Algebra booleana

$\{0, 1\}$ $\{T, F\}$

AND, OR, NOT

Tabella di verità

input → output

Ingressi	uscita
combinazioni degli ingressi	uscita corrispondente

AND binario

x_1	x_2	z
0	0	0
0	1	0
1	0	0
1	1	1

OR binario

x_1	x_2	z
0	0	0
0	1	1
1	0	1
1	1	1

NOT unario

x_1	z
0	1
1	0

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PROPRIETÀ

$+ \Leftrightarrow$ OR $\quad \cdot \Leftrightarrow$ AND $\quad \bar{x} \Leftrightarrow$ not

A	B	C	$(A+B)$	$(A+B)+C$	A	$(B+C)$	$A+(B+C)$
0	0	0	0	0	0	0	0
0	1	0	1	1	0	1	1
1	0	0	1	1	1	0	1
1	1	0	1	1	1	1	1
0	0	1	0	1	0	1	1
0	1	1	1	1	0	1	1
1	0	1	1	1	1	1	1
1	1	1	1	1	1	1	1

Complementazione $\quad A + \bar{A} = 1 \qquad A \cdot \bar{A} = 0$

Involuzione $\quad \bar{\bar{A}} = A$

Potenza identica $\quad A + A = A \qquad A \cdot A = A$

Unione & Intersezione $\quad A + 0 = A \qquad A \cdot 0 = 0$

$A + 1 = 1 \qquad A \cdot 1 = A$

Commutatività $\quad A + B = B + A \qquad A \cdot B = B \cdot A$

Associatività $\quad (A+B)+C = A+(B+C) \qquad (A \cdot B) \cdot C = A \cdot (B \cdot C)$

Distributività $\quad A(B+C) = AB + AC \qquad A + (BC) = (A+B) \cdot (A+C)$

De Morgan $\quad \overline{A+B} = \bar{A} \cdot \bar{B} \qquad \overline{A \cdot B} = \bar{A} + \bar{B}$

SOMMATORE di 2 bit

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x_1 □ +

x_2 □ =

ow □ □ z

↑ overflow

X_1	X_2	Z	ow
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$Z =$ expr dell'alg. booleana

$OW =$ " " " —

Somma di prodotti

$$Z = \overline{X_1} \cdot X_2 + X_1 \cdot \overline{X_2}$$

$X_1 X_2$	$\overline{X_1}$	$\overline{X_1} X_2$	$\overline{X_2}$	$X_1 \overline{X_2}$	+
0 0	1	0	1	0	0
0 1	1	1	0	0	1
1 0	0	0	1	1	1
1 1	0	0	0	0	0

(1) ∀ degli "1" nella colonna delle uscite prendo un prodotto (AND) degli ingressi negati se l'ingresso è a ∅ o normali altrimenti

(2) i termini che ottengo li "sommo" (OR)

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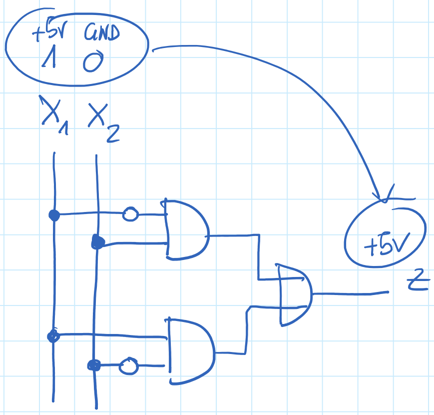
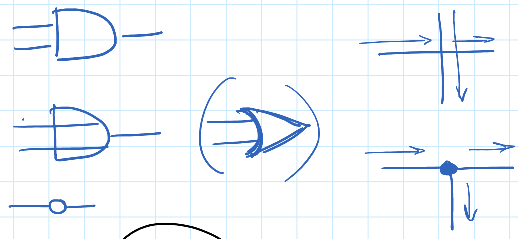
$$Z = \overline{X_1} X_2 + X_1 \overline{X_2}$$

