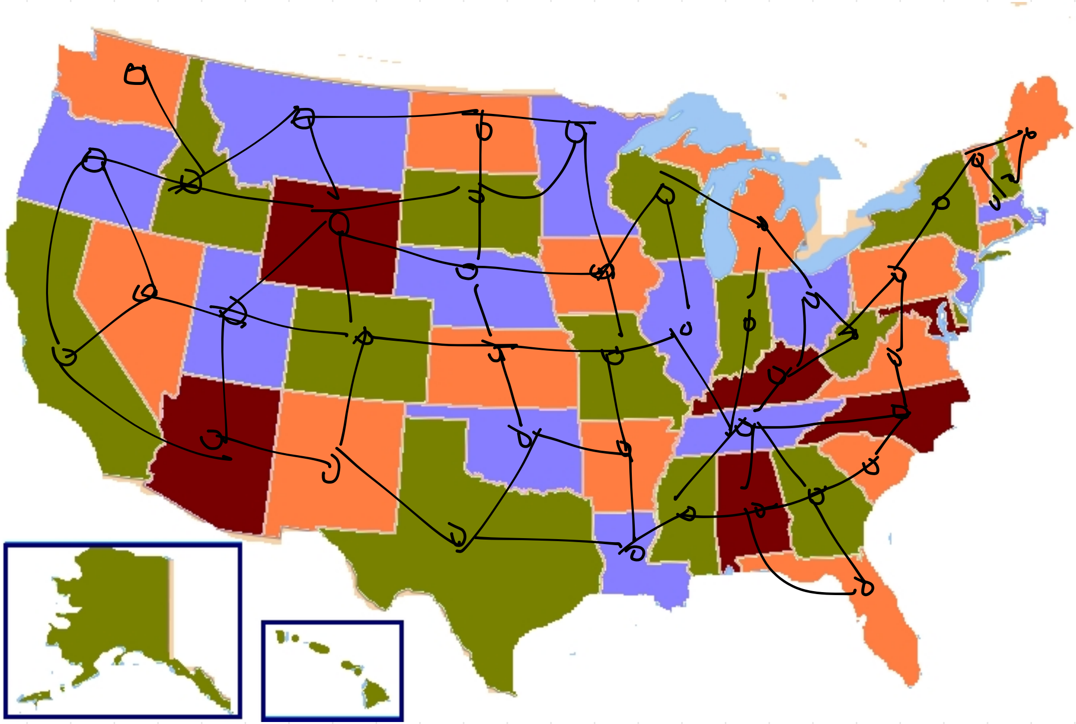
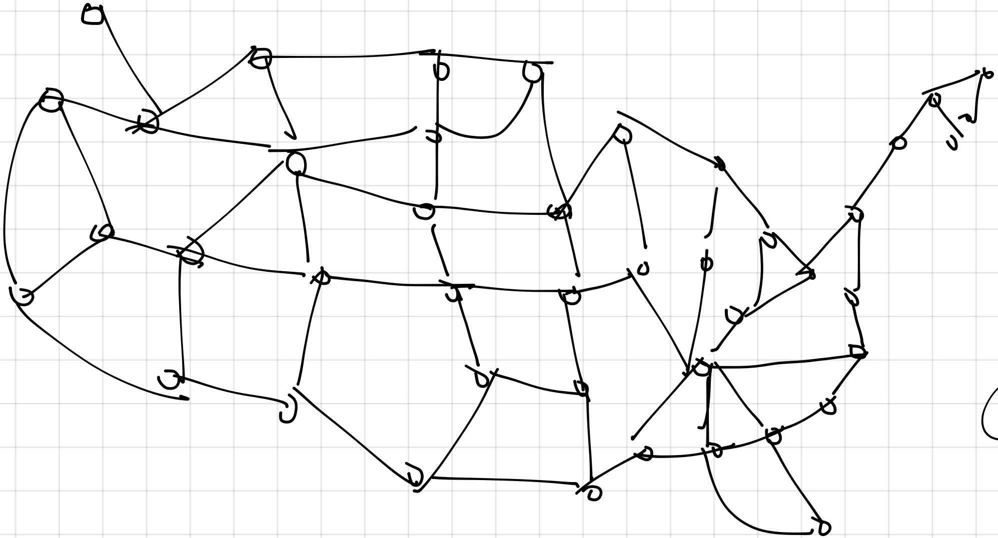




$G = (V, E)$

k-Coloring : $\chi : V \to 1, 2, \dots, k$

s.t. $\forall (u, v) \in E : \chi(u) \neq \chi(v)$

Decision

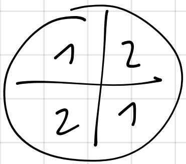
Graph $G=(V,E)$, an integer k

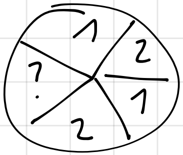
Q: Has G a k-coloring?

