

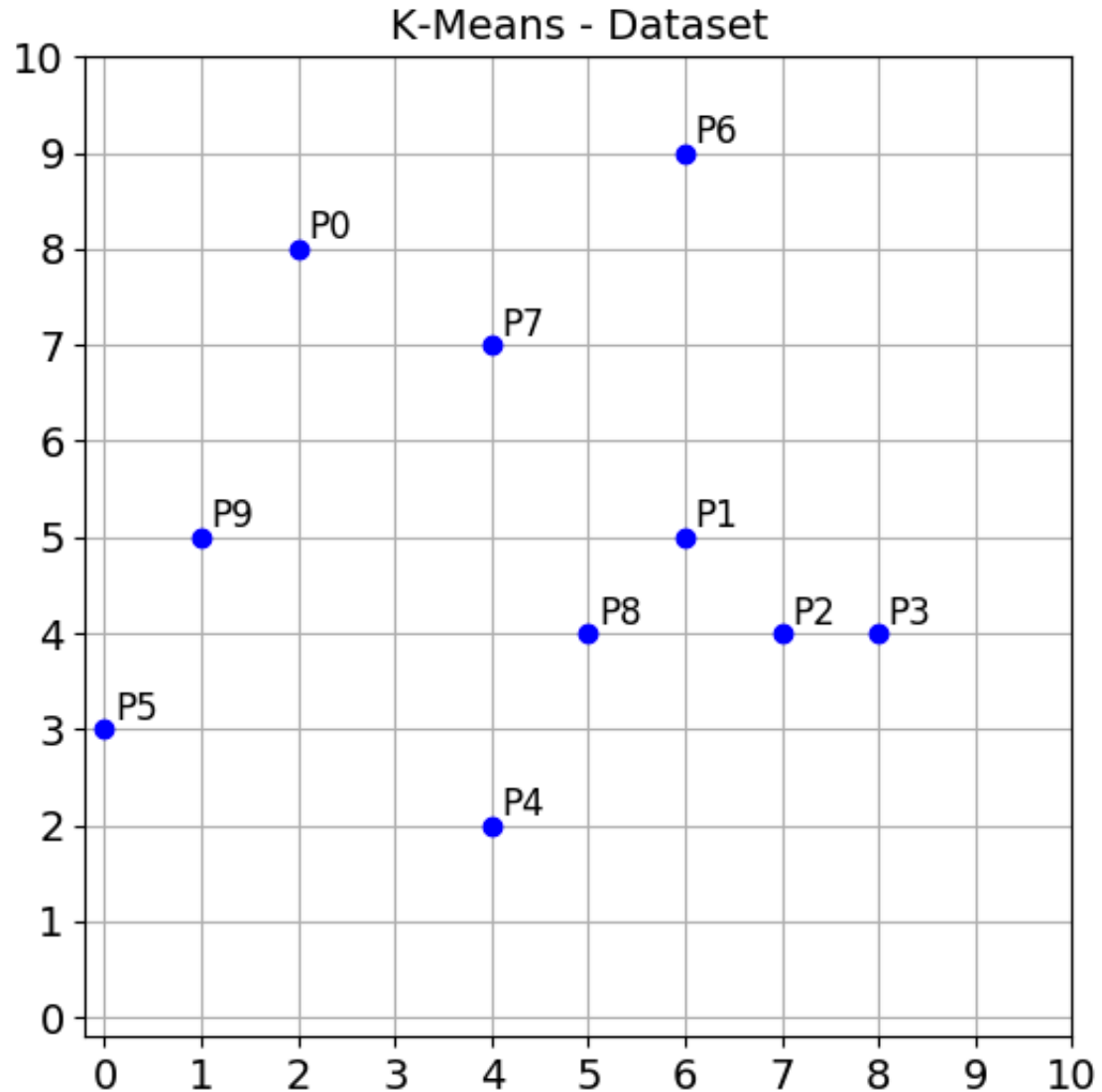
Ex. - Clustering

# K-means simulation

**Initial centroids:**

$C1 = P2 = (7,4)$

$C2 = P8 = (5,4)$



# Solution: Identify the **Bisecting** lines dividing the plane between pairs of centroids

## Cluster1

P0,P7,P9,P8,P5,P4,P1,P6

## Cluster2

P2,P3

## Centrod1:

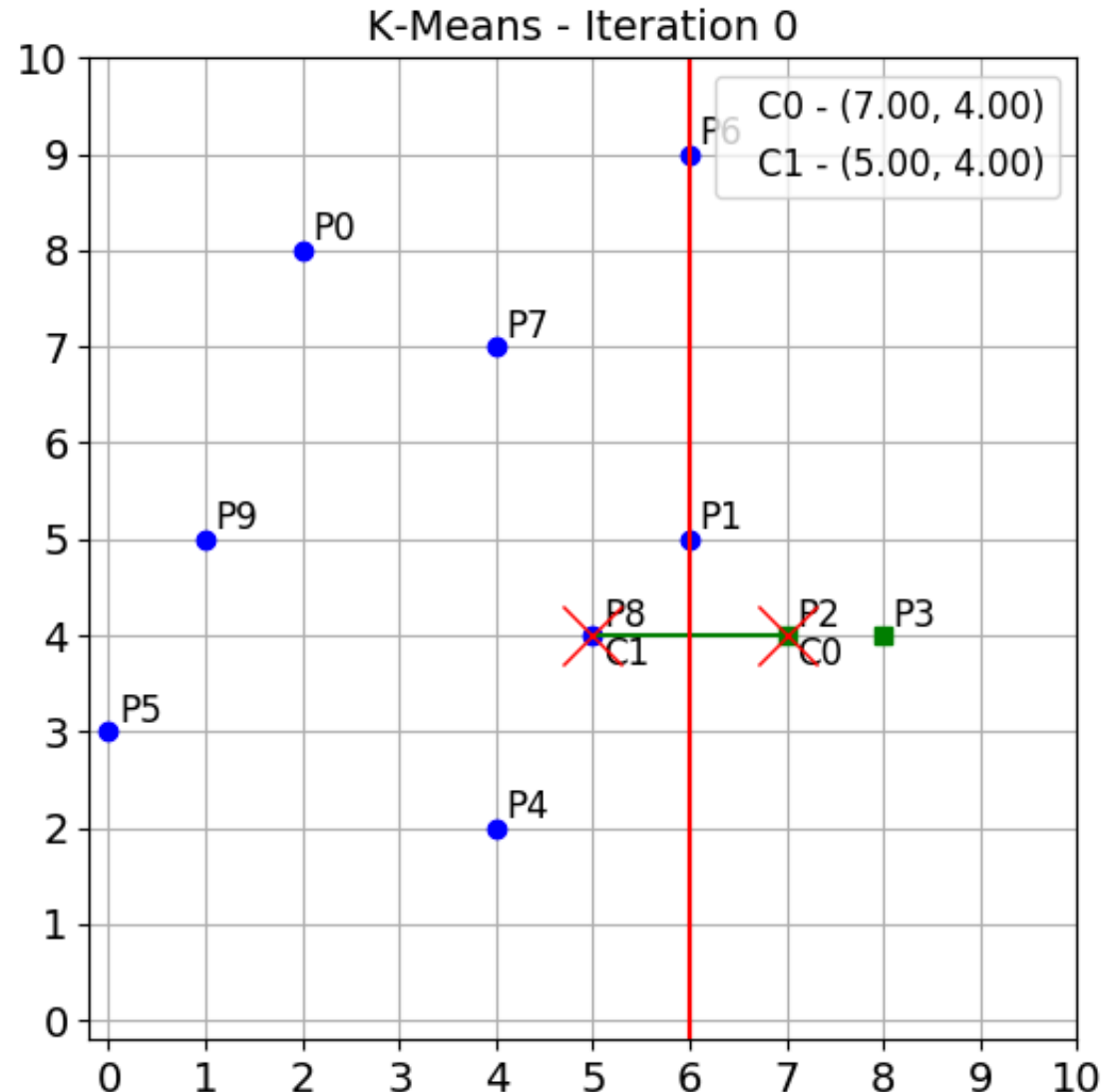
$$X1 = (0+1+2+4+4+5+6+6)/8 = 3.5$$

$$Y1 = (2+3+4+5+5+7+8+9)/8 = 5.38$$

## Centrod2:

$$X2 = (7+8)/2 = 7.5$$

$$Y2 = (4+4)/2 = 4$$



# Solution: Identify the **Bisecting** lines dividing the plane between pairs of centroids

## Cluster1

P0,P7,P9,P8,P5,P4,P6

## Cluster2

P1,P2,P3

