Models of computation (MOD) 2016/17
Exam – January 16, 2018

[**Ex. 1**] Let us extend IMP with the command

$$\textbf{keep } x \textbf{ along } c$$

whose denotational semantics is

$$\mathcal{C}[\![\textbf{keep } x \textbf{ along } c]\!]\sigma \stackrel{\text{def}}{=} (\lambda\sigma_1.\ \sigma_1(x) = \sigma(x) \to \sigma_1, \sigma)^* \, (\mathcal{C}[\![c]\!]\sigma)$$

1. Define the operational semantics for the new construct.
2. Extend the proofs of correctness and completeness between the operational and the denotational semantics to account for the new construct.
3. Prove that $\langle \textbf{while } x = 0 \textbf{ do } (\textbf{keep } x \textbf{ along } x := x + 1), \sigma[0/x]\rangle \not\to$ using the rule for divergence.

[**Ex. 2**] Consider the HOFL term

$$t \stackrel{\text{def}}{=} \textbf{rec } f.\ \lambda x.\ (\ \textbf{fst}(\ f(\textbf{fst } x, 0)\ )\ ,\ \textbf{snd}(\ f(0, \textbf{snd } x)\ )\ )$$

1. Find the principal type of t.
2. Compute the (lazy) denotational semantics of t.

[**Ex. 3**] Let us consider the CCS processes

$$p \stackrel{\text{def}}{=} \textbf{rec } x.(\alpha.x|\beta.\textbf{nil}) \quad q \stackrel{\text{def}}{=} \textbf{rec } y.\overline{\beta}.y \quad r \stackrel{\text{def}}{=} \textbf{rec } z.\alpha.z$$

1. Draw (at least in part) the LTS of the process $s \stackrel{\text{def}}{=} (p|q)\backslash\beta$.
2. Show that s and r are not strong bisimilar.
3. Show that s and r are weak bisimilar.

Hint: You can assume that $p|\textbf{nil} = p$.

[**Ex. 4**] Dennis is an orphan with three aunties Alice, Barbara and Carol. He spends each night at home of one of them. Each morning he draws a dice to let the fate decide where to sleep the next night. If he slept at Alices's place, and the dice is odd, then he'll spend the next night at Barbara's, otherwise at Carol's. If he slept at Barbara's, and the dice yields a prime number, then he'll remain at Barbara's; if it is 1 he will move to Carol's; otherwise to Alice's. If he slept at Carol's, he'll choose where to spend the next night with equal probability.

1. Model the system as a DTMC.
2. Where is it more likely to find Dennis at night?
3. If Dennis sleeps at Alice's on sunday, what it the probability that he sleeps at Alice's also on wednesday?