Overview

Complexity of LTL model checking

main steps of automata-based LTL model checking:

construction of an NBA $\mathcal{A}$ for $\neg\varphi$

persistence checking in the product $\mathcal{T} \otimes \mathcal{A}$

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The **LTL** model checking problem is **PSPACE**-complete

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given:	finite transition system $\mathcal{T}$
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we show

- just for fun: **coNP**-hardness
- **PSPACE**-completeness