## Centrod1:

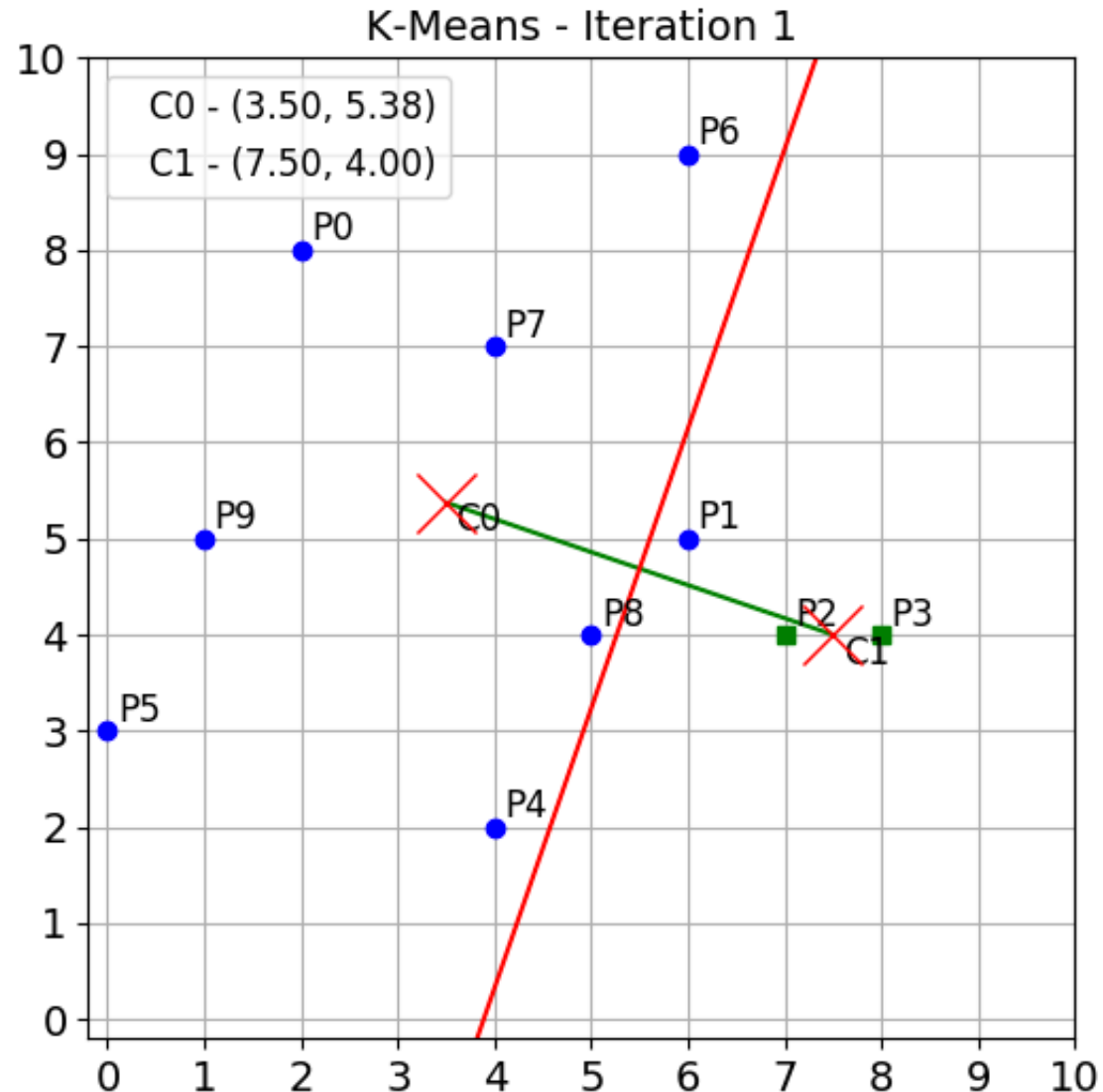
$$X1 = (0+1+2+4+4+5+6)/7 = 3.14$$

$$Y1 = (2+3+4+5+7+8+9)/7 = 5.43$$

## Centrod2:

$$X2 = (6+7+8)/3 = 7$$

$$Y2 = (5+4+4)/3 = 4.33$$



# Solution: Identify the **Bisecting** lines dividing the plane between pairs of centroids

## Cluster1

P0,P7,P9,P5,P4,P6

## Cluster2

P1,P2,P3,P8

## Centrod1:

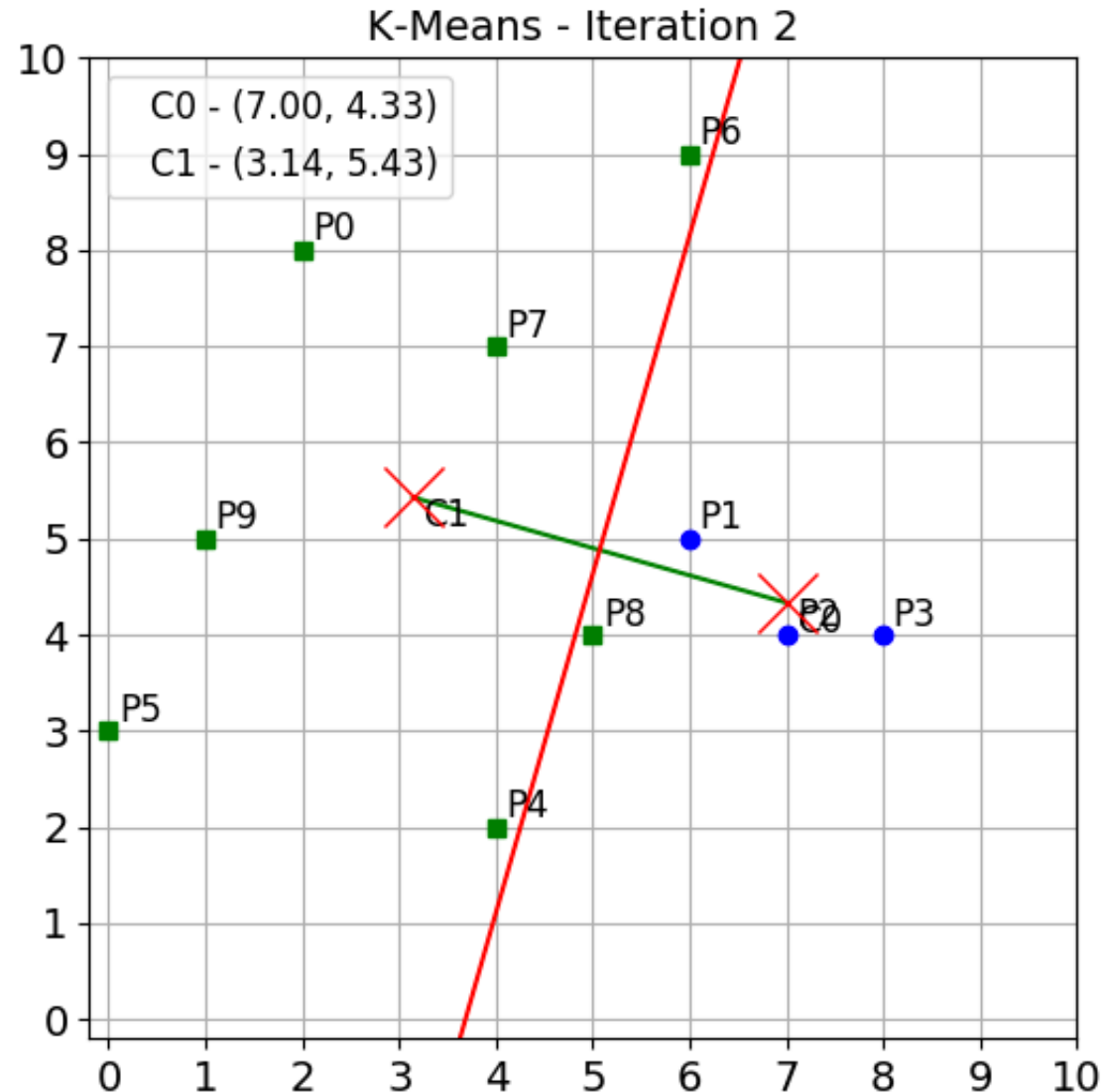
$$X1 = (0+1+2+4+4+6)/6 = 2.83$$

$$Y1 = (2+3+5+7+8+9)/6 = 5.67$$

## Centrod2:

$$X2 = (6+7+8+5)/4 = 6.5$$

$$Y2 = (5+4+4+4)/4 = 4.25$$



# Solution: Identify the **Bisecting** lines dividing the plane between pairs of centroids

## Cluster1

P0,P7,P9,P5,P6

