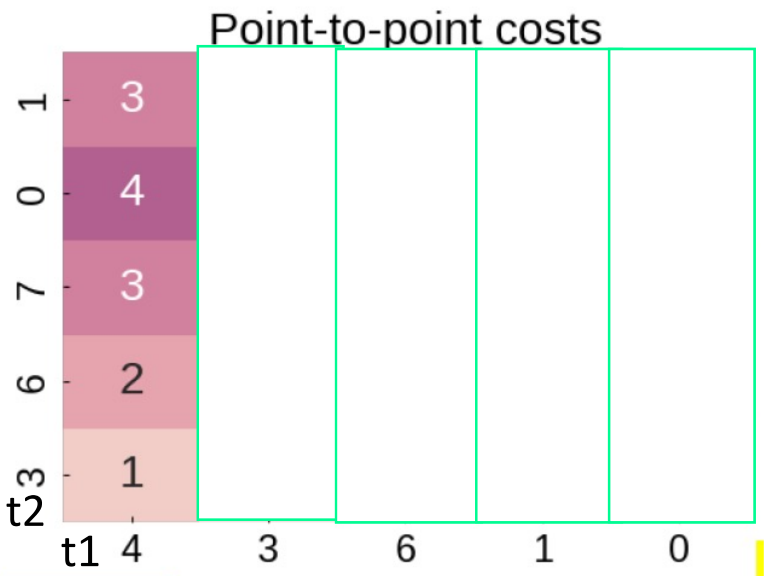


Result: 4

DTW – Exercise 1 - Solution

t1	< 4, 3, 6, 1, 0 >
t2	< 3, 6, 7, 0, 1 >

- A)



Result: 4

DTW – Exercise 1 - Solution

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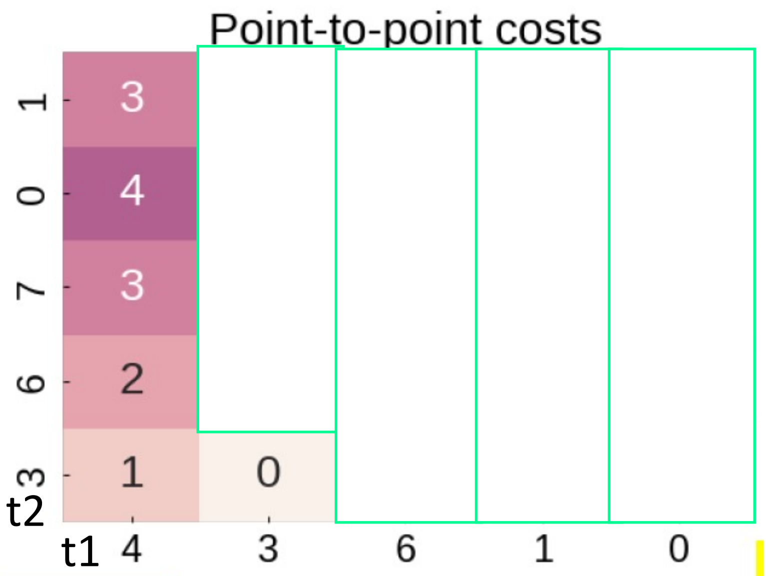
- A)

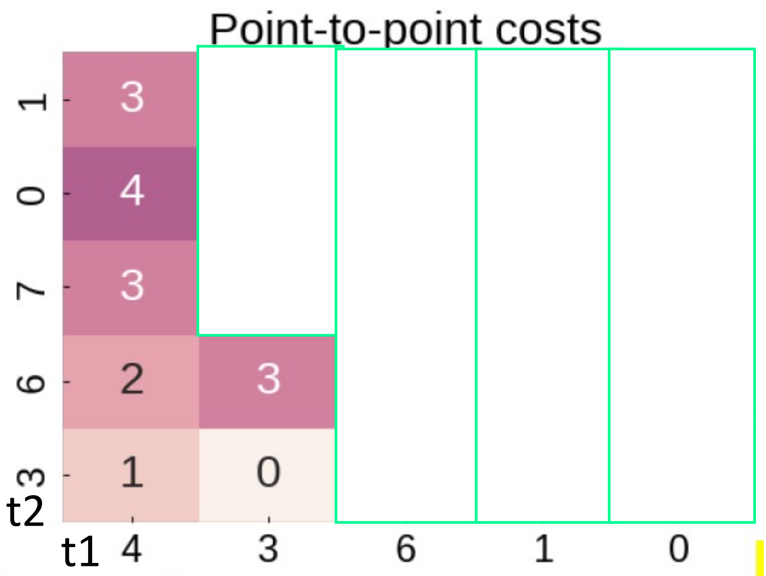


Result: 4

DTW – Exercise 1 - Solution

t1	< 4, 3, 6, 1, 0 >
t2	< 3, 6, 7, 0, 1 >

- A)



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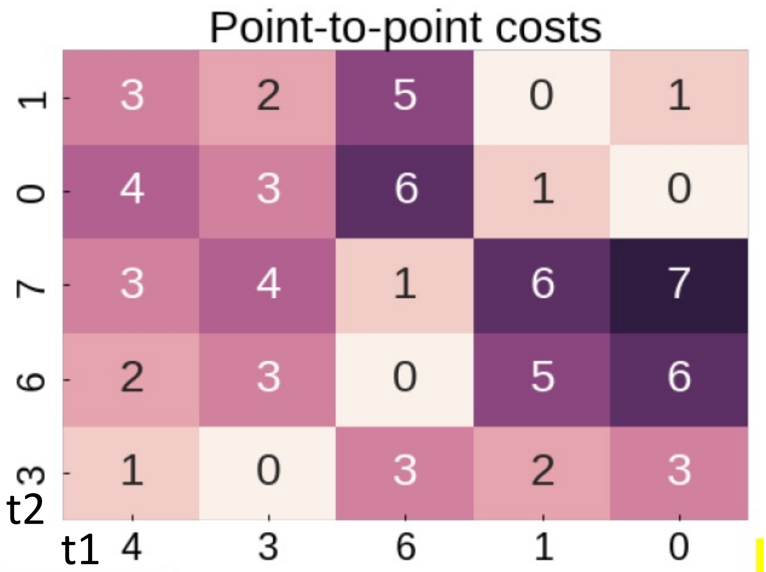

