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Guarding and Searching
Polyhedral Environments

Abstract

We study some visibility problems in 3-dimensional computational geometry.

As a preliminary structural result, we prove the existence of a compatible tetrahedralization of any pair of tetrahedralized 0-genus polyhedra which combinatorially refines both tetrahedralizations, thus settling and generalizing an open problem from CCCG 2009. Additionally we compute explicit polynomial bounds on the output size, provided that the given tetrahedralizations are shellable.

In order to cope with the lack of linear-size partitions of orthogonal polyhedra into convex parts, we introduce a representation method based on symmetric differences of cuboids, and prove that it yields a linear-size structure. Such structure gives rise to a notion of shape complexity that naturally enables the study of fixed-parameter tractability of visibility problems.

We then consider the ART GALLERY PROBLEM (AGP) for polyhedra, both with vertex-guards and edge-guards, proving its NP-completeness and APX-hardness, even when restricted to orthogonal polyhedra. By studying some suitably enriched aspect graphs of polyhedra, we show that the set of reflex edges is a solution to AGP. We further propose a polynomial-time discretization technique and apply it to obtain an $O(\log n)$-approximate algorithm for AGP. The same technique would also yield an improved $O(\log \mathrm{OPT})$-approximate algorithm, if the VC-dimensions of certain SET COVER instances turned out to be bounded by a constant, which we conjecture to be true at least for edge-guards in orthogonal polyhedra.

Finally we model the 3-DIMENSIONAL SEARCHLIGHT SCHEDULING PROBLEM for polyhedra, with *searchplanes* as opposed to 1-dimensional rays. We prove that two of its variants are NP-hard and APX-hard, even with severe restrictions; then we discuss the extendibility to our model of some well-known properties of its 2-dimensional version. We also give a heuristic to place searchplanes in order to make a given polyhedron searchable: if r is the number of reflex edges, we show that $2r$ searchplanes are enough for general polyhedra, whereas just r searchplanes suffice for orthogonal polyhedra. This further corroborates a general feeling emerging throughout our work, that edge-guards in polyhedra are powerful enough to act as the natural counterparts of vertex-guards in polygons.