## Cluster2

P1,P2,P3,P8,P4

## Centrod1:

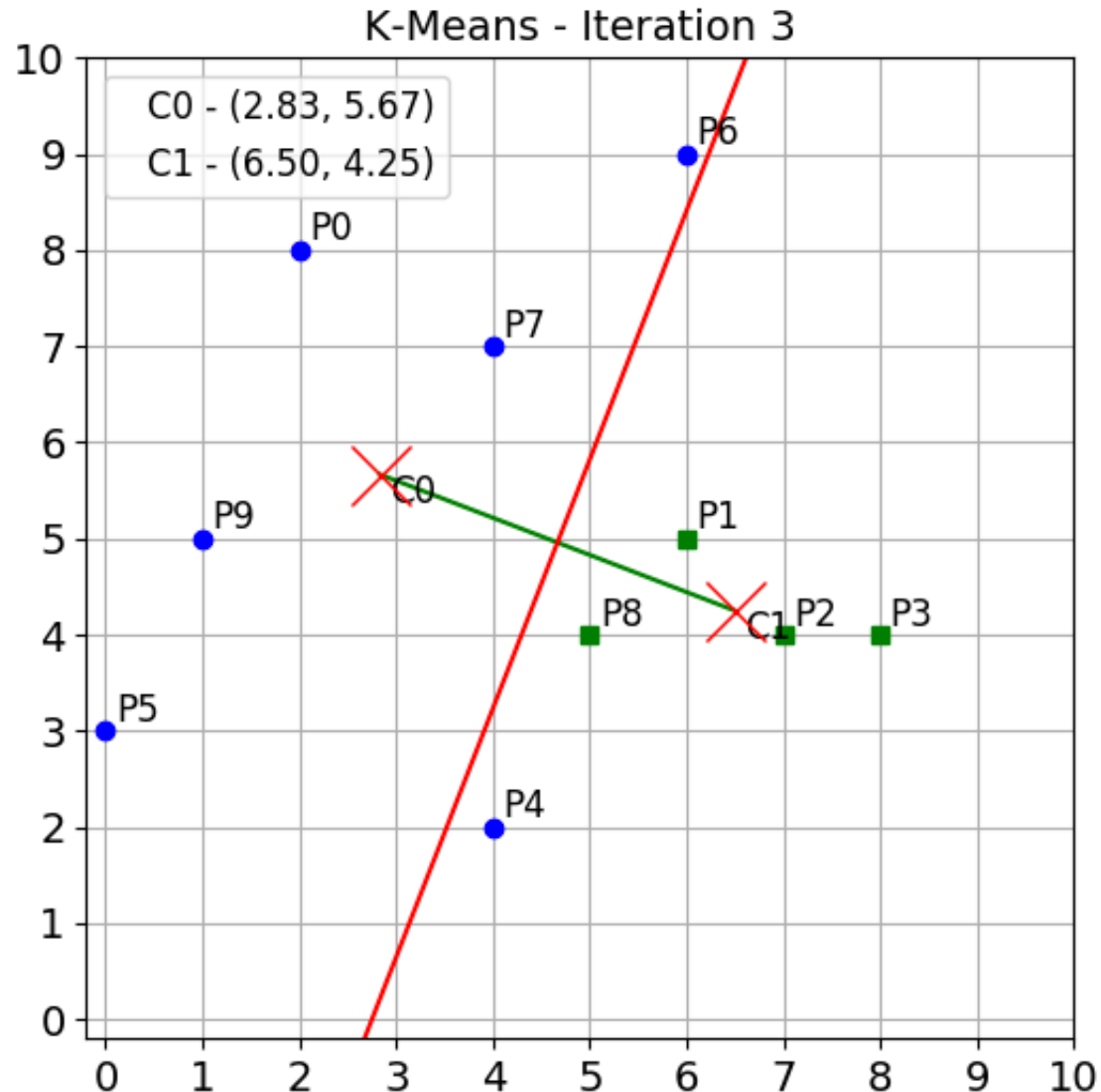
$$X1 = (0+1+2+4+6)/5 = 2.6$$

$$Y1 = (3+5+7+8+9)/5 = 6.4$$

## Centrod2:

$$X2 = (6+7+8+5+4)/5 = 6$$

$$Y2 = (5+4+4+4+2)/5 = 3.8$$

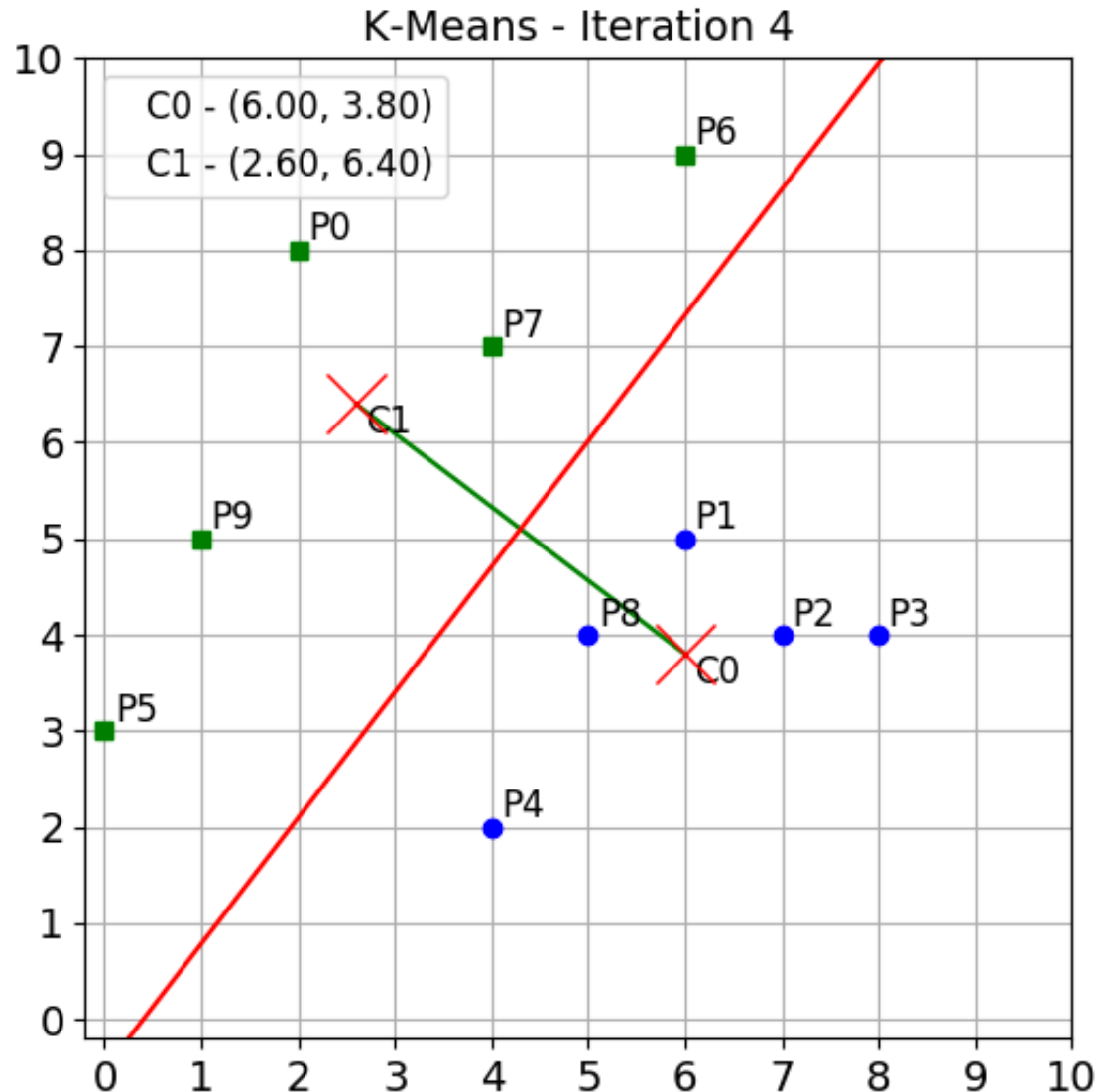


# Solution: Identify the **Bisecting** lines dividing the plane between pairs of centroids

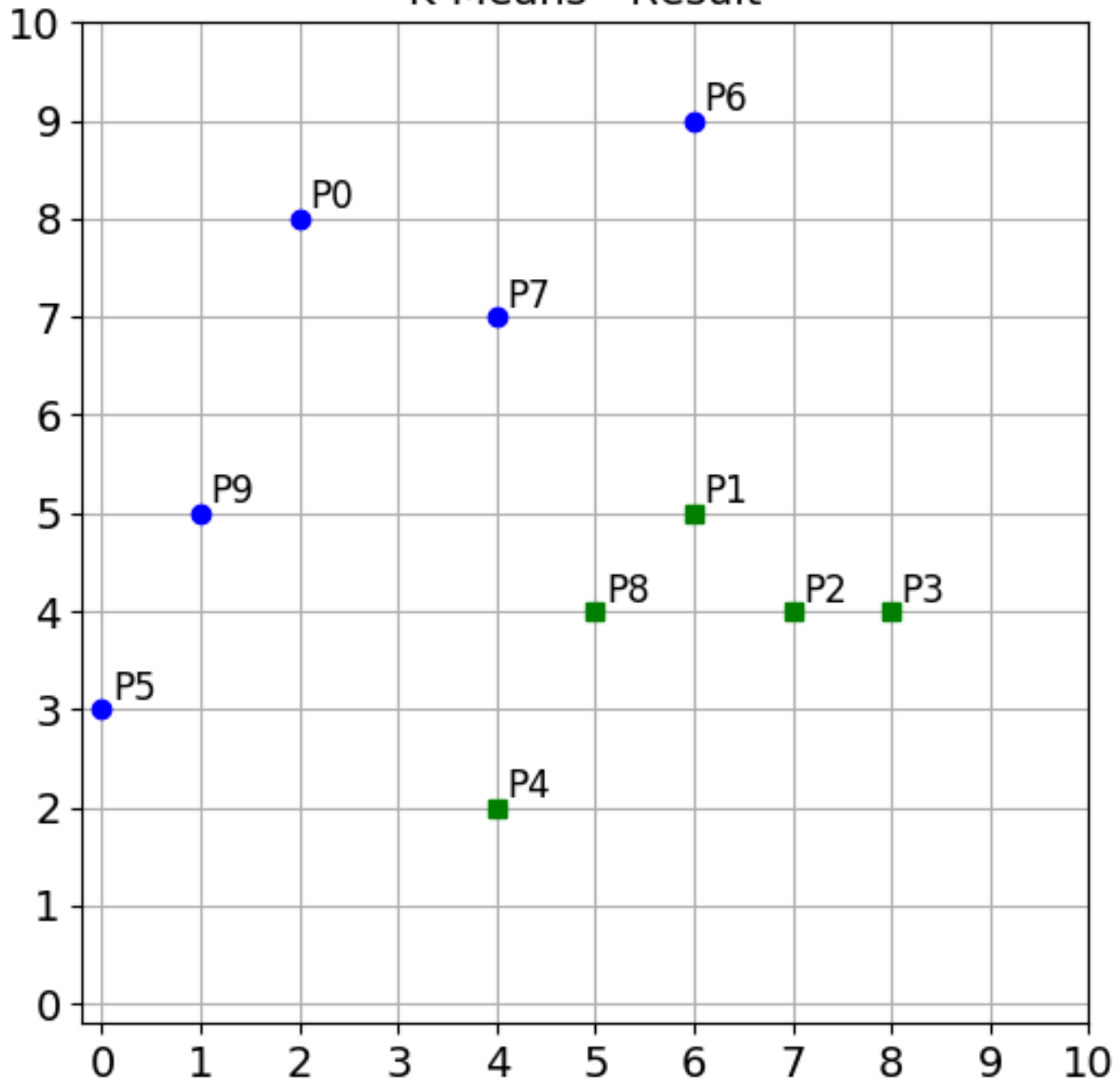
**Cluster1**  
P0,P7,P9,P5,P6

**Cluster2**  
P1,P2,P3,P8,P4

The cluster composition does not change, so K-means stops



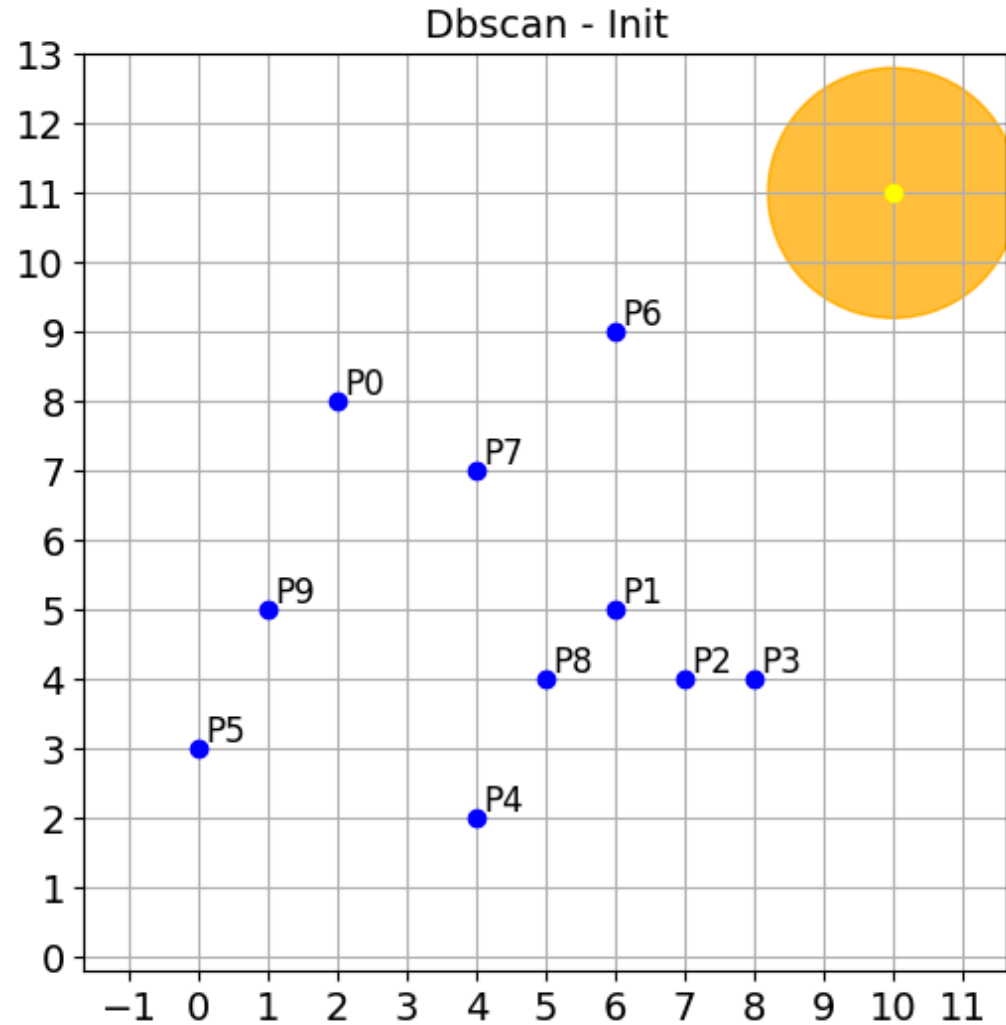
K-Means - Result





# DBSCAN - Simulation

- Eps = 1.8
- MinPoints=3
  - (included the point)