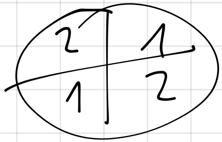
PLANAR GRAPHS

$k \geq 4$: answer = YES (because of 4-col theorem)

$k = 2$: Y. N







$\text{COLOR}(u, c)$: $c \in \{1, 2\}$

$X(u) := c$; $\bar{c} = \{1, 2\} - c$

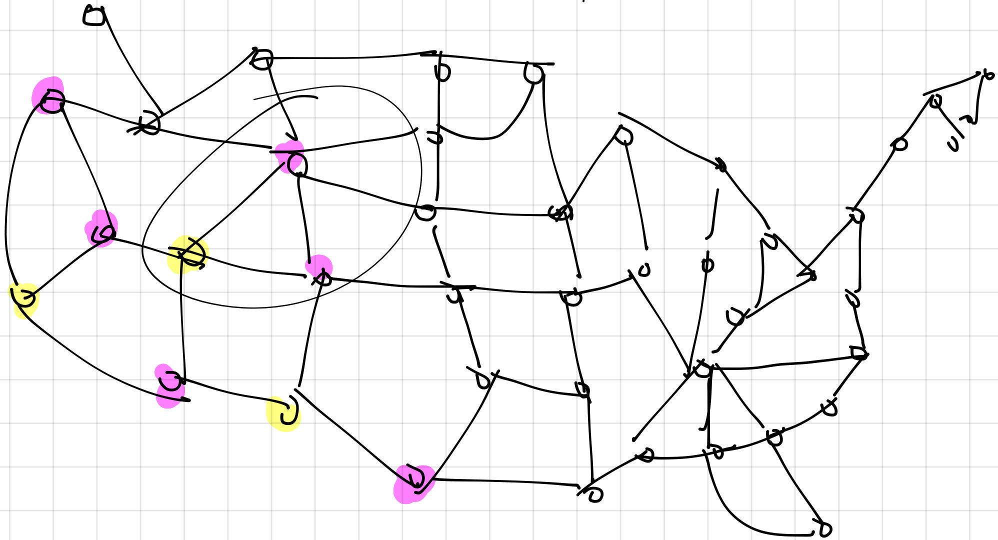
for $v \in adj(u)$ **do**

if $X(v)$ is undefined **then**

$\text{COLOR}(v, \bar{c})$

else if $X(v) = c$ **then** it's not 2-col

NO



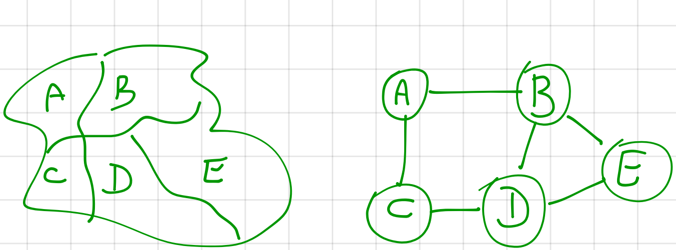
$\boxed{K=3}$ $\in$ NPC   3-COL

proof   3-SAT $\prec$ 3-COL   (too long here)

What we see here is   3-COL $\propto$ 3-SAT

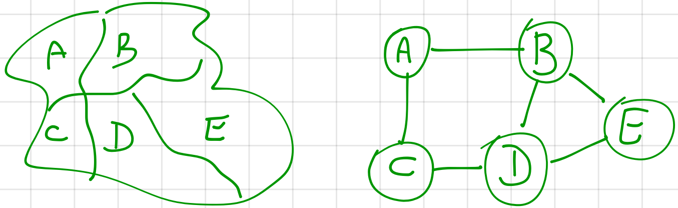
Reduction: show how expressive are Boolean formulas

planar graph



3-COL $\propto$ SAT $\propto$ 3-SAT

we know this



Boolean variables:

$x_A^1 \quad x_A^2 \quad x_A^3$: if country A has color 1, 2, 3

$\vdots$

$x_E^1 \quad x_E^2 \quad x_E^3$

Formulas

$$\varphi(x,y,z) \equiv (x \wedge \bar{y} \wedge \bar{z}) \vee (\bar{x} \wedge y \wedge \bar{z}) \vee (\bar{x} \wedge \bar{y} \wedge z)$$

$\varphi(x,y,z)$ is true ~~iff~~ exactly one of x, y, z is true

We need to transform φ into CNF : φ

$$F = \hat{\varphi}(x_A^1, x_A^2, x_A^3) \wedge \cdots \wedge \hat{\varphi}(x_E^1, x_E^2, x_E^3) \wedge \boxed{\cdots}$$

$$\tau(x_0, y_0, z_0, x_1, y_1, z_1) \equiv$$

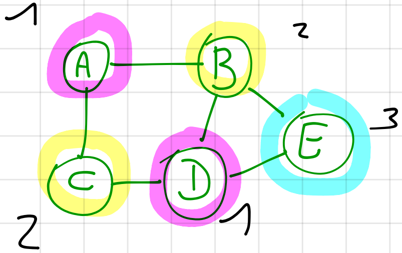
$$(x_0 \wedge \overline{x_1}) \vee (y_0 \wedge \overline{y_1}) \vee (z_0 \wedge \overline{z_1})$$

$$\boxed{\tau(x_A^1, x_A^2, x_A^3, x_B^1, x_B^2, x_B^3) \to \exists \text{ edge } (A, B)}$$

is true iff

A and B have different colors

$\to$ #?



$X_A^1 = T \quad X_A^2 = X_A^3 = F$

$X_B^2 = T$ - - - . .

$X_C^2 = T$. - -

$F = \underbrace{\hat{\varphi}(X_A^1, X_A^2, X_A^3)} \wedge \cdots \wedge \underbrace{\hat{\varphi}(X_E^1, X_E^2, X_E^3)} \wedge$

$\to \hat{\tau}(A, B) \wedge \hat{\tau}(A, C) \wedge \hat{\tau}(C, D) \wedge \hat{\tau}(B, D) \wedge \hat{\tau}(B, E) \wedge \hat{\tau}(D, E)$

where $\hat{\varphi}(A, B)$ is the CNF of τ(- . -)