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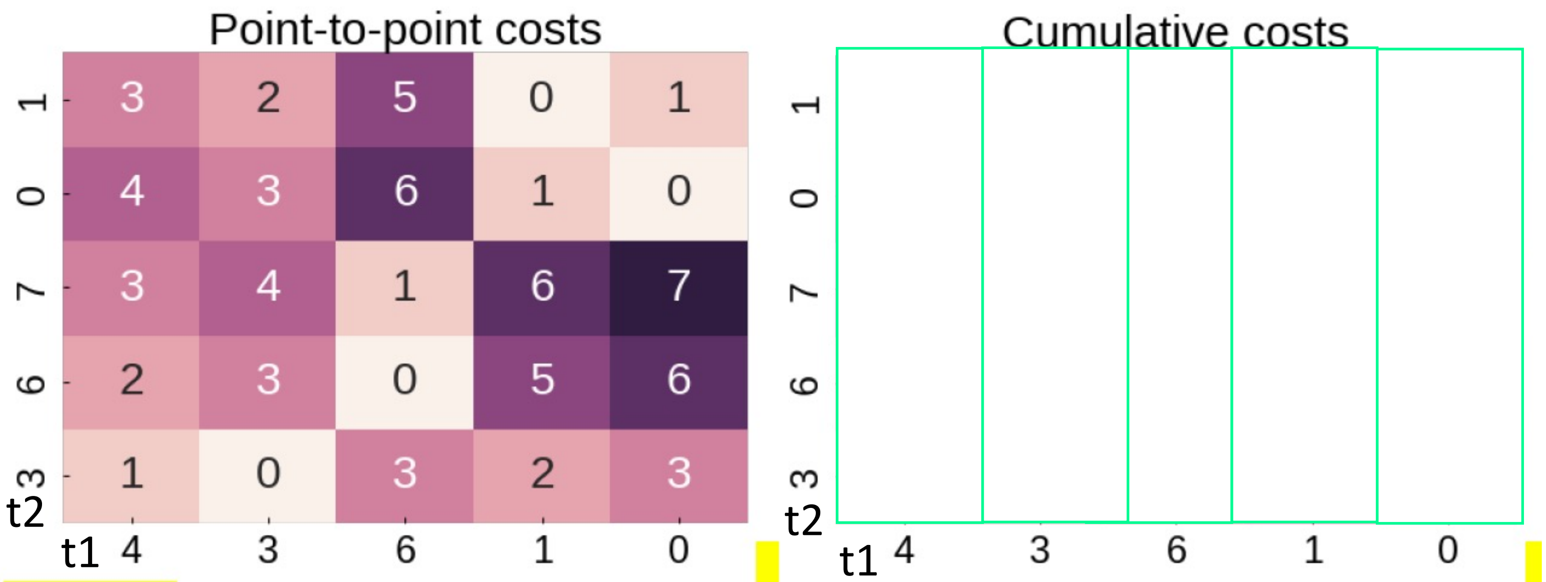


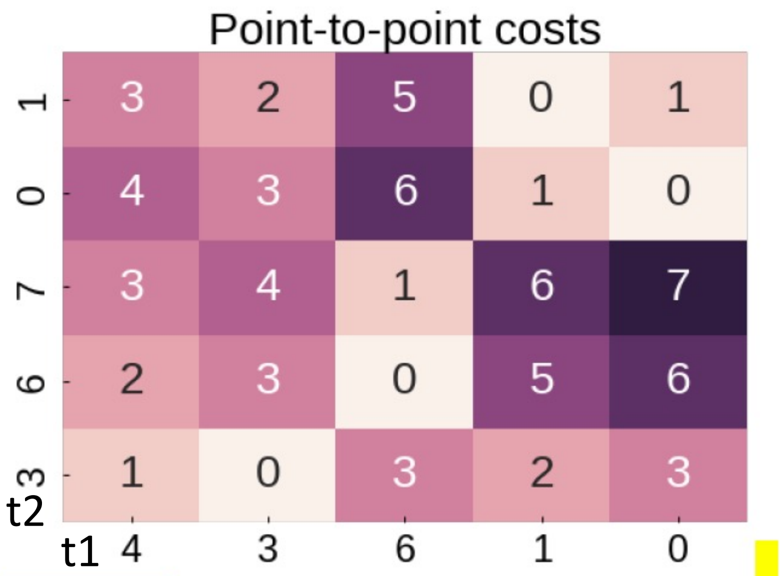
Result: 4

$$\gamma(i,j) = d(q_i,c_j) + \min\{\gamma(i-1,j-1), \gamma(i-1,j), \gamma(i,j-1)\}$$

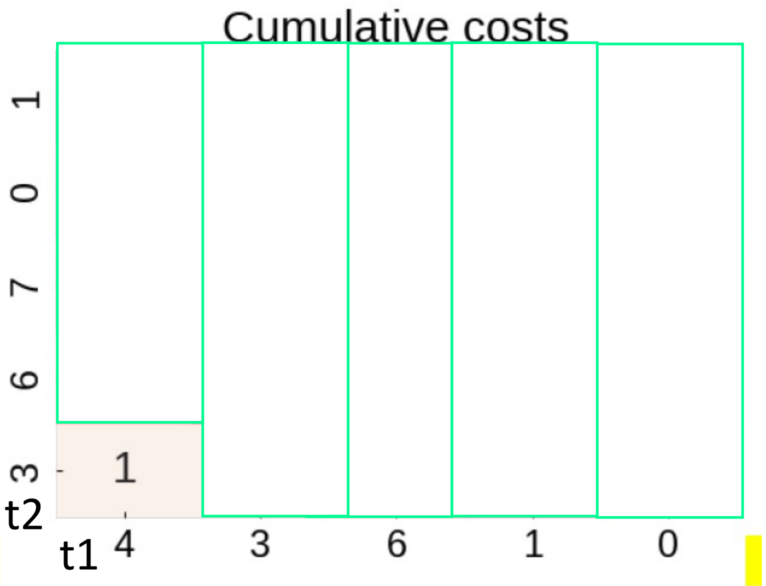
DTW – Exercise 1 - Solution

t1	< 4, 3, 6, 1, 0 >
t2	< 3, 6, 7, 0, 1 >

- A)



Result: 4



$$\gamma(i,j) = d(q_i,c_j) + \min\{\gamma(i-1,j-1), \gamma(i-1,j), \gamma(i,j-1)\}$$