# DBSCAN

- Eps = 1.8
- MinPoints=3
  - (included the point)

## CORE POINTS:

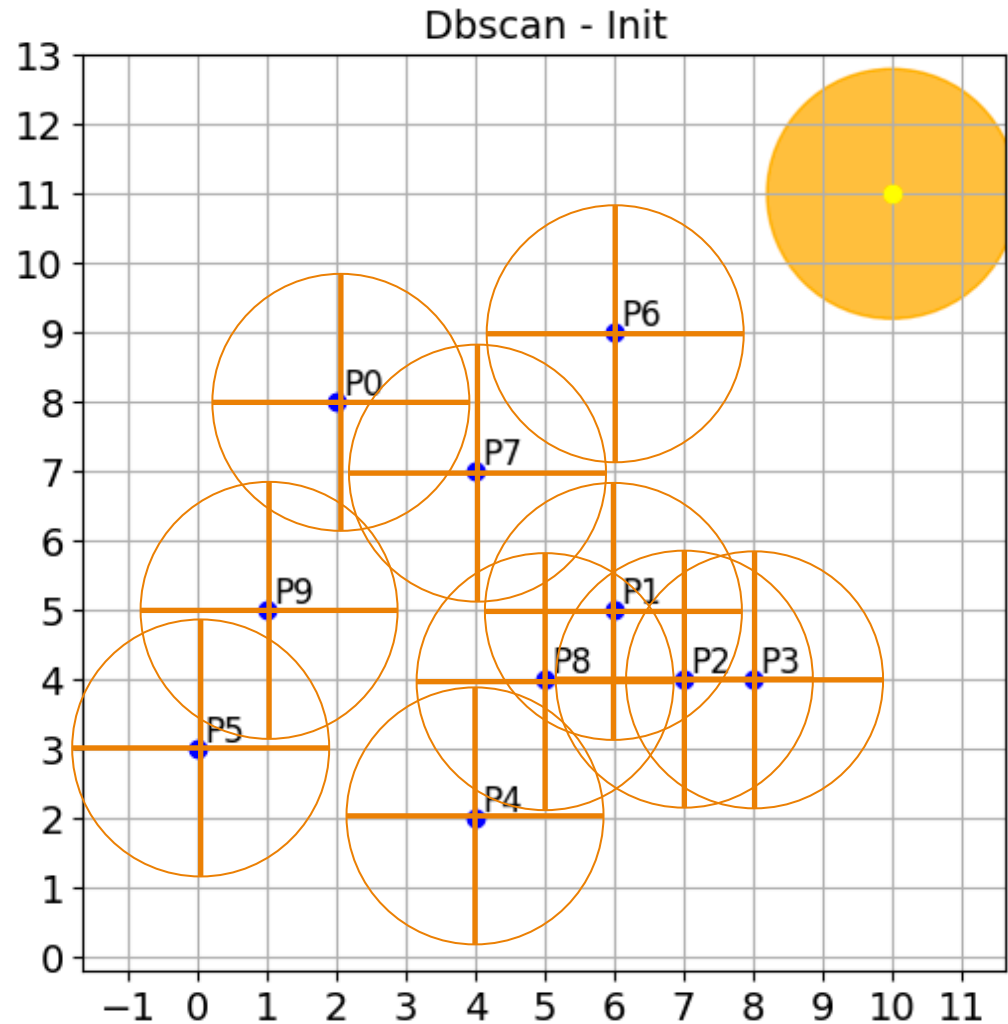
- P1
- P2

## BORDER POINTS

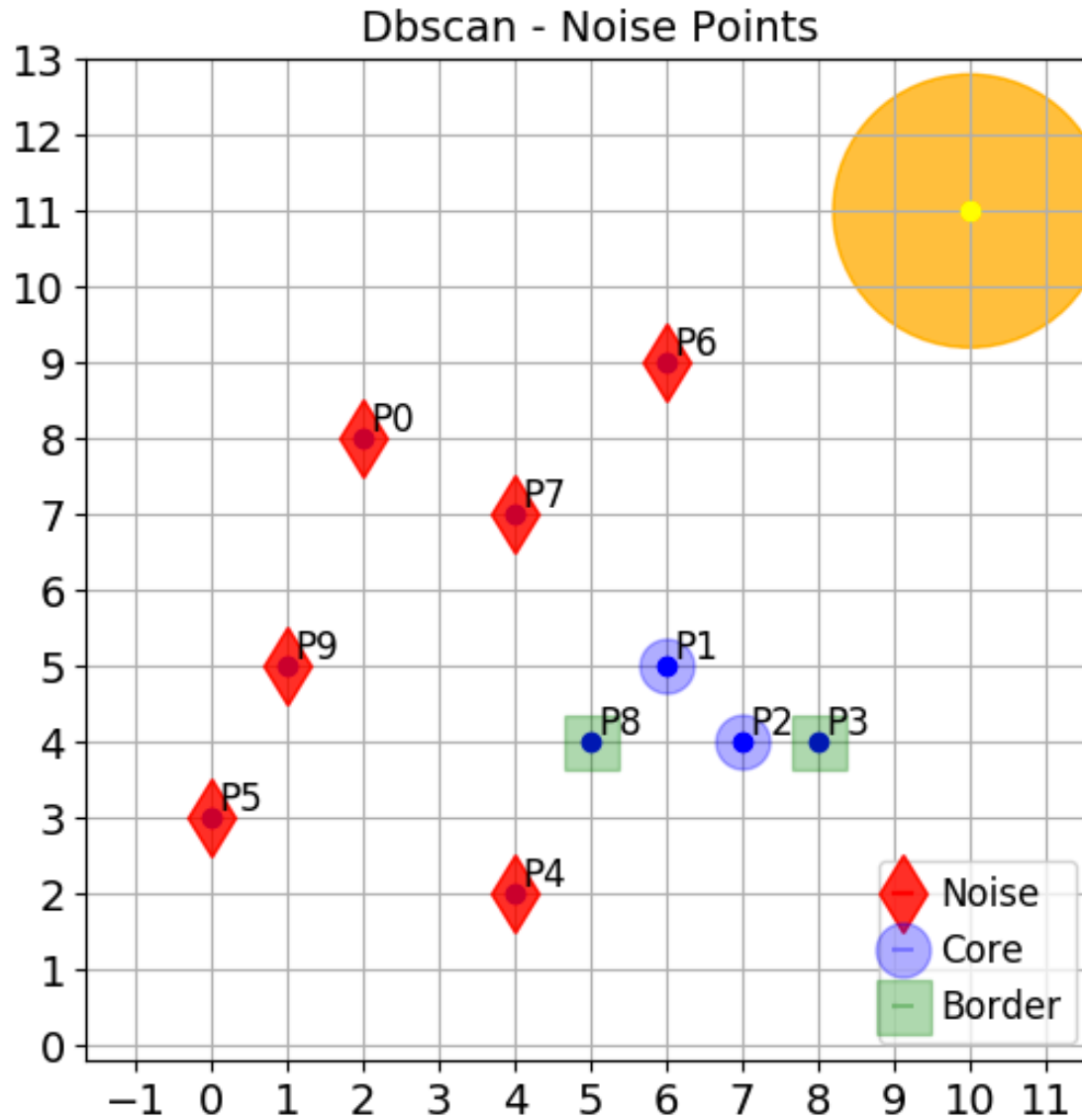
- P3
- P8

## NOISE POINTS

- P4, P5, P9, P0, P6, P7

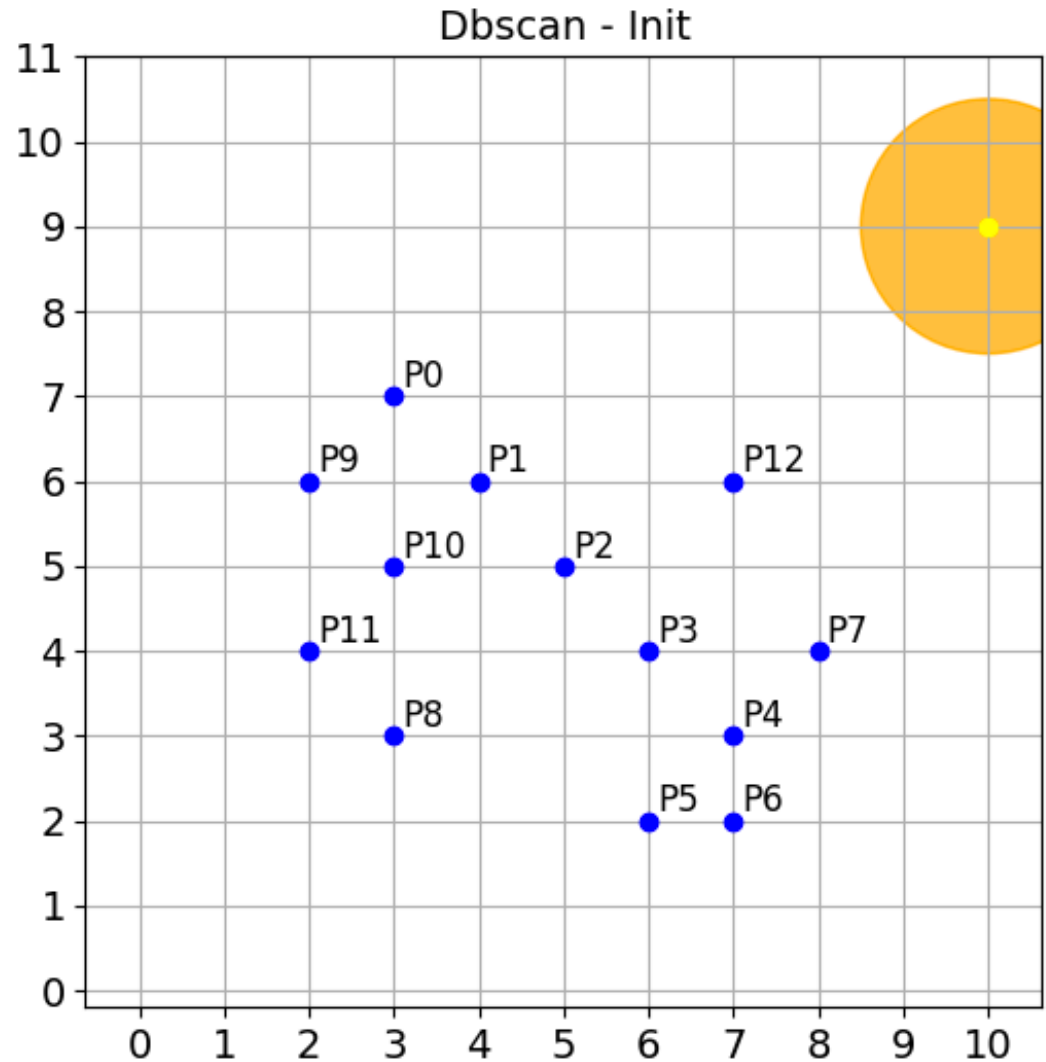


# DBSCAN



# DBSCAN EX. 2

- Eps = 1.5
- MinPoints=3
  - (included the point)



# DBSCAN 2

- Eps = 1.8
- MinPoints=3
  - (included the point)

