Models of computation (MOD) 2014/15
Exam – Jan. 20, 2016

[**Ex. 1**] Add to IMP the atomic interleaving construct

$$c_1 \;|||\; c_2$$

that (non-deterministically) either executes c_1 before c_2 or c_2 before c_1. For example, the execution of $x := 1 \;|||\; x := 2$ in σ can lead to $\sigma[2/x]$ or to $\sigma[1/x]$, depending on the order in which the assignments are evaluated.

1. Define the operational semantics for the new expression.
2. Redefine, if necessary, the formal definition of abstract semantics $\sim$ (taking into account that the semantics is no longer deterministic).
3. Is the command $x := 0$ operationally equivalent to the command c below? Explain.

$$c \stackrel{\text{def}}{=} x := 1 \;;\; (\textbf{while}\ x > 0\ \textbf{do}\ (x := 0 \;|||\; x := x + 1))$$

4. Is it true that, for all commands c_1, c_2, c_3, the commands $c_1 \;|||\; (c_2 \;|||\; c_3)$ and $(c_1 \;|||\; c_2) \;|||\; c_3$ are equivalent with respect to $\sim$?

[**Ex. 2**] Consider the binary relation $\preceq$ defined over the the set of positive natural numbers with infinite $\{1, 2, 3, ..., \infty\}$ such that

$$n \preceq m \quad \Leftrightarrow \quad m = \infty \vee (n, m \neq \infty \wedge n \text{ divides } m).$$

1. Is it a partial order with bottom?
2. Is it complete?
3. Are the functions below monotone? If so, are they continuous?

$$succ(n) \stackrel{\text{def}}{=} \begin{cases} n+1 & \text{if } n \neq \infty \\ \infty & \text{otherwise} \end{cases} \qquad dup(n) \stackrel{\text{def}}{=} \begin{cases} 2 \cdot n & \text{if } n \neq \infty \\ \infty & \text{otherwise} \end{cases}$$

[**Ex. 3**] Let us consider the HOFL term

$$t \stackrel{\text{def}}{=} \textbf{rec}\ f.\ \lambda x.\ \lambda y.\ \textbf{if}\ x\ \textbf{then}\ 0\ \textbf{else}\ (f\ y\ x)$$

1. Find the principal type of t.
2. Compute the denotational semantics of t.

[**Ex. 4**] Suppose we were to modify the operational semantics of the PEPA cooperation combinator by substituting the usual synchronization rule with the one below: discuss the weaknesses of such a definition.
Hint: consider the consequences at the level of the apparent rate.

$$\frac{P \xrightarrow{(\alpha, r_1)} P' \quad Q \xrightarrow{(\alpha, r_2)} Q' \quad \alpha \in L}{P \underset{L}{\bowtie} Q \xrightarrow{(\alpha, r)} P' \underset{L}{\bowtie} Q'} \quad \text{where } r = \min\{r_1, r_2\}$$