DTW – Exercise 1 - Solution

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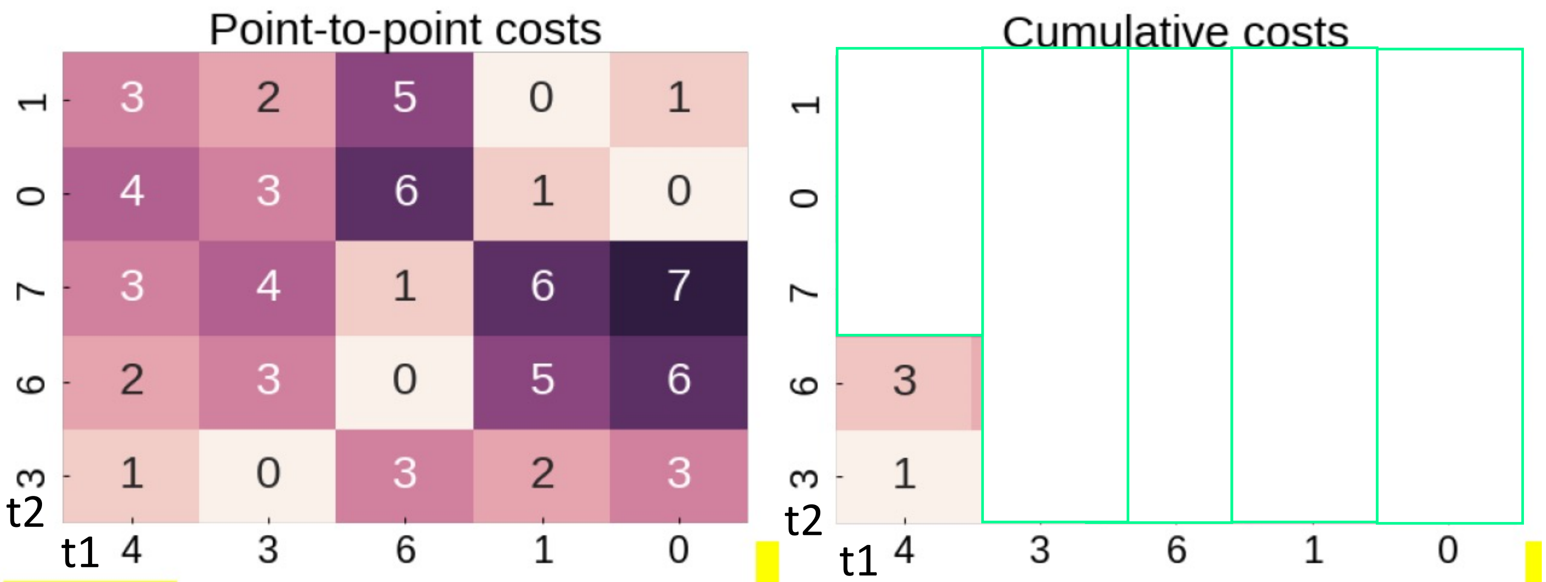

Result: 4

$$\gamma(i,j) = d(q_i,c_j) + \min\{\ \gamma(i-1,j-1),\ \gamma(i-1,j\ ),\ \gamma(i,j-1)\ \}$$

DTW – Exercise 1 - Solution

t1	< 4, 3, 6, 1, 0 >
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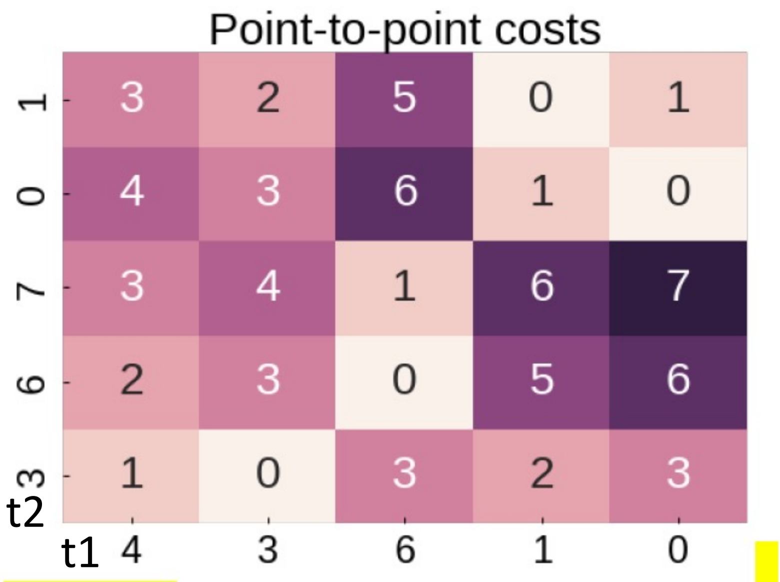
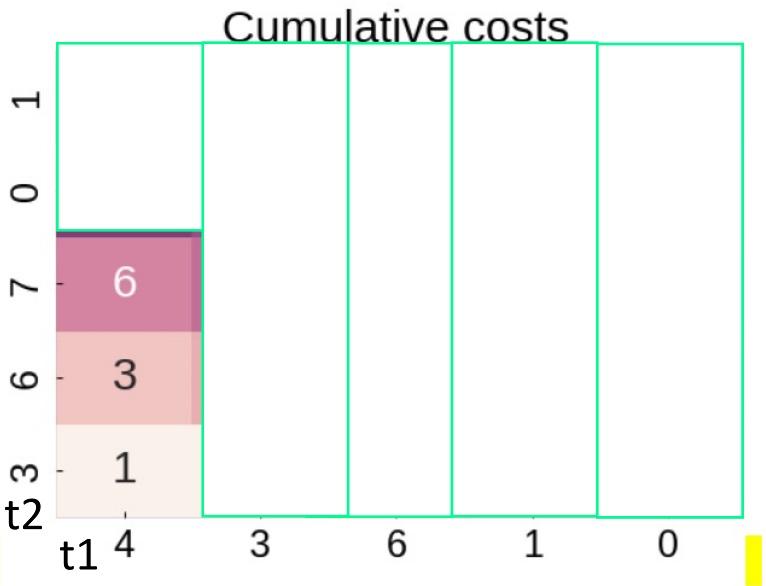
- A)



Result: 4



$$\gamma(i,j) = d(q_i,c_j) + \min\{ \gamma(i-1,j-1), \gamma(i-1,j), \gamma(i,j-1) \}$$