## CORE POINTS:

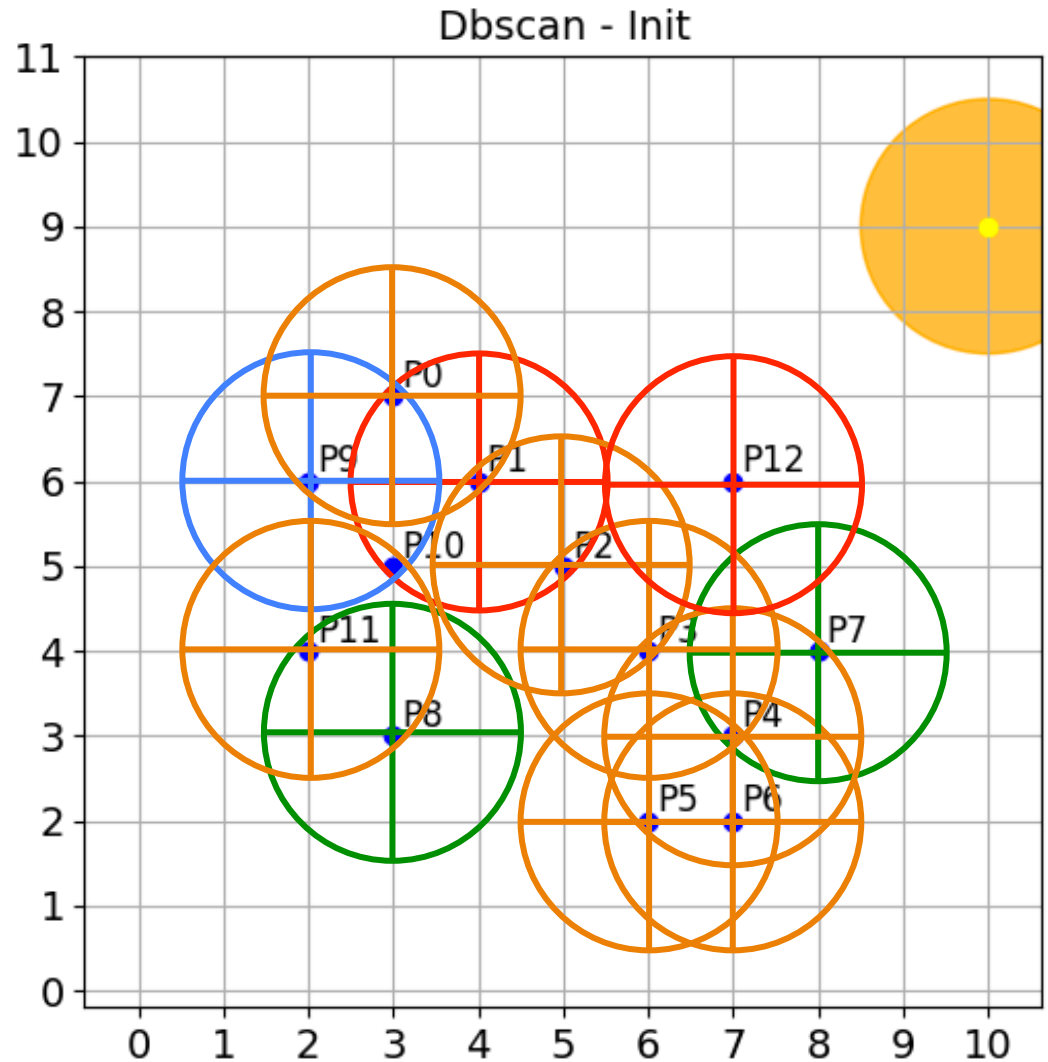
- P5
- P6
- P4
- P3
- P2
- P1
- P0
- P9
- P11

## BORDER POINTS

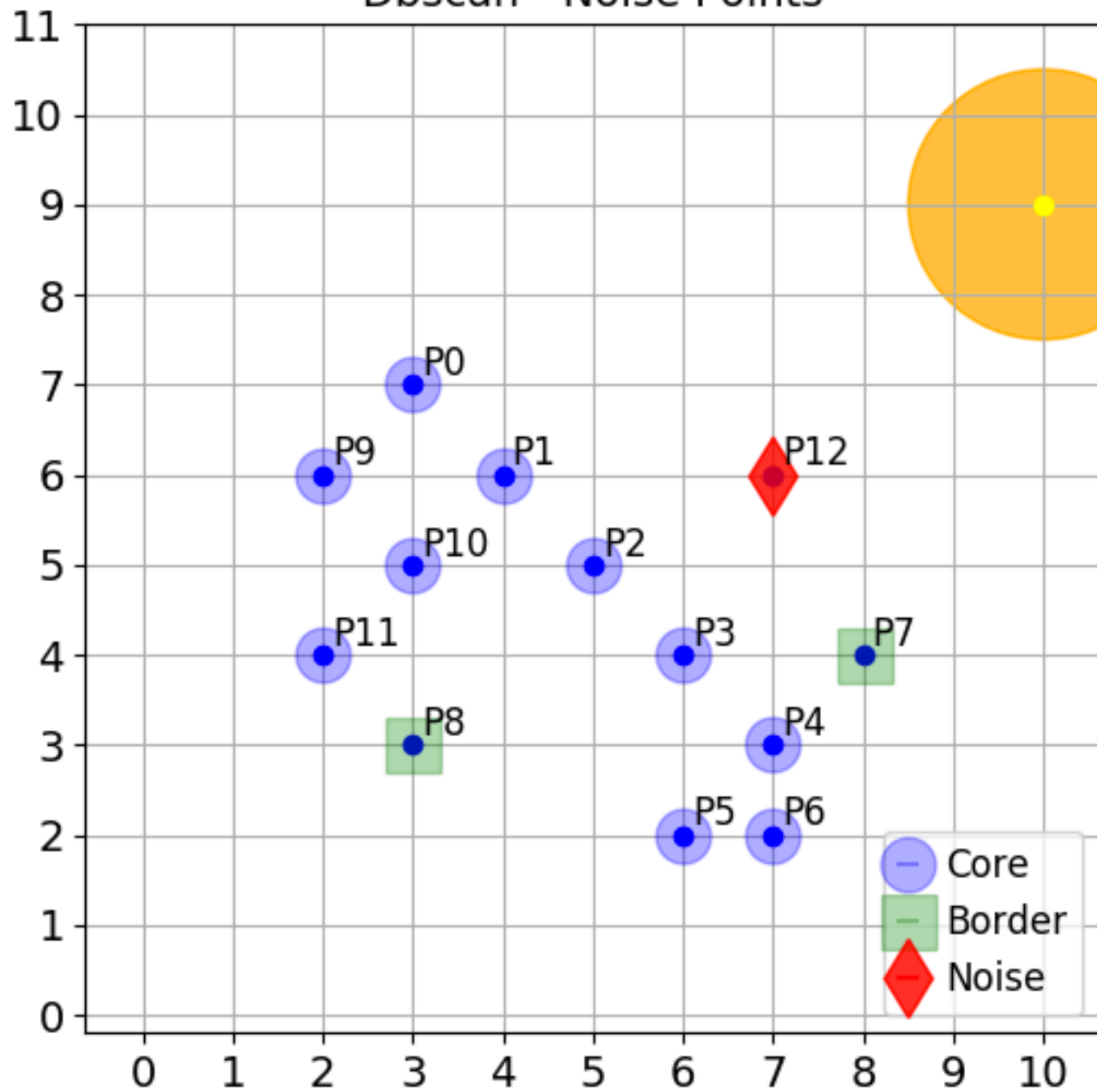
- P8
- P7

## NOISE POINTS

- P12



# Dbscan - Noise Points



# Hierarchical

|    |   |   |   |   |
|----|---|---|---|---|
| P0 | X | 1 | Y | 3 |
| P1 | X | 5 | Y | 5 |
| P2 | X | 4 | Y | 3 |
| P3 | X | 4 | Y | 1 |
| P4 | X | 3 | Y | 1 |
| P5 | X | 3 | Y | 2 |

