Models of computation (MOD) 2016/17
Exam – Sept. 7, 2017

[Ex. 1] Let the set $\mathsf{loc}(a)$ of all locations that appear in the IMP expression a be defined by structural recursion as follows:

$$\mathsf{loc}(n) \stackrel{\text{def}}{=} \emptyset \qquad \mathsf{loc}(x) \stackrel{\text{def}}{=} \{x\} \qquad \mathsf{loc}(a_0 \text{ op } a_1) \stackrel{\text{def}}{=} \mathsf{loc}(a_0) \cup \mathsf{loc}(a_1)$$

1. Prove that $\forall a \in \mathbf{Aexp}.\ \forall \sigma \in \Sigma.\ \forall m \in \mathbb{Z}.\ \forall y \notin \mathsf{loc}(a)$:

$$\mathcal{A}\,[\![a]\!]\,(\sigma[^m/_y]) = \mathcal{A}\,[\![a]\!]\,\sigma$$

2. Prove by rule induction that $\forall a \in \mathbf{Aexp}.\ \forall \sigma \in \Sigma.\ \forall n \in \mathbb{Z}$:

$$\langle a, \sigma \rangle \to n \ \Rightarrow\ (\,\forall m \in \mathbb{Z}.\ \forall y \notin \mathsf{loc}(a).\ \langle a, \sigma[^m/_y] \rangle \to n\,)$$

3. Given two locations x, y find an expression a with $y \notin \mathsf{loc}(a)$ such that the command $y := a; x := a$ is not denotationally equivalent to the command $x := a; y := a$.

[Ex. 2] Let $(D, \sqsubseteq)$ be a flat $\text{CPO}_\perp$ and $f : D \to D$ a function.

1. Prove that if f is monotone then f is continuous.
   Hint: Note that any chain in a flat domain is finite.
2. Prove that if f is monotone then $\text{fix } f = f(\perp)$.
   Hint: Prove that if f is monotone then $f(f(\perp)) = f(\perp)$.

[Ex. 3] Extend the operational semantics of HOFL with the additional rule

$$\frac{t_0 \to 0}{t_0 \times t_1 \to 0}$$

1. Show a closed term t and a canonical form c such that $t \to c$ in the extended operational semantics but $[\![t]\!]\,\rho \neq [\![c]\!]\,\rho$ for any ρ.
2. Change the denotational semantics of product by letting

$$[\![t_0 \times t_1]\!]\,\rho \stackrel{\text{def}}{=} \mathit{Cond}([\![t_0]\!]\,\rho, \lfloor 0 \rfloor, [\![t_0]\!]\,\rho \mathbin{\underline{\times}_\perp} [\![t_1]\!]\,\rho)$$

   Find two terms t_0 and t_1 such that $t_0 \times t_1$ is not denotationally equivalent to $t_1 \times t_0$ (according to the above denotational semantics).
3. Prove that $t \to c$ (in the extended operational semantics) implies $\forall \rho.\ [\![t]\!]\,\rho = [\![c]\!]\,\rho$ (in the modified denotational semantics).
   Hint: Consider only the relevant cases.

[Ex. 4] Let us consider the CCS processes

$$p \stackrel{\text{def}}{=} \mathbf{rec}\, x.(\alpha.x + \overline{\beta}.\mathbf{nil}) \quad q \stackrel{\text{def}}{=} \mathbf{rec}\, y.(\overline{\alpha}.\mathbf{nil} + \beta.y) \quad r \stackrel{\text{def}}{=} (p|q)\backslash\alpha\backslash\beta \quad s \stackrel{\text{def}}{=} \mathbf{rec}\, z.\tau.z$$

1. Draw the LTS of the process r.
2. Show that r and s are weak bisimilar but not strong bisimilar.