DTW – Exercise 1 - Solution

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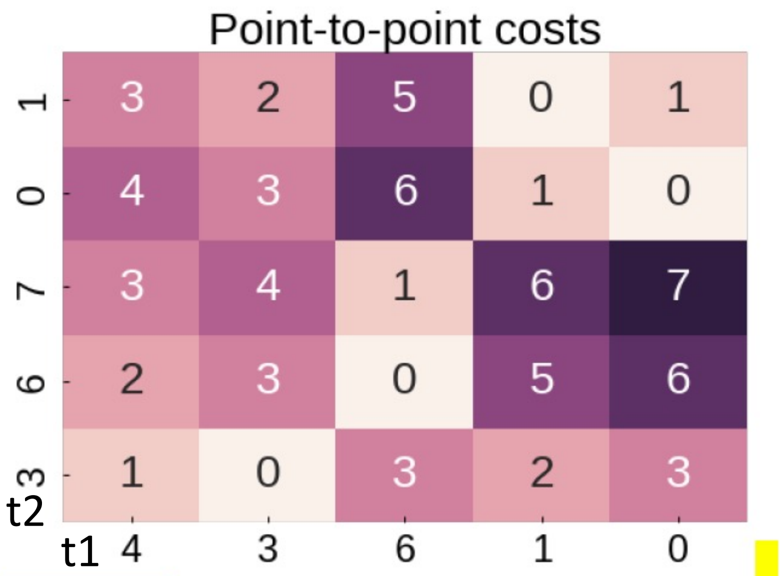
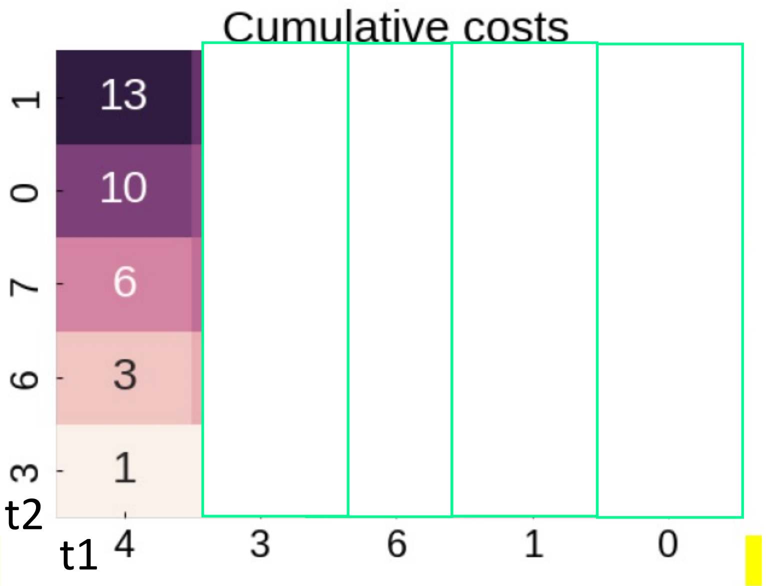
t2	< 3, 6, 7, 0, 1 >

- A)



Result: 4



$$\gamma(i,j) = d(q_i,c_j) + \min\{ \gamma(i-1,j-1), \gamma(i-1,j), \gamma(i,j-1) \}$$

DTW – Exercise 1 - Solution

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t2	< 3, 6, 7, 0, 1 >

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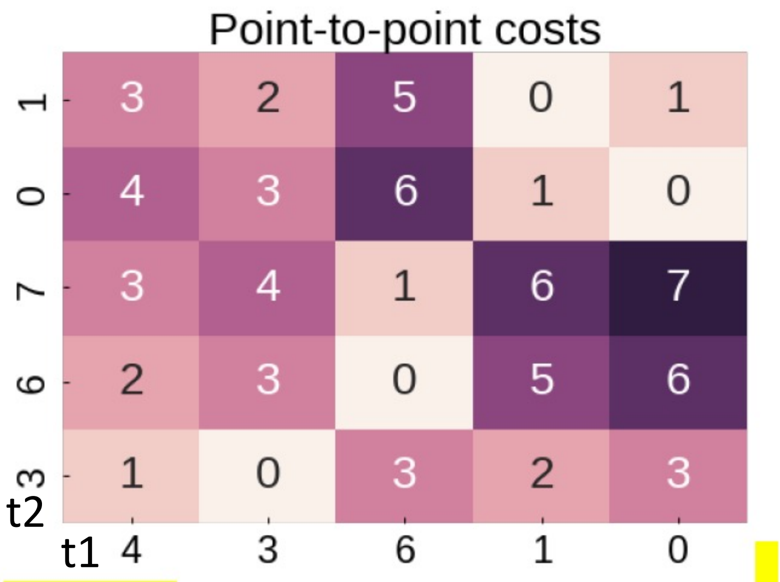
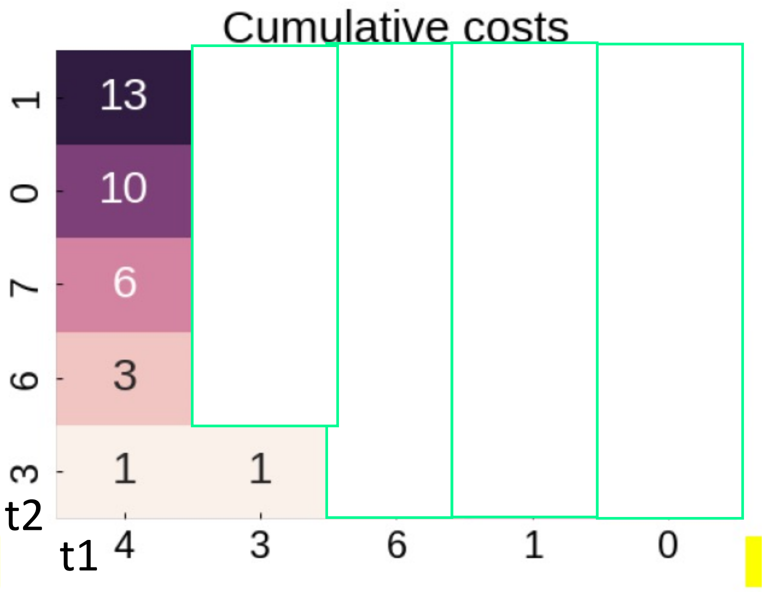