## Euclidean Distance

$$\left( (x_0 - x_1)^2 + (y_0 - y_1)^2 \right)^{1/2}$$

## Distance Matrix

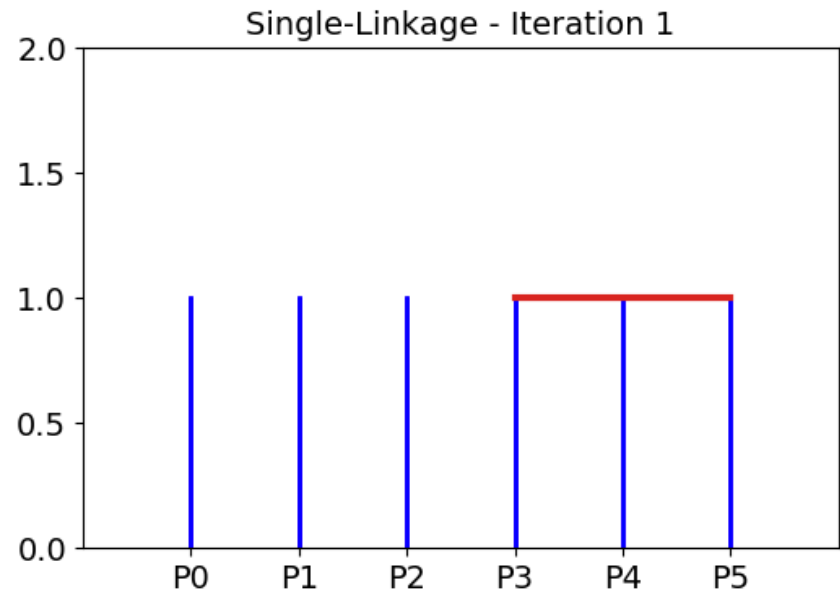
|    | (0,) | (1,) | (2,) | (3,) | (4,) | (5,) |
|----|------|------|------|------|------|------|
| P0 | 0.0  | 4.47 | 3.0  | 3.61 | 2.83 | 2.24 |
| P1 | 4.47 | 0.0  | 2.24 | 4.12 | 4.47 | 3.61 |
| P2 | 3.0  | 2.24 | 0.0  | 2.0  | 2.24 | 1.41 |
| P3 | 3.61 | 4.12 | 2.0  | 0.0  | 1.0  | 1.41 |
| P4 | 2.83 | 4.47 | 2.24 | 1.0  | 0.0  | 1.0  |
| P5 | 2.24 | 3.61 | 1.41 | 1.41 | 1.0  | 0.0  |

# Hierarchical: Single-LINK

Distance Matrix

|   | (0,) | (1,) | (2,) | (3,) | (4,) | (5,) |
|---|------|------|------|------|------|------|
| 0 | 0.0  | 4.47 | 3.0  | 3.61 | 2.83 | 2.24 |
| 1 | 4.47 | 0.0  | 2.24 | 4.12 | 4.47 | 3.61 |
| 2 | 3.0  | 2.24 | 0.0  | 2.0  | 2.24 | 1.41 |
| 3 | 3.61 | 4.12 | 2.0  | 0.0  | 1.0  | 1.41 |
| 4 | 2.83 | 4.47 | 2.24 | 1.0  | 0.0  | 1.0  |
| 5 | 2.24 | 3.61 | 1.41 | 1.41 | 1.0  | 0.0  |

Minimum Distance





# Hierarchical: Single-LINK

Distance Matrix

|         | (0,) | (1,) | (2,) | (3, 4, 5) |
|---------|------|------|------|-----------|
| 0       | 0.0  | 4.47 | 3.0  |           |
| 1       | 4.47 | 0.0  | 2.24 |           |
| 2       | 3.0  | 2.24 | 0.0  |           |
| (3,4,5) |      |      |      | 0.0       |

$D([3,4,5], 0) =$

**Min Distance**

|   | (0,) | (1,) | (2,) | (3,) | (4,) | (5,) |
|---|------|------|------|------|------|------|
| 0 | 0.0  | 4.47 | 3.0  | 3.61 | 2.83 | 2.24 |
| 1 | 4.47 | 0.0  | 2.24 | 4.12 | 4.47 | 3.61 |
| 2 | 3.0  | 2.24 | 0.0  | 2.0  | 2.24 | 1.41 |
| 3 | 3.61 | 4.12 | 2.0  | 0.0  | 1.0  | 1.41 |
| 4 | 2.83 | 4.47 | 2.24 | 1.0  | 0.0  | 1.0  |
| 5 | 2.24 | 3.61 | 1.41 | 1.41 | 1.0  | 0.0  |

# Hierarchical: Single-LINK

Distance Matrix

|         | (0,) | (1,) | (2,) | (3, 4, 5) |
|---------|------|------|------|-----------|
| 0       | 0.0  | 4.47 | 3.0  | 2.24      |
| 1       | 4.47 | 0.0  | 2.24 |           |
| 2       | 3.0  | 2.24 | 0.0  |           |
| (3,4,5) | 2.24 |      |      | 0.0       |

$D([3,4,5], 1) =$

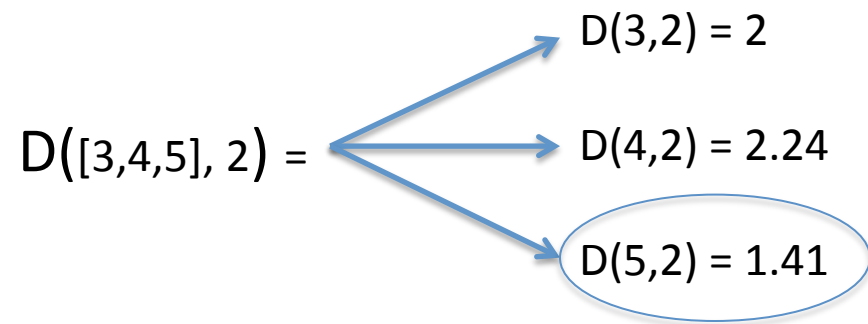
**Min Distance**

|   | (0,) | (1,) | (2,) | (3,) | (4,) | (5,) |
|---|------|------|------|------|------|------|
| 0 | 0.0  | 4.47 | 3.0  | 3.61 | 2.83 | 2.24 |
| 1 | 4.47 | 0.0  | 2.24 | 4.12 | 4.47 | 3.61 |
| 2 | 3.0  | 2.24 | 0.0  | 2.0  | 2.24 | 1.41 |
| 3 | 3.61 | 4.12 | 2.0  | 0.0  | 1.0  | 1.41 |
| 4 | 2.83 | 4.47 | 2.24 | 1.0  | 0.0  | 1.0  |
| 5 | 2.24 | 3.61 | 1.41 | 1.41 | 1.0  | 0.0  |

# Hierarchical: Single-LINK

Distance Matrix

|         | (0,) | (1,) | (2,) | (3, 4, 5) |
|---------|------|------|------|-----------|
| 0       | 0.0  | 4.47 | 3.0  | 2.24      |
| 1       | 4.47 | 0.0  | 2.24 | 3.61      |
| 2       | 3.0  | 2.24 | 0.0  |           |
| (3,4,5) | 2.24 | 3.61 |      | 0.0       |



|   | (0,) | (1,) | (2,) | (3,) | (4,) | (5,) |
|---|------|------|------|------|------|------|
| 0 | 0.0  | 4.47 | 3.0  | 3.61 | 2.83 | 2.24 |
| 1 | 4.47 | 0.0  | 2.24 | 4.12 | 4.47 | 3.61 |
| 2 | 3.0  | 2.24 | 0.0  | 2.0  | 2.24 | 1.41 |
| 3 | 3.61 | 4.12 | 2.0  | 0.0  | 1.0  | 1.41 |
| 4 | 2.83 | 4.47 | 2.24 | 1.0  | 0.0  | 1.0  |
| 5 | 2.24 | 3.61 | 1.41 | 1.41 | 1.0  | 0.0  |

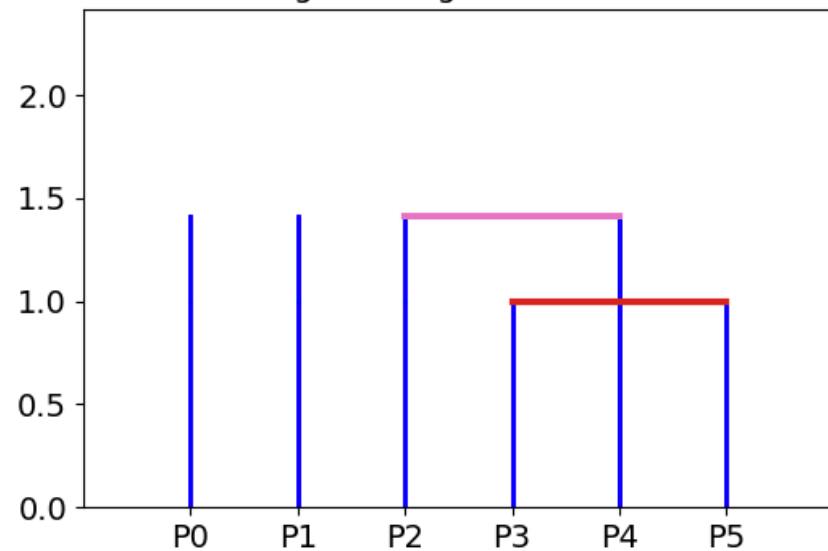
# Hierarchical: Single-LINK

Distance Matrix

|         | (0,) | (1,) | (2,) | (3, 4, 5) |
|---------|------|------|------|-----------|
| 0       | 0.0  | 4.47 | 3.0  | 2.24      |
| 1       | 4.47 | 0.0  | 2.24 | 3.61      |
| 2       | 3.0  | 2.24 | 0.0  | 1.41      |
| (3,4,5) | 2.24 | 3.61 | 1.41 | 0.0       |

