Silhouette Coefficients

Given the points:
P0 = {1,0}
P1 = {1,1}
P2 = {1,2}
P3 = {2,3}
P4 = {2,2}
P5 = {1,2}
P6 = {3,1}
P7= {3,3}
P8 = {2,1}

Suppose this cluster compositions:

Cluster 1 ={P0, P1}
Cluster 2 ={P2, P3, P4, P5},
Cluster 3 = {P6, P7, P8}

Consider the point P0={1,0}
- Calculate its average distance to all other points in it's cluster, i.e. Cluster 1

$$\mathbf{a1 = \sqrt{(1-1)^2 + (0-1)^2} = \sqrt{(0+1)} = \sqrt{1} = 1}$$

- Now for the object {1,0} in Cluster 1 calculate its average distance from all the objects in Cluster 2 and Cluster 3 and take the **minimum** average distance.

So for Cluster 2

{1,0} ----> {1,2} = distance = $\sqrt{((1-1)^2 + (0-2)^2)} = \sqrt{(0+4)} = \sqrt{4} = 2$
{1,0} ----> {2,3} = distance = $\sqrt{((1-2)^2 + (0-3)^2)} = \sqrt{(1+9)} = \sqrt{10} = 3.16$
{1,0} ----> {2,2} = distance = $\sqrt{((1-2)^2 + (0-2)^2)} = \sqrt{(1+4)} = \sqrt{5} = 2.24$
{1,0} ----> {1,2} = distance = $\sqrt{((1-1)^2 + (0-2)^2)} = \sqrt{(0+4)} = \sqrt{4} = 2$

Therefore, the average distance of point P0 = {1,0} in Cluster 1 to all the points in cluster 2 is:

(2+3.16+2.24+2)/4 = 2.325

Similarly, for Cluster 3:
{1,0} ----> {3,1} = distance = √((1-3)^2 + (0-1)^2) =√(4+1)=√5=2.24
{1,0} ----> {3,3} = distance = √((1-3)^2 + (0-3)^2) =√(4+9)=√13=3.61
{1,0} ----> {2,1} = distance = √((1-2)^2 + (0-1)^2) =√(1+1)=√2=2.24

Therefore, the **average distance** of point P0={1,0} in Cluster 1 to all the points in Cluster 3 is:

(2.24+3.61+2.24)/3 = 2.7

Now, the **minimum** average distance of the point P0={1,0} in Cluster 1 to the other Clusters 2 and 3 is:

b1 =2.325

The silhouette coefficient S1 is:

s1= (b1-a1)/max(a1, b1) = (2.325 -1)/2.325)= 0.5699

You need to do this computation for every point.

For computing the **cluster silhouette coefficients** of points belonging to the cluster: compute the **average** silhouette coefficient of all points.

For computing the **clustering silhouette coefficients**: compute the **average** silhouette coefficient of all points in the data.

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