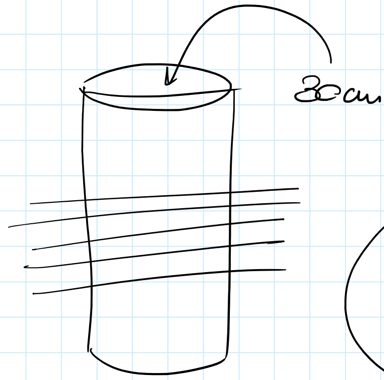


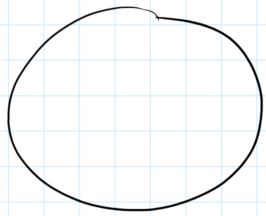
AND

OR

NOT

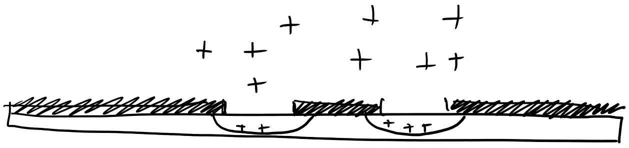


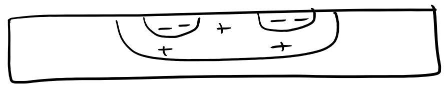


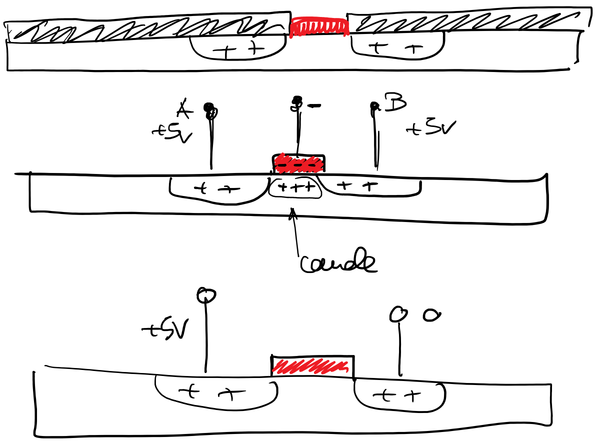


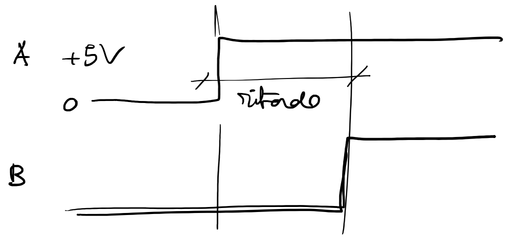
$1 = +5V$

$\phi = GND \quad \phi V$






A
B
+5V
+5V
canale
+5V


A
+5V
0
ritardo
B

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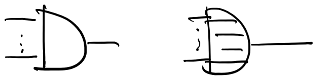
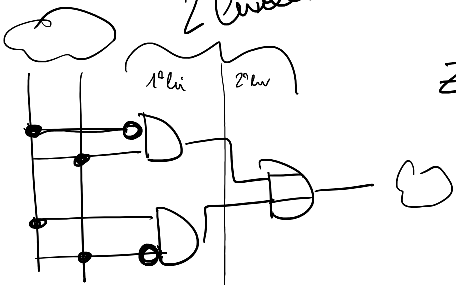
al più 8 ingressi

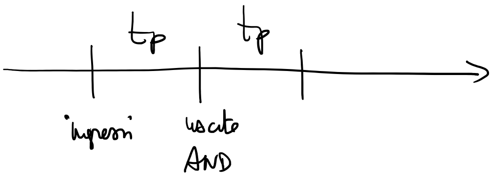


ritardo di $1\, t_p$

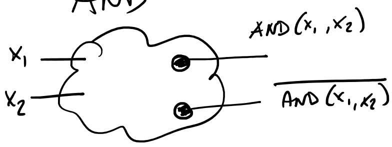
2 livelli



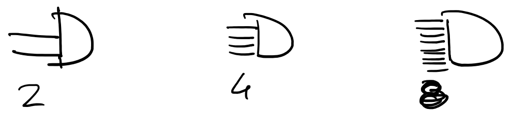
$$Z = \underbrace{\bar{x}_1 x_2 + x_1 \bar{x}_2}$$

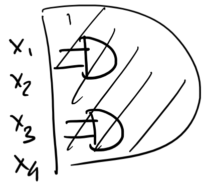


AND

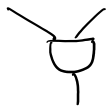


$$(AB)C = A(BC)$$





$$x_1 \cdot x_2 \cdot x_3 \cdot x_4$$
$$=$$
$$x_1 \cdot x_2 \qquad x_3 \cdot x_4$$

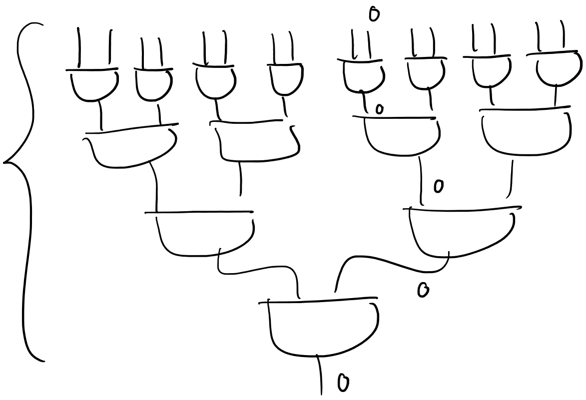


porte da 2 ingressi

16 ingressi

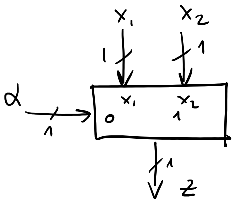
livelli?