**Minimum Distance**

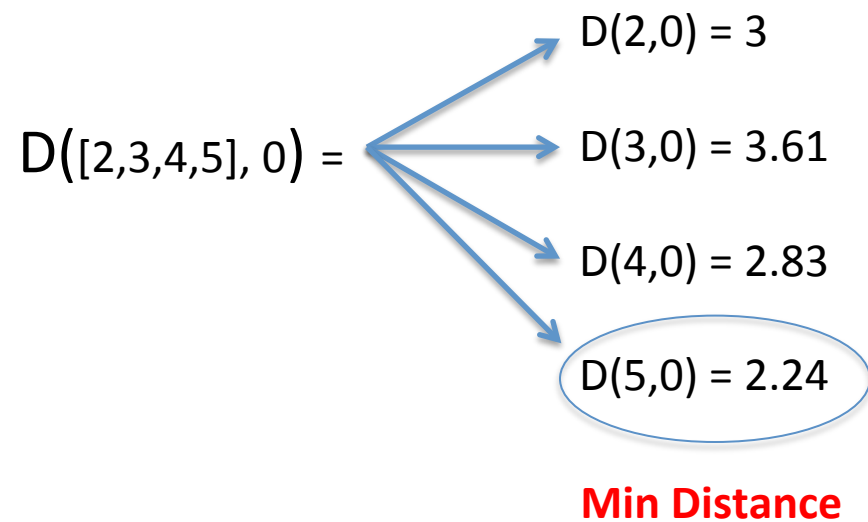
Single-Linkage - Iteration 2



# Hierarchical: Single-LINK

Distance Matrix

|           | ((0,)) | ((1,)) | ((2,), (3, 4, 5)) |
|-----------|--------|--------|-------------------|
| 0         | 0.0    | 4.47   |                   |
| 1         | 4.47   | 0.0    |                   |
| (2,3,4,5) |        |        | 0.0               |



|   | (0,) | (1,) | (2,) | (3,) | (4,) | (5,) |
|---|------|------|------|------|------|------|
| 0 | 0.0  | 4.47 | 3.0  | 3.61 | 2.83 | 2.24 |
| 1 | 4.47 | 0.0  | 2.24 | 4.12 | 4.47 | 3.61 |
| 2 | 3.0  | 2.24 | 0.0  | 2.0  | 2.24 | 1.41 |
| 3 | 3.61 | 4.12 | 2.0  | 0.0  | 1.0  | 1.41 |
| 4 | 2.83 | 4.47 | 2.24 | 1.0  | 0.0  | 1.0  |
| 5 | 2.24 | 3.61 | 1.41 | 1.41 | 1.0  | 0.0  |

# Hierarchical: Single-LINK

Distance Matrix

|           | ((0,)) | ((1,)) | ((2,), (3, 4, 5)) |
|-----------|--------|--------|-------------------|
| 0         | 0.0    | 4.47   | 2.24              |
| 1         | 4.47   | 0.0    |                   |
| (2,3,4,5) | 2.24   |        | 0.0               |

$D([2,3,4,5], 1) =$ 

- $D(2,1) = 2.24$
- $D(3,1) = 4.12$
- $D(4,1) = 4.47$
- $D(5,1) = 3.61$

**Min Distance**

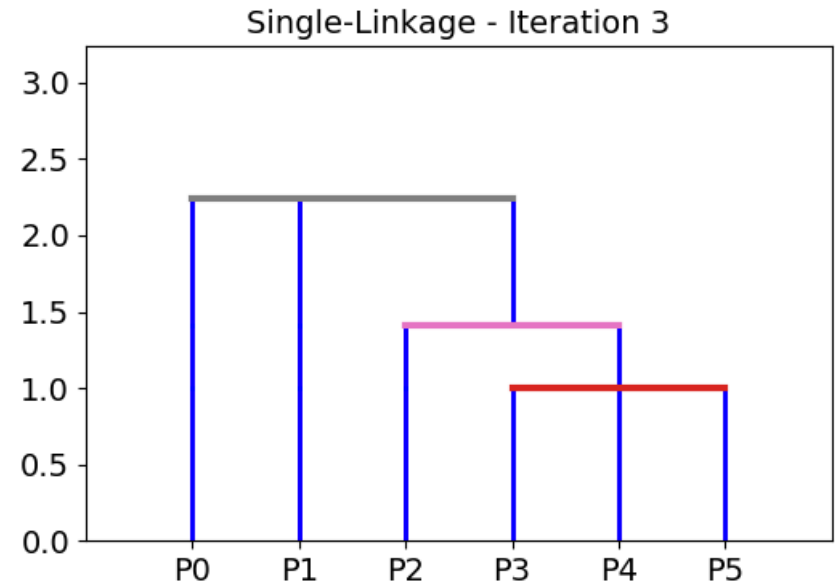
|   | (0,) | (1,) | (2,) | (3,) | (4,) | (5,) |
|---|------|------|------|------|------|------|
| 0 | 0.0  | 4.47 | 3.0  | 3.61 | 2.83 | 2.24 |
| 1 | 4.47 | 0.0  | 2.24 | 4.12 | 4.47 | 3.61 |
| 2 | 3.0  | 2.24 | 0.0  | 2.0  | 2.24 | 1.41 |
| 3 | 3.61 | 4.12 | 2.0  | 0.0  | 1.0  | 1.41 |
| 4 | 2.83 | 4.47 | 2.24 | 1.0  | 0.0  | 1.0  |
| 5 | 2.24 | 3.61 | 1.41 | 1.41 | 1.0  | 0.0  |

# Hierarchical: Single-LINK

Distance Matrix

|           | ((0,)) | ((1,)) | ((2,), (3, 4, 5)) |
|-----------|--------|--------|-------------------|
| 0         | 0.0    | 4.47   | 2.24              |
| 1         | 4.47   | 0.0    | 2.24              |
| (2,3,4,5) | 2.24   | 2.24   | 0.0               |

**Minimum Distance**



# Hierarchical – Complete Link

|    |   |   |   |   |
|----|---|---|---|---|
| P0 | X | 1 | Y | 3 |
| P1 | X | 5 | Y | 5 |
| P2 | X | 4 | Y | 3 |
| P3 | X | 4 | Y | 1 |
| P4 | X | 3 | Y | 1 |
| P5 | X | 3 | Y | 2 |

**Euclidean Distance**

$$\left( (x_0 - x_1)^2 + (y_0 - y_1)^2 \right)^{1/2}$$

**Distance Matrix**

|    | (0,) | (1,) | (2,) | (3,) | (4,) | (5,) |
|----|------|------|------|------|------|------|
| P0 | 0.0  | 4.47 | 3.0  | 3.61 | 2.83 | 2.24 |
| P1 | 4.47 | 0.0  | 2.24 | 4.12 | 4.47 | 3.61 |
| P2 | 3.0  | 2.24 | 0.0  | 2.0  | 2.24 | 1.41 |
| P3 | 3.61 | 4.12 | 2.0  | 0.0  | 1.0  | 1.41 |
| P4 | 2.83 | 4.47 | 2.24 | 1.0  | 0.0  | 1.0  |
| P5 | 2.24 | 3.61 | 1.41 | 1.41 | 1.0  | 0.0  |



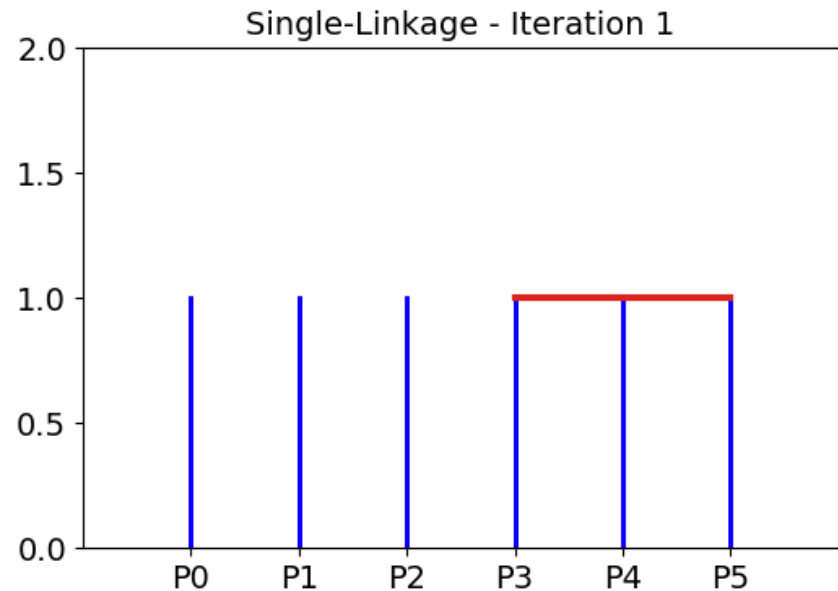
# Hierarchical: Complete-LINK

Distance Matrix

|   | (0,) | (1,) | (2,) | (3,) | (4,) | (5,) |
|---|------|------|------|------|------|------|
| 0 | 0.0  | 4.47 | 3.0  | 3.61 | 2.83 | 2.24 |
| 1 | 4.47 | 0.0  | 2.24 | 4.12 | 4.47 | 3.61 |
| 2 | 3.0  | 2.24 | 0.0  | 2.0  | 2.24 | 1.41 |
| 3 | 3.61 | 4.12 | 2.0  | 0.0  | 1.0  | 1.41 |
| 4 | 2.83 | 4.47 | 2.24 | 1.0  | 0.0  | 1.0  |
| 5 | 2.24 | 3.61 | 1.41 | 1.41 | 1.0  | 0.0  |

Minimum Distance

First Step **equal** to SINGLE LINK



# Hierarchical: Complete-LINK

Distance Matrix

|         | (0,) | (1,) | (2,) | (3, 4, 5) |
|---------|------|------|------|-----------|
| 0       | 0.0  | 4.47 | 3.0  |           |
| 1       | 4.47 | 0.0  | 2.24 |           |
| 2       | 3.0  | 2.24 | 0.0  |           |
| (3,4,5) |      |      |      | 0.0       |

$D([3,4,5], 0) =$ 

- $D(3,0) = 3.61$
- $D(4,0) = 2.83$
- $D(5,0) = 2.24$

**Max Distance**

|   | (0,) | (1,) | (2,) | (3,) | (4,) | (5,) |
|---|------|------|------|------|------|------|
| 0 | 0.0  | 4.47 | 3.0  | 3.61 | 2.83 | 2.24 |
| 1 | 4.47 | 0.0  | 2.24 | 4.12 | 4.47 | 3.61 |
| 2 | 3.0  | 2.24 | 0.0  | 2.0  | 2.24 | 1.41 |
| 3 | 3.61 | 4.12 | 2.0  | 0.0  | 1.0  | 1.41 |
| 4 | 2.83 | 4.47 | 2.24 | 1.0  | 0.0  | 1.0  |
| 5 | 2.24 | 3.61 | 1.41 | 1.41 | 1.0  | 0.0  |

# Hierarchical: Complete-LINK

Distance Matrix

|         | (0,) | (1,) | (2,) | (3, 4, 5) |
|---------|------|------|------|-----------|
| 0       | 0.0  | 4.47 | 3.0  | 3.61      |
| 1       | 4.47 | 0.0  | 2.24 |           |
| 2       | 3.0  | 2.24 | 0.0  |           |
| (3,4,5) | 3.61 |      |      | 0.0       |

