Models of computation (MOD) 2015/16
Exam – Sept. 6, 2016

[**Ex. 1**] Suppose one wants to insert some measure of efficiency in the operational semantics of IMP.

1. Redefine the operational semantics of IMP commands in such a way that the transition predicate takes the form

$$\langle c, \sigma, n \rangle \rightsquigarrow \sigma'$$

with the meaning that "the command c, when executed in the state σ converges to the state σ' by performing at most n assignments."

2. Then, prove that for all c, σ, σ':

$$\langle c, \sigma \rangle \to \sigma' \;\Rightarrow\; \exists n \in \mathbb{N}.\ \langle c, \sigma, n \rangle \rightsquigarrow \sigma'.$$

[**Ex. 2**] Let $\mathcal{D} = (\mathbb{N} \cup S, \sqsubseteq)$ be a CPO that extends the PO of natural numbers $\mathcal{N} = (\mathbb{N}, \leq)$ (i.e., such that for any $n, m \in \mathbb{N}$ we have $n \leq m \Rightarrow n \sqsubseteq m$). Prove that, no matter how the set S and the relation $\sqsubseteq$ are defined, any two infinite chains of natural numbers have the same set of upper bounds (and thus the same lub) in $\mathcal{D}$.
Hint: Remember that an infinite chain is a chain that contains infinitely many distinct elements.
[**Ex. 3**] The CCS process $p_{\alpha,\beta} \stackrel{\text{def}}{=} \textbf{rec}\ x.\ (\alpha.\overline{\beta}.x)$ forwards incoming messages on channel α to channel β.

1. Draw the LTS for the process $q \stackrel{\text{def}}{=} (p_{\alpha,\gamma} \mid p_{\gamma,\beta})\backslash\gamma$ obtained by composing two forwarders (see Fig. 1).

2. Prove that q is not weakly bisimilar to $p_{\alpha,\beta}$.
Hint: Show that Alice has a winning strategy against Bob in the weak bisimulation game.

3. Prove that q is weakly bisimilar to $p_{\alpha,\beta} \mid p_{\alpha,\beta}$.

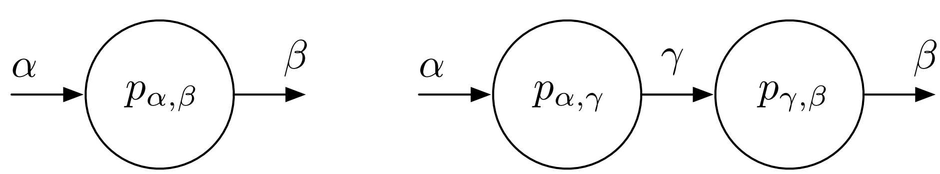

Figure 1: Diagrams illustrating CCS processes in Exercise 3

[**Ex. 4**] Prove, by rule induction, that according to the operational semantics of the π-calculus we have that for all processes p, p' and any label α:

$$p \xrightarrow{\alpha} p' \quad \Rightarrow \quad (\ (\ fn(\alpha) \subseteq fn(p)\ )\ \wedge\ (\ fn(p') \subseteq fn(p) \cup bn(\alpha)\ )\ ).$$

Hint: The most interesting rules to consider are (ComL), (Res), (Open), and (CloseL). You may skip the proof details for all the other rules.