$\lceil \lg_2 16 \rceil$



FUNZIONI (BOOLEANE)

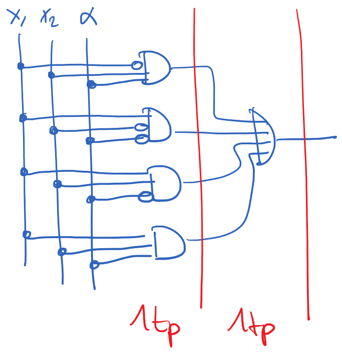
COMMUTATORE



x_1	x_2	α	z
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

4 "1"

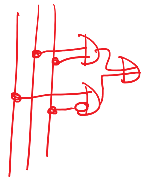
$$Z = \bar{x}_1 x_2 \alpha + x_1 \bar{x}_2 \bar{\alpha} + x_1 x_2 \bar{\alpha} + x_1 x_2 \alpha$$



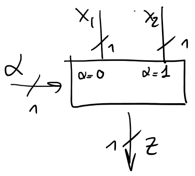
$$\bar{x}_1 x_2 \alpha + x_1 x_2 \alpha$$

$$\underbrace{(\bar{x}_1 + x_1)}_{1} x_2 \alpha = x_2 \alpha$$

1tp 1tp



$$Z = x_2 \alpha + \underbrace{x_1 \bar{x}_2 \bar{\alpha} + x_1 x_2 \bar{\alpha}}_{(x_1 \bar{\alpha})\underbrace{(\bar{x}_2 + x_2)}_{1}} = x_2 \alpha + x_1 \bar{\alpha}$$



x_1	x_2	α	z
0	–	0	0
1	–	0	1
–	0	1	0
–	1	1	1

non specificati
"don't care"

$z = \underbrace{\quad}\ \underbrace{\quad}$

nei prodotti
non tutti gli ingressi
non specificati

$z = x_1\bar{\alpha} + x_2\alpha$

Karnaugh

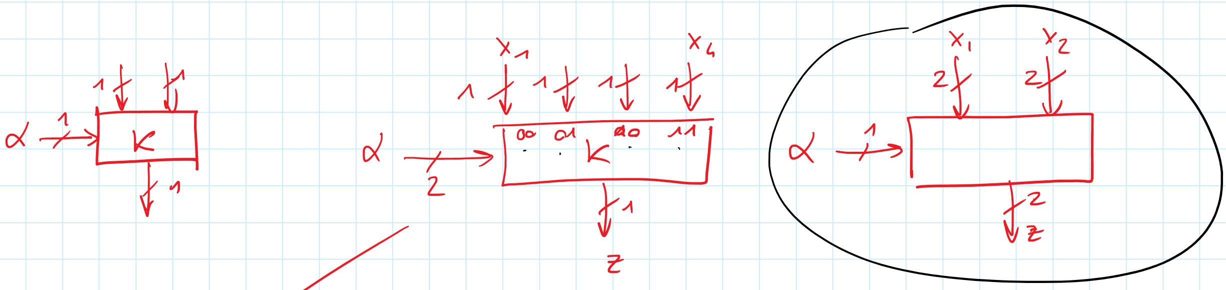
$\alpha \backslash x_1x_2$	00	01	11	10
0	0	0	1	1
1	0	1	1	0

$\bar{\alpha}x_1\cancelto{1}{(x_2+\bar{x}_2)}$

$\alpha x_2\cancel{(x_1+\bar{x}_1)}$

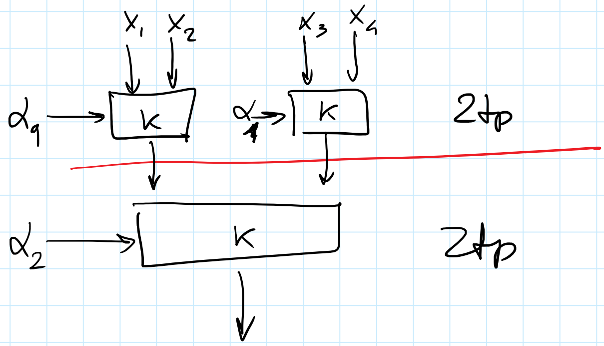
$+ \Big\} \ \bar{\alpha}x_1 + \alpha x_2$

Co



x_1	x_2	x_2	x_3	$\alpha_1 \alpha_2$	z
1	–	–	–	00	1
~~0~~	~~–~~	~~–~~	~~–~~	~~00~~	~~0~~
–	1	–	–	01	1
–	–	1	–	10	1
–	–	–	1	11	1

VS.



<8 ingressi AND
~~XAND~~
<8 ingressi OR $\Longrightarrow$ 2tp !?

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