$D([3,4,5], 1) =$

**Max Distance**

|   | (0,) | (1,) | (2,) | (3,) | (4,) | (5,) |
|---|------|------|------|------|------|------|
| 0 | 0.0  | 4.47 | 3.0  | 3.61 | 2.83 | 2.24 |
| 1 | 4.47 | 0.0  | 2.24 | 4.12 | 4.47 | 3.61 |
| 2 | 3.0  | 2.24 | 0.0  | 2.0  | 2.24 | 1.41 |
| 3 | 3.61 | 4.12 | 2.0  | 0.0  | 1.0  | 1.41 |
| 4 | 2.83 | 4.47 | 2.24 | 1.0  | 0.0  | 1.0  |
| 5 | 2.24 | 3.61 | 1.41 | 1.41 | 1.0  | 0.0  |

# Hierarchical: Complete-LINK

Distance Matrix

|         | (0,) | (1,) | (2,) | (3, 4, 5) |
|---------|------|------|------|-----------|
| 0       | 0.0  | 4.47 | 3.0  | 3.61      |
| 1       | 4.47 | 0.0  | 2.24 |           |
| 2       | 3.0  | 2.24 | 0.0  |           |
| (3,4,5) | 3.61 | 4.47 |      | 0.0       |

$$D([3,4,5], 2) = \begin{cases} D(3,2) = 2 \\ D(4,2) = 2.24 \\ D(5,2) = 1.41 \end{cases}$$

**Max Distance**

|   | (0,) | (1,) | (2,) | (3,) | (4,) | (5,) |
|---|------|------|------|------|------|------|
| 0 | 0.0  | 4.47 | 3.0  | 3.61 | 2.83 | 2.24 |
| 1 | 4.47 | 0.0  | 2.24 | 4.12 | 4.47 | 3.61 |
| 2 | 3.0  | 2.24 | 0.0  | 2.0  | 2.24 | 1.41 |
| 3 | 3.61 | 4.12 | 2.0  | 0.0  | 1.0  | 1.41 |
| 4 | 2.83 | 4.47 | 2.24 | 1.0  | 0.0  | 1.0  |
| 5 | 2.24 | 3.61 | 1.41 | 1.41 | 1.0  | 0.0  |

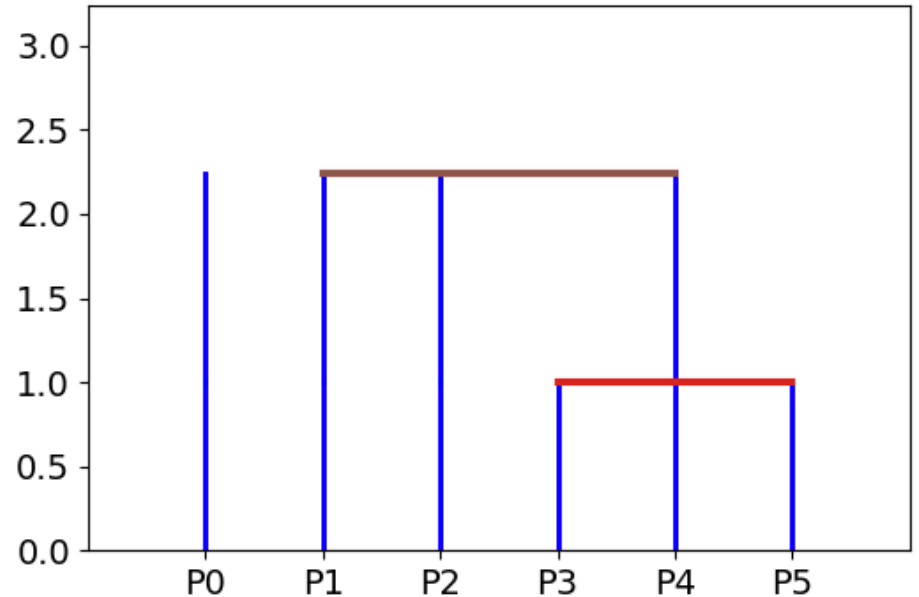
# Hierarchical: Complete-LINK

Distance Matrix

|         | (0,) | (1,) | (2,) | (3, 4, 5) |
|---------|------|------|------|-----------|
| 0       | 0.0  | 4.47 | 3.0  | 3.61      |
| 1       | 4.47 | 0.0  | 2.24 | 4.47      |
| 2       | 3.0  | 2.24 | 0.0  | 2.24      |
| (3,4,5) | 3.61 | 4.47 | 2.24 | 0.0       |

**Minimum Distance**

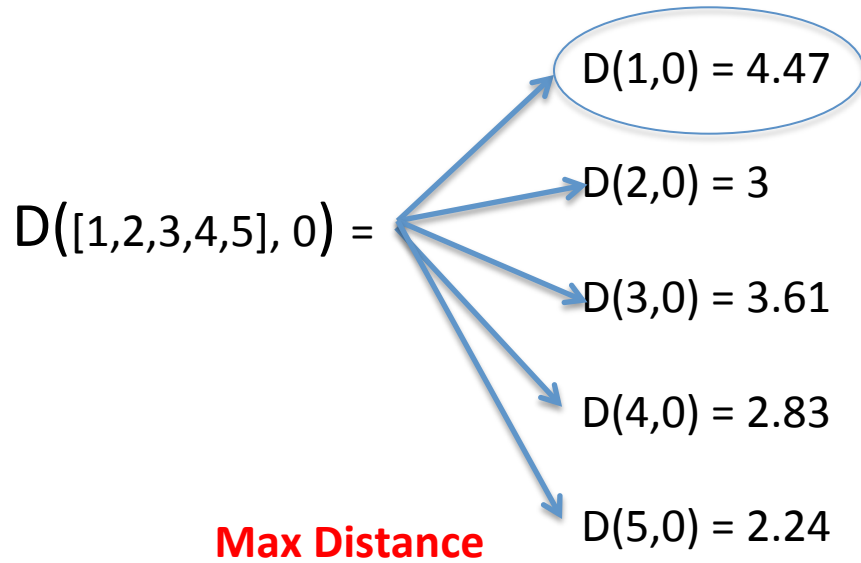
Complete-Linkage - Iteration 2



# Hierarchical: Complete-LINK

Distance Matrix

|             | ((0,)) | ((1,), (2,), (3, 4, 5)) |
|-------------|--------|-------------------------|
| 0           | 0.0    |                         |
| (1,2,3,4,5) |        | 0.0                     |



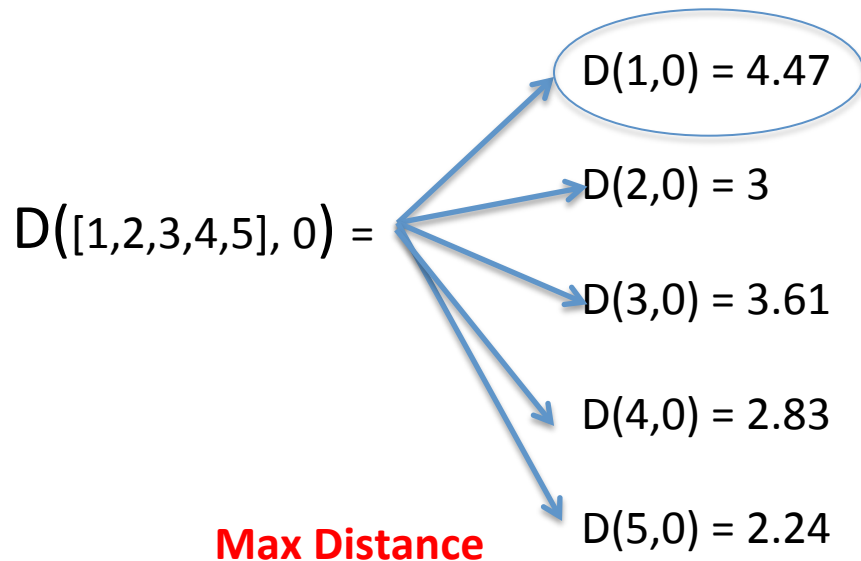
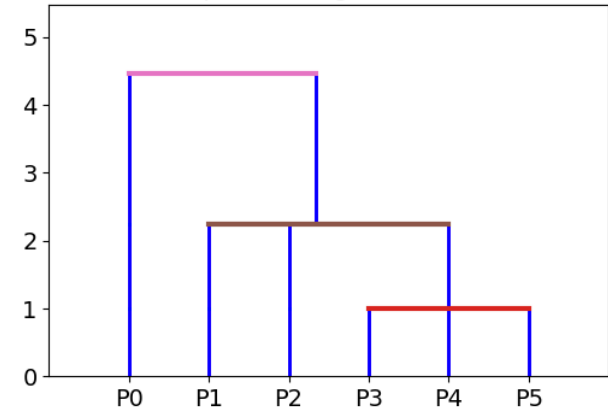
|   | (0,) | (1,) | (2,) | (3,) | (4,) | (5,) |
|---|------|------|------|------|------|------|
| 0 | 0.0  | 4.47 | 3.0  | 3.61 | 2.83 | 2.24 |
| 1 | 4.47 | 0.0  | 2.24 | 4.12 | 4.47 | 3.61 |
| 2 | 3.0  | 2.24 | 0.0  | 2.0  | 2.24 | 1.41 |
| 3 | 3.61 | 4.12 | 2.0  | 0.0  | 1.0  | 1.41 |
| 4 | 2.83 | 4.47 | 2.24 | 1.0  | 0.0  | 1.0  |
| 5 | 2.24 | 3.61 | 1.41 | 1.41 | 1.0  | 0.0  |

# Hierarchical: Complete-LINK

Distance Matrix

|             | ((0,)) | ((1,), (2,), (3, 4, 5)) |
|-------------|--------|-------------------------|
| 0           | 0.0    | 4.47                    |
| (1,2,3,4,5) | 4.47   | 0.0                     |

Complete-Linkage - Iteration 3



|   | (0,) | (1,) | (2,) | (3,) | (4,) | (5,) |
|---|------|------|------|------|------|------|
| 0 | 0.0  | 4.47 | 3.0  | 3.61 | 2.83 | 2.24 |
| 1 | 4.47 | 0.0  | 2.24 | 4.12 | 4.47 | 3.61 |
| 2 | 3.0  | 2.24 | 0.0  | 2.0  | 2.24 | 1.41 |
| 3 | 3.61 | 4.12 | 2.0  | 0.0  | 1.0  | 1.41 |
| 4 | 2.83 | 4.47 | 2.24 | 1.0  | 0.0  | 1.0  |
| 5 | 2.24 | 3.61 | 1.41 | 1.41 | 1.0  | 0.0  |