# Models of computation (MOD) 2014/15
Exam – June 10, 2015

[**Ex. 1**] Add to IMP the command

$$\textbf{check } c_0 \textbf{ and } c_1 \textbf{ on } x$$

with the following informal meaning: if the executions of $c_0$ and $c_1$ in the current memory lead to the same value $v$ for $x$ then the memory is updated by assigning $v$ to $x$, otherwise it is left unchanged.

1. Define the operational semantics of the new command.
2. Define the denotational semantics of the new command.
3. Extend the proof of equivalence of the operational and denotational semantics of IMP to take into account the new command.

[**Ex. 2**] Let $\mathcal{D} = (D, \sqsubseteq)$ be a CPO. A *down-closed set of* $\mathcal{D}$ is a set $S \subseteq D$ such that

$$\forall d, s \in D.\ d \sqsubseteq_D s \wedge s \in S \Rightarrow d \in S$$

Let $\mathcal{D}\downarrow$ denote the set of all down-closed sets of $\mathcal{D}$, ordered by inclusion. Obviously $\mathcal{D}\downarrow$ is a partial order and has a bottom element (the empty set).

1. Prove that $\mathcal{D}\downarrow$ is complete by showing that the limit of a chain of down-closed sets is a down-closed set.
2. Let $(\cdot)^{\downarrow} : D \to \mathcal{D}\downarrow$ be the function defined by:

$$s^{\downarrow} \stackrel{\text{def}}{=} \{\ d \ \mid\ d \sqsubseteq_D s\ \}$$

   Prove that $(\cdot)^{\downarrow}$ is monotone but not necessarily continuous.

[**Ex. 3**] Compute the type, the canonical form and the denotational semantics of the HOFL term

$$t \stackrel{\text{def}}{=} \textbf{rec } f.\ \lambda x.\ \textbf{if } x \textbf{ then } 0 \textbf{ else } (f(x) \times f(x))$$

[**Ex. 4**] Let $\mathcal{P}$ denote the set of all (closed) CCS processes.

1. Prove that $\forall p, q \in \mathcal{P}.\ p|q \approx q|\tau.p$, where $\approx$ denotes weak bisimilarity, by showing that the relation $R$ below is a weak bisimulation:

$$R \stackrel{\text{def}}{=} \{\ (p|q, q|\tau.p) \ \mid\ p, q \in \mathcal{P}\ \} \cup \{\ (p|q, q|p) \ \mid\ p, q \in \mathcal{P}\ \}$$

2. Then exhibit two processes $p$ and $q$ and a context $C[\cdot]$ showing that $s \stackrel{\text{def}}{=} p|q$ and $t \stackrel{\text{def}}{=} q|\tau.p$ are not weak observational congruent.
   *Hint:* remind that: $s \cong t$ iff $s \approx t \ \wedge\ \forall r.\ s + r \approx t + r$, where $\cong$ is the weak observational congruence.