Models of computation (MOD) 2014/15
Exam – July 1, 2015

[**Ex. 1**] Add to IMP the command

$$\textbf{fix } a \textbf{ of } c$$

that keeps executing c until the value of the expression a is not changed by the last execution of c (for example **fix** x **of** $x := x + 1$ diverges for any σ, while **fix** x **of** $(x := 1; y := y + 1)$ terminates after having incremented y by 2 if initially $\sigma(x) \neq 1$ or by 1 otherwise).

1. Define the operational semantics of the new command.
2. Define the denotational semantics of the new command, paying attention to the introduction of lifting whenever necessary.
   Hint: defines the semantics as the fixpoint of a suitable function $\Gamma_{a,c}$.
3. Extend the proof of completeness of the denotational semantics to take into account the new command.

[**Ex. 2**] Let $\mathcal{D} = (D, \sqsubseteq)$ be a CPO. Let us consider the pair $(\wp(D), \preceq)$, where

$$S \preceq S' \quad \Leftrightarrow \quad \forall s \in S.\ \exists s' \in S'.\ s \sqsubseteq s'$$

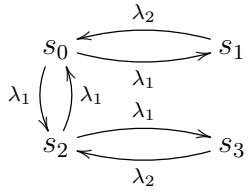
Is $(\wp(D), \preceq)$ a $\mathrm{CPO}_\perp$?

[**Ex. 3**] Let us consider the following definition

$$f(x) \stackrel{\text{def}}{=} \textbf{if } x = 0 \textbf{ then } 1 \textbf{ else } (1 + f(x - 1))$$

1. Define a (recursive) HOFL term t that corresponds to the above definition and prove that it has type $int \to int$.
2. Prove that for any $n \geq 0$ it holds $(t\ n) \to n + 1$ using the eager operational semantics.

[**Ex. 4**] Let us consider the CTMC



1. What is the probability to sojourn in s_0 for some time r?
2. Assume $\lambda_2 > 2\lambda_1$: are there some bisimilar states?