Result: 4



$$\gamma(i,j) = d(q_i,c_j) + \min\{\, \gamma(i-1,j-1),\ \gamma(i-1,j),\ \gamma(i,j-1) \,\}$$

t1	< 4, 3, 6, 1, 0 >
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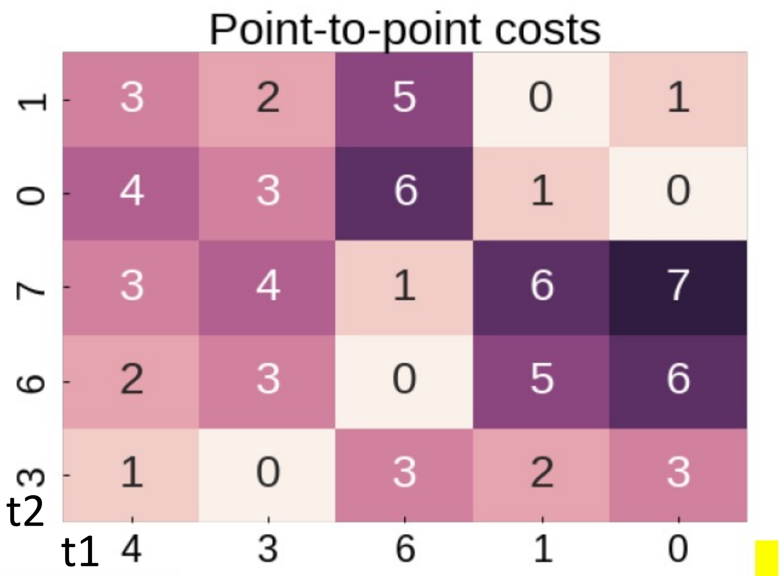
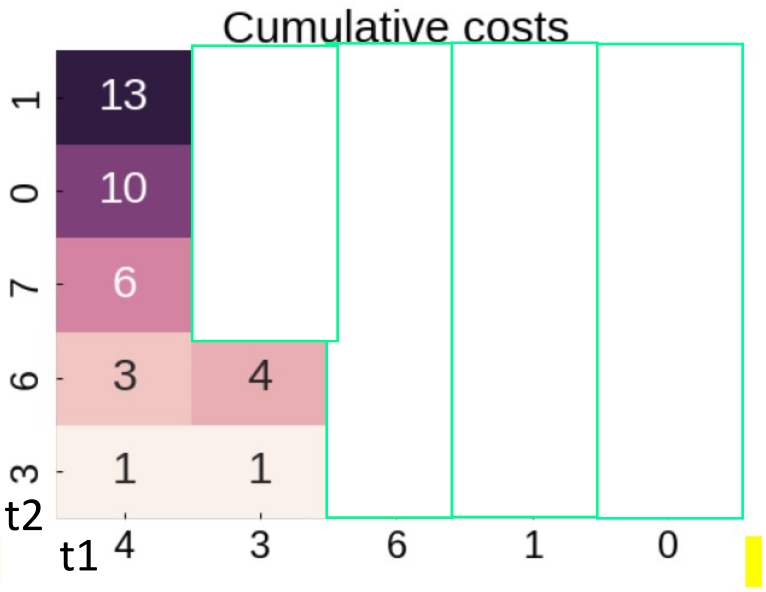
DTW – Exercise 1 - Solution

- A)



Result: 4



$$\gamma(i,j) = d(q_i,c_j) + \min\{ \gamma(i\text{-}1,j\text{-}1), \gamma(i\text{-}1,j), \gamma(i,j\text{-}1) \}$$

DTW – Exercise 1 - Solution

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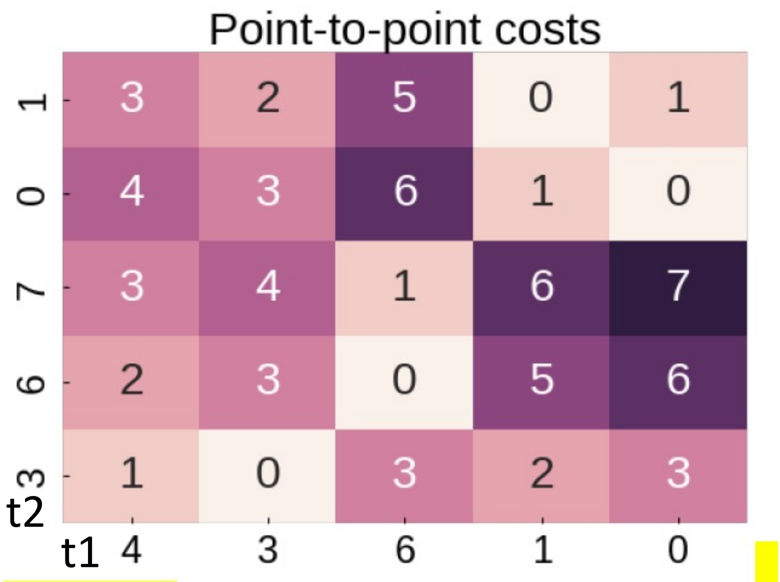
t2	< 3, 6, 7, 0, 1 >

- A)



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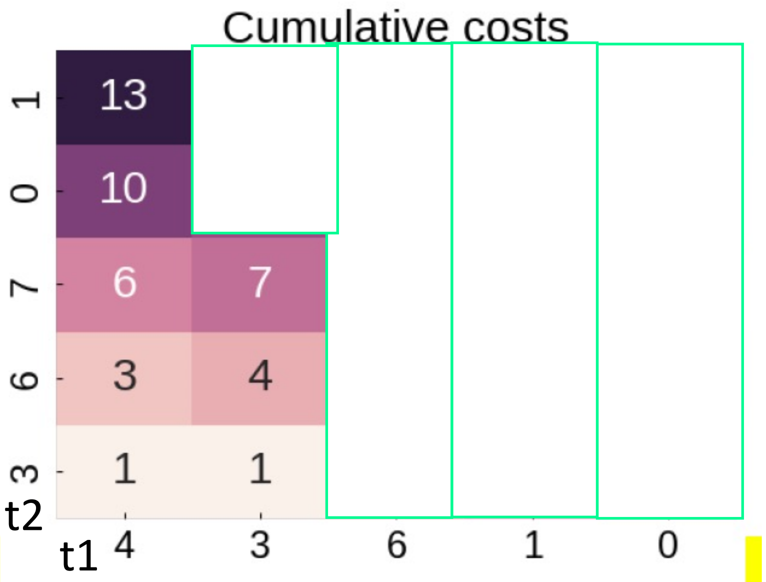

$$\gamma(i,j) = d(q_i,c_j) + \min\{ \gamma(i-1,j-1), \gamma(i-1,j), \gamma(i,j-1) \}$$

