Models of computation (MOD) 2014/15
Exam – Feb. 10, 2016

[**Ex. 1**] Add to IMP the conditional construct

$$\textbf{if } c_1 \textbf{ ends do } c_2$$

that computes the memory obtained by executing c_2 in the current memory σ if the execution of c_1 in σ terminates (and it diverges otherwise). For example, the execution of **if** $x := 0$ **ends do** $x := x + 1$ in σ produces $\sigma[(\sigma(x) + 1)/x]$.

1. Define the operational semantics for the new construct.
2. Extend the proof by rule induction seen in the course to prove that the evaluation of commands is deterministic.
3. Is it true that for any $b \in \mathit{Bexp}$ and $c \in \mathit{Com}$ the two commands below are operationally equivalent, i.e., that $c' \sim c''$? Explain.

$$c' \stackrel{\text{def}}{=} \textbf{if } \neg b \textbf{ then } c \textbf{ else skip} \qquad c'' \stackrel{\text{def}}{=} \textbf{if } (\textbf{while } b \textbf{ do } c) \textbf{ ends do } c$$

[**Ex. 2**] Consider the binary relation $\preceq$ defined over the the set of positive natural numbers with infinite $\{1, 2, 3, ..., \infty\}$ such that

$$x \preceq y \quad \Leftrightarrow \quad y = \infty \vee (x, y \neq \infty \wedge \exists k > 0.\ y = x^k).$$

1. Prove that $\preceq$ is a partial order relation. Is there a bottom element?
2. Is the partial order complete?
3. Are the functions below monotone? If so, are they continuous?

$$\mathit{square}(x) \stackrel{\text{def}}{=} \begin{cases} x^2 & \text{if } x \neq \infty \\ \infty & \text{otherwise} \end{cases} \qquad \mathit{dup}(x) \stackrel{\text{def}}{=} \begin{cases} 2 \cdot x & \text{if } x \neq \infty \\ \infty & \text{otherwise} \end{cases}$$

[**Ex. 3**] Let us consider the three CCS processes

$$p \stackrel{\text{def}}{=} \textbf{rec } X.\ \alpha.(\beta.X + \gamma.X) \qquad r \stackrel{\text{def}}{=} \alpha.\ \textbf{rec } Z.\ (\beta.\alpha.Z + \gamma.\alpha.Z)$$
$$q \stackrel{\text{def}}{=} \textbf{rec } Y.\ (\alpha.\beta.Y + \alpha.\gamma.Y)$$

Draw the corresponding LTSs. Are they strong bisimilar? Explain.

[**Ex. 4**] The character of a videogame runs forward on a path with three lanes (left, mid, right). Suppose that an automatic controller is designed such that, at each turn, the character can either stay in its current lane with probability $p = \frac{1}{2}$ or move to an adjacent lane (with equal probability to move left or right if currently in the mid lane).

1. Draw the DTMC for the controller and show that it is ergodic.
2. Compute the steady state probability of being in the mid lane.