Titolo nota 11/03/2016

TEOREMA DELL' ESPERTO

$$\begin{cases} T(n) = a\,T\left(\frac{n}{b}\right) + f(n) \\ T(n) = \Theta(1) \quad \text{per } n = 1 \end{cases} \qquad a \geq 1 \quad b > 1$$

$$\boxed{n = b^i}$$

metodo iterativo

$$T(n) = a\,T\left(\frac{n}{b}\right) + f(n) \qquad T\left(\frac{n}{b}\right) = a\,T\left(\frac{n}{b^2}\right) + f\left(\frac{n}{b}\right)$$

$$T(n) = a\left(a\,T\left(\frac{n}{b^2}\right) + f\left(\frac{n}{b}\right)\right) + f(n) \qquad T\left(\frac{n}{b^2}\right) = a\,T\left(\frac{n}{b^3}\right) + f\left(\frac{n}{b^2}\right)$$

$$T\left(\frac{n}{b^i}\right) = \Theta(1)$$

$$T(n) = a\left(a\left(a\,T\left(\frac{n}{b^3}\right) + f\left(\frac{n}{b^2}\right)\right) + f\left(\frac{n}{b}\right)\right) + f(n)$$

$$T(n) = a\left(a\left(a \ldots \left(a\,T\left(\frac{n}{b^i}\right) + f\left(\frac{n}{b^{i-1}}\right)\right) + f\left(\frac{n}{b^{i-2}}\right)\right) \ldots + f\left(\frac{n}{b}\right)\right) + f(n)$$

$$T(n) = a^i\, T\left(\frac{n}{b^i}\right) \qquad \sum$$

$$\frac{n}{b^i} = 1$$

$$n = b^i$$

$$\log_b n = \log_b b^i$$

$$i = \log_b n$$

$$= a^i\,\Theta(1) + a^{i-1} f\left(\frac{n}{b^{i-1}}\right) + a^{i-2} f\left(\frac{n}{b^{i-2}}\right) + \dots + a f\left(\frac{n}{b}\right)$$

$$+ a^0 f\left(\frac{n}{b^0}\right)$$

$$= a^{\log_b n} \cdot \Theta(1) + \underbrace{\sum_{i=0}^{\log_b n - 1} a^i f\left(\frac{n}{b^i}\right)}$$

$$\parallel$$

$$= \underbrace{\Theta\left(n^{\log_b a}\right)}_{\text{Risolvi}} + \underbrace{\sum_{i=0}^{\log_b a - 1} \boxed{a^i f\left(\boxed{\frac{n}{b^i}}\right)}}_{\text{Combina e Dividi}}$$

$$n^{\log_b a} \;?\; f(n)$$

1) pesa di più $\Theta\left(n^{\log_b a}\right)$

2) se $\Theta\left(n^{\log_b a}\right) : f(n)$

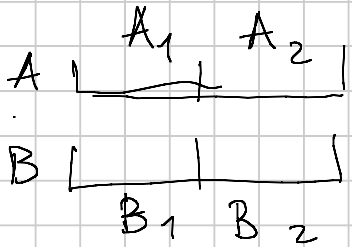
3) $f(n)$ pesa di più

MOLTIPLICAZIONE

carta e penna $\Theta(n^2)$

MOLTEGIZIA $\Theta(n^2)$

MOL_DI ?



A e B di n cifre

bit model (RAM)

A^n

$$\begin{cases} T(n) = \Theta(1) & \text{se } n = 1 \\ T(n) = 4T\left(\frac{n}{2}\right) + \Theta(n) \end{cases}$$

somme e traslazioni

Dividi et Impera

1. Dividi A e B di n cifre in A_1, B_1, A_2, B_2 di n/2 cifre
- Risolvi ricorsivamente
- Combina

$A = 1587 = 15 \cdot 10^2 + 87$

$1500 + 87$

$$A * B = \left(A_1 \cdot 10^{n/2} + A_2\right) * \left(B_1\, 10^{n/2} + B2\right)$$

$$= A_1 B_1 \cdot 10^n + A_1 B_2\, 10^{n/2} + A_2 B_1\, 10^{n/2} + A_2 B_2$$

$$T(n) = 4T\left(\frac{n}{2}\right) + \Theta(n)$$

A $\underbrace{15}_{A_1}\ \underbrace{87}_{A_2}$

$a = 4 \quad b = 2 \quad n^{\log_2 4} = n^2$

$$\begin{array}{r} 15 \\ 87 \\ \hline 102 \end{array}$$ 3 cifre

$n \ ; \ n^2 \qquad f(n) = O(n^{2-\varepsilon}) \ \text{per}\ \varepsilon \leq 1$

caso 1) $T(n) = \Theta(n^2)$

$$(A_1 + A_2)(B_1 + B_2) = (A_1B_1 + A_2B_2 + A_2B_1 + A_1B_2)$$

$$A_2B_1 + A_1B_2 = (A_1 + A_2)(B_1 + B_2) - A_1B_1 - A_2B_2$$

$$A * B = A_1B_1\,10^n + \left((\underbrace{A_1 + A_2}_{}) (B_1 + B_2) - A_1B_1 - A_2B_2\right)10^{n/2} + A_2B_2$$

