Models of computation (MOD) 2016/17

Second Mid-Term Exam – June 1, 2017

[**Ex. 1**] Consider the HOFL term

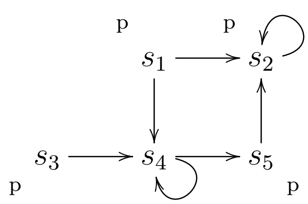
$$t \stackrel{\text{def}}{=} \mathbf{rec}\ f.\ \lambda x.\ \lambda y.\ \mathbf{if}\ (x \times y)\ \mathbf{then}\ 0\ \mathbf{else}\ ((f\ (y \times y))\ 1)$$

1. [**Optional**] Write the derivation tree that proves $t : int \to int \to int$.
2. Prove that the term $((t\ (1 \times 1))\ 1)$ has no canonical form.
3. Compute the denotational semantics of t.

[**Ex. 2**] Let $\simeq$ denote strong bisimilarity of CCS processes.

1. Find a process r and a permutation ψ such that $(r[\psi])\backslash\alpha \not\simeq (r\backslash\alpha)[\psi]$.
2. Let ϕ be a (injective) permutation such that $\phi(\alpha) = \alpha$. Prove that for any CCS processes p, q we have that $p \simeq q$ implies $(p[\phi])\backslash\alpha \simeq (q\backslash\alpha)[\phi]$.

[**Ex. 3**] Write the μ-calculus formula φ that corresponds to the CTL formula $E\ G\ p$. Then evaluate the semantics of φ over the LTS below.



[**Ex. 4**] In the Land of Oz there are never two nice days in a row. Moreover:

- If they have a nice day, they have snow or rain the next day with equal probability.
- If they have snow or rain, they have the same the next day with probability $\frac{1}{2}$.
- If they have snow or rain, the next day is nice with probability $\frac{1}{4}$.

Use a DTMC to model the weather conditions of the Land of Oz.

1. Write down the transition matrix that defines the DTMC and draw the corresponding PTS.
2. Check that the DTMC is ergodic and find the probability to have a rainy day on the long run.
3. If friday is a snowy day, what is the probability that sunday is nice?