DTW – Exercise 1 - Solution

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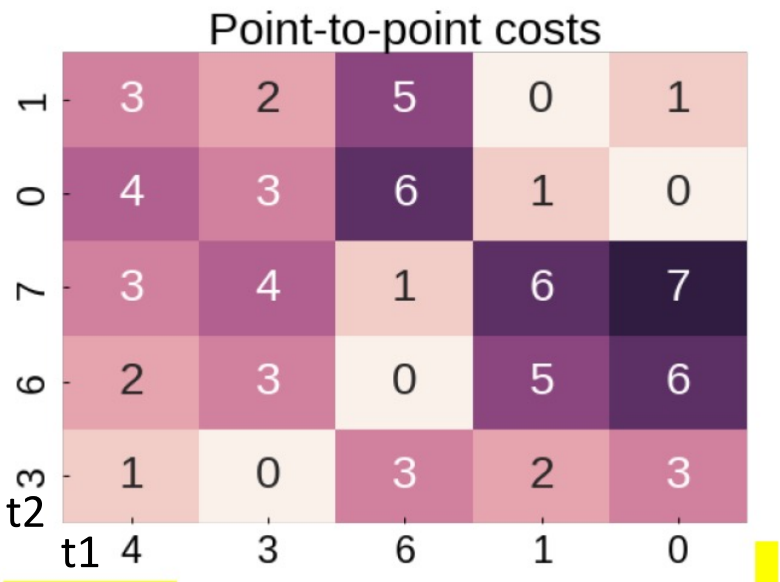
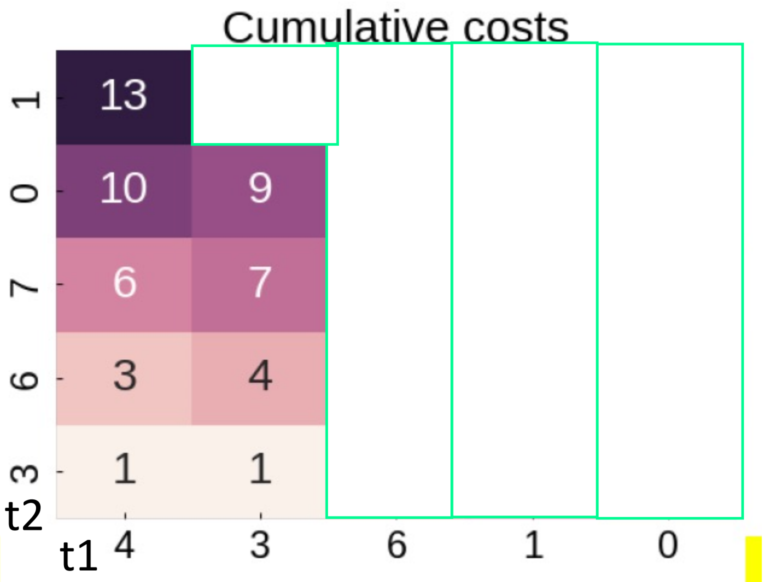


Result: 4



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DTW – Exercise 1 - Solution

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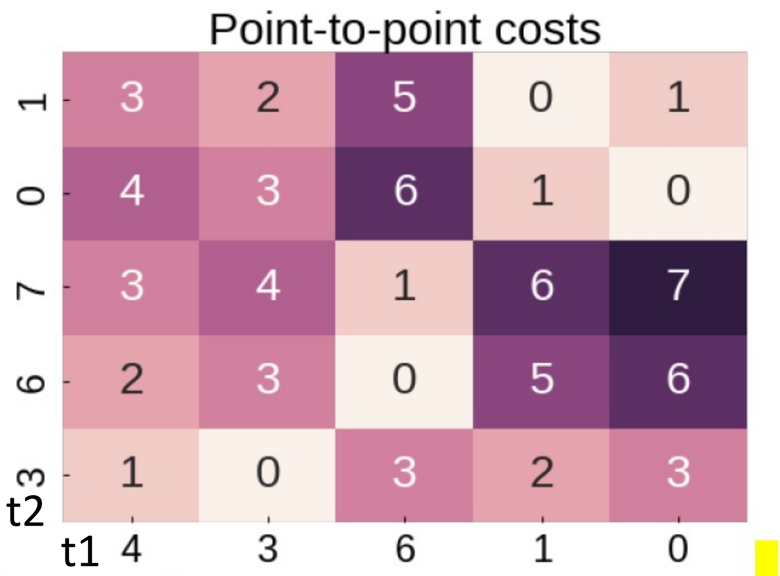
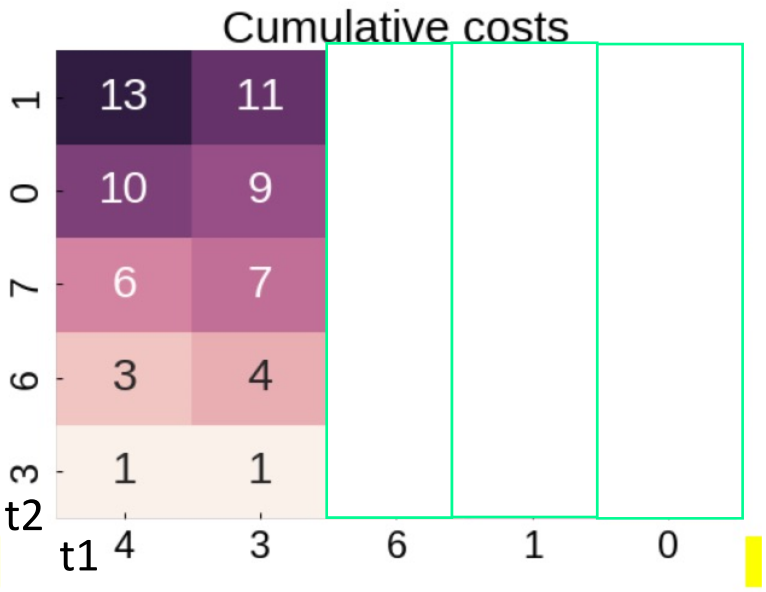
- A)