($n/2 + 1$ cifre

MOLTRAPIDA (X, Y, n)

Risolvi dir. $\Theta(1)$ — if (n == 1) return X * Y

else {

dividi $\Theta(1)$ — " dividi X in X_1 e X_2 di n/2 cifre"

" dividi Y in Y_1 e Y_2 di n/2 cifre"

Risolvi ricorsivamente —

A = MOLTRAPIDA (X_1, Y_1, n/2);

B = MOLTRAPIDA (X_2, Y_2, n/2);

C = MOLTRAPIDA ($X_1 + X_2$, $Y_1 + Y_2$, n/2);

Combina — return ($A\,10^n + (C - A - B)\,10^{n/2} + B$);

}

$T(n) = \Theta(1)$ per $n = 1$

$T(n) = 3T\left(\frac{n}{2}\right) + \Theta(n)$

$a = 3 \quad b = 2$

$n^{\log_2 3} = n^{1.59}$

$n = O\left(n^{1.59-\varepsilon}\right)$

per $\varepsilon \leq 0.59$

caso 1

$$\boxed{T(n) = \Theta\left(n^{1.59}\right)}$$

MOLTIPLICAZIONE MATRICI

A e B $n \times n$ input size : n

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ & & \\ & & \end{bmatrix} + \begin{bmatrix} b_{11} & & \\ & & \\ & & \end{bmatrix} =$$

$S = A + B$ $\Theta(n^2)$

$C = A \times B$ = prodotto righe x colonne

$$c_{ij} = \sum_{k=1}^{n} a_{ik} \cdot b_{kj}$$

$$A \; 2 \begin{bmatrix} & & \\ \cdot & \cdot & \cdot \\ & & \end{bmatrix} B \begin{bmatrix} \cdot & & \\ \cdot & & \\ \cdot & & \end{bmatrix} = \overset{C}{\begin{bmatrix} & & \\ c_{21} & & \\ & & \end{bmatrix}}$$

Calcolare C richiede $\Theta(n^2)$ somme $\Theta(n^3)$ mol

Divide et Impera

NO

$$T(n) = 8\,T\left(\frac{n}{2}\right) + \Theta(n^2)$$

$$T(1) = \Theta(1)$$

$$A \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times B \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = C \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

$$C_{11} = A_{11} \cdot B_{11} + A_{12} \cdot B_{21}$$

$$C_{12} = A_{11} \cdot B_{12} + A_{12} \cdot B_{22}$$

$$C_{21} = A_{21} \cdot B_{11} + A_{22} \cdot B_{21}$$

$$C_{22} = A_{21} \cdot B_{12} + A_{22} \cdot B_{22}$$

$a = 8 \quad b = 2$

$$n^{\log_2 8} = n^3$$

$$f(n) = O(n^{3-\varepsilon}) \quad \varepsilon \leq 1$$

$$T(n) = \Theta(n^3)$$

$X_1 = (A_{11} + A_{22}) \cdot (B_{11} + B_{22})$

$X_2 = (A_{21} + A_{22}) \cdot B_{11}$

$X_3 = A_{11} \cdot (B_{12} - B_{22})$

$X_4 = A_{22} \cdot (B_{21} - B_{11})$

$X_5 = (A_{11} + A_{12}) \cdot B_{22}$

$X_6 = (A_{11} - A_{21})(B_{11} + B_{12})$

$X_7 = (A_{12} - A_{22})(B_{21} + B_{22})$

$C_{11} = X_1 + X_2 - X_5 - X_7$

$C_{12} = X_3 + X_5$

$C_{21} = X_2 + X_4$

$C_{22} = X_1 + X_3 - X_2 - X_6$

$\Omega(n^2)$

$T(n) = 7\, T\left(\frac{n}{2}\right) + 18\, n^2$

$\Theta(n^2)$

$n^{\log_2 a} = n^{\log_2 7} = n^{2.81}$

STRASSEN

$n^2 \leq n^{2.81}$

$T(n) = \Theta(n^{2.81})$

$f(n) = O(n^{2.81-\varepsilon})$

$\varepsilon \leq 0.81$

Prodotti di sottomatrici di size $n/2$

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VINOGRAD $\Theta(n^{2.376})$

$\Downarrow$

limite inferiore $\Omega(n^2)$

Bini, Lotti, Romani, Capovani $\Theta(n^{2.79})$

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