Result: 4

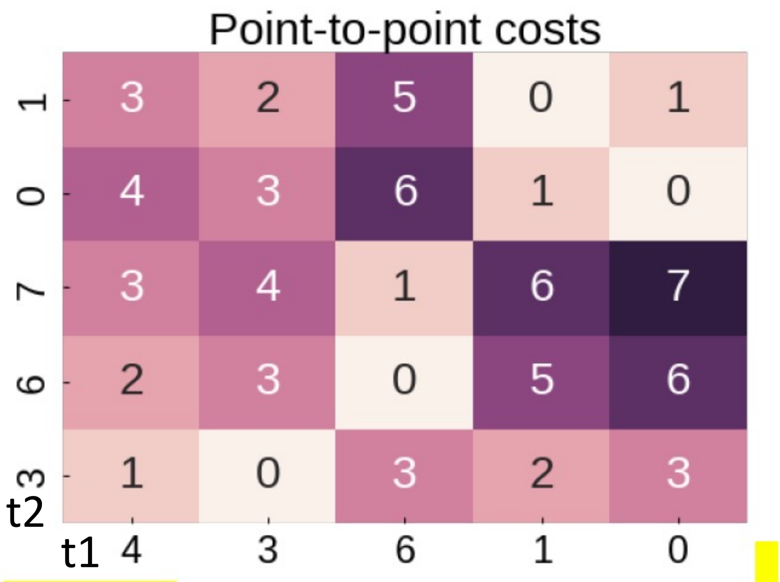


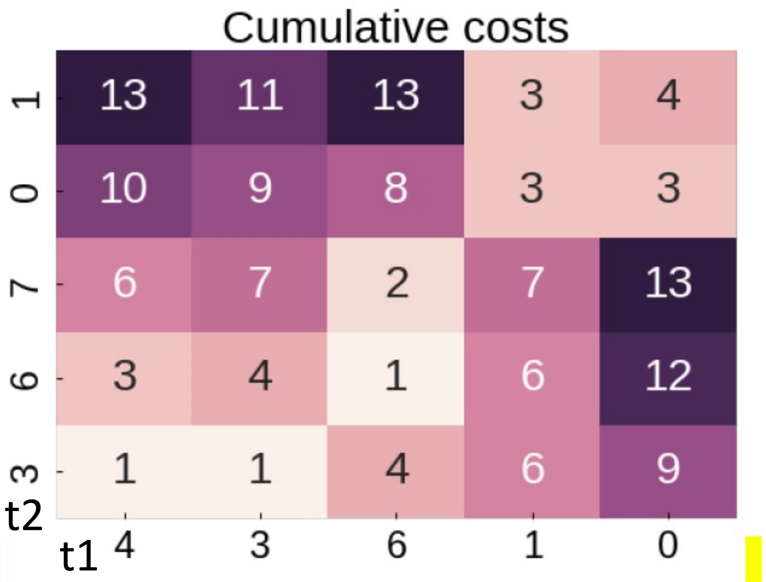
$$\gamma(i,j) = d(q_i,c_j) + \min\{\ \gamma(i-1,j-1),\ \gamma(i-1,j\ ),\ \gamma(i,j-1)\ \}$$

DTW – Exercise 1 - Solution

t1	< 4, 3, 6, 1, 0 >
t2	< 3, 6, 7, 0, 1 >

- A)





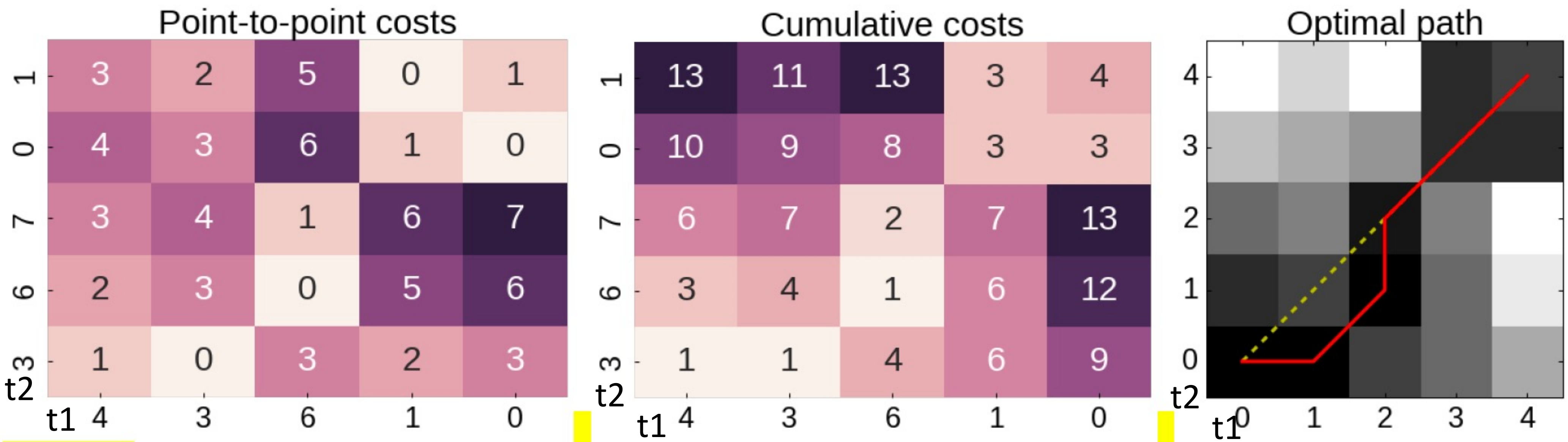
Result: 4

$$\gamma(i,j) = d(q_i,c_j) + \min\{ \gamma(i-1,j-1), \gamma(i-1,j), \gamma(i,j-1) \}$$

t1	< 4, 3, 6, 1, 0 >
t2	< 3, 6, 7, 0, 1 >

DTW – Exercise 1 - Solution

- A)



Result: 4

- B) No. Because the DTW optimal path remains inside the band of size r=1
- C) Yes. The optimal path in one direction is the same in the opposite direction. Though, the cumulative costs matrix might look different.

DTW – Exercise 2

- Given the following time series:

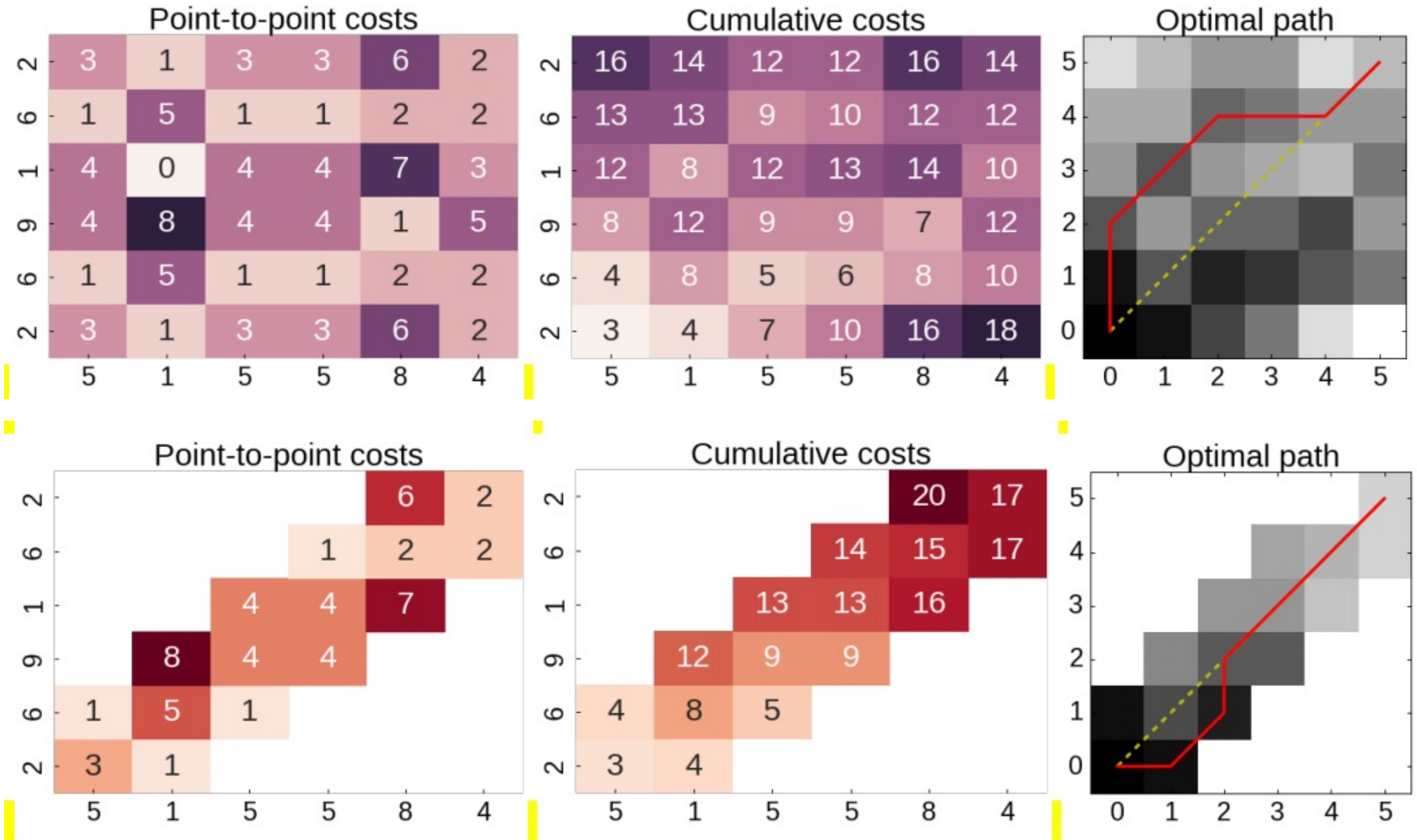
t	=	< 2, 6, 9, 1, 6, 2 >
q	=	< 5, 1, 5, 5, 8, 4 >

compute

- (i) their Manhattan and Euclidean distance,
- (ii) their DTW, and (iii) their DTW with Sakoe-Chiba band of size r=1 (i.e. all cells at distance <= 1 from the diagonal are allowed).
- For points (ii) and (iii) show the cost matrix and the optimal path found.

DTW – Exercise 2 - Solution

- Euclidean = sqrt(74) = 8.6, Manhattan = 20
- DTW = 14
- DTW r=1 = 17



DTW – Exercise 3

- Given the following time series:

ID	Time series
W	< 6, 11, 13, 15 >
X	< 10, 7, 7, 12, 14, 17 >
Y	< 9, 11, 14, 13, 20 >

- Compute the distances among all pairs of time series adopting a Dynamic Time Warping distance, and computing the distances between single points as d(x,y) = | x – y |. For each pair of time series compared also show the matrix used to compute the final result.

DTW – Exercise 3 - Solution

ID	Time series
W	< 6, 11, 13, 15 >
X	< 10, 7, 7, 12, 14, 17 >
Y	< 9, 11, 14, 13, 20 >

W – X

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
[1,]	(4) 4	(1) 5	(1) 6	(6) 12	(8) 20	(11) 31
[2,]	(1) 5	(4) 8	(4) 9	(1) 7	(3) 10	(6) 16
[3,]	(3) 8	(5) 11	(5) 14	(1) 8	(1) 8	(4) 12
[4,]	(5) 13	(8) 16	(8) 19	(3) 11	(4) 9	(2) 10

W – Y

	[,1]	[,2]	[,3]	[,4]	[,5]
[1,]	(3) 3	(5) 8	(8) 16	(7) 23	(14) 37
[2,]	(2) 5	(0) 3	(3) 6	(2) 8	(9) 17
[3,]	(5) 9	(2) 5	(1) 4	(0) 4	(7) 11
[4,]	(6) 15	(4) 9	(1) 5	(2) 6	(5) 9

X – Y

	[,1]	[,2]	[,3]	[,4]	[,5]
[1,]	(1) 1	(1) 2	(4) 6	(3) 9	(10) 19
[2,]	(2) 3	(4) 5	(7) 9	(6) 12	(13) 22
[3,]	(2) 5	(4) 7	(7) 12	(6) 15	(13) 25
[4,]	(3) 8	(1) 6	(2) 8	(1) 9	(8) 17
[5,]	(5) 13	(3) 9	(0) 6	(1) 7	(6) 13
[6,]	(8) 21	(6) 15	(3) 9	(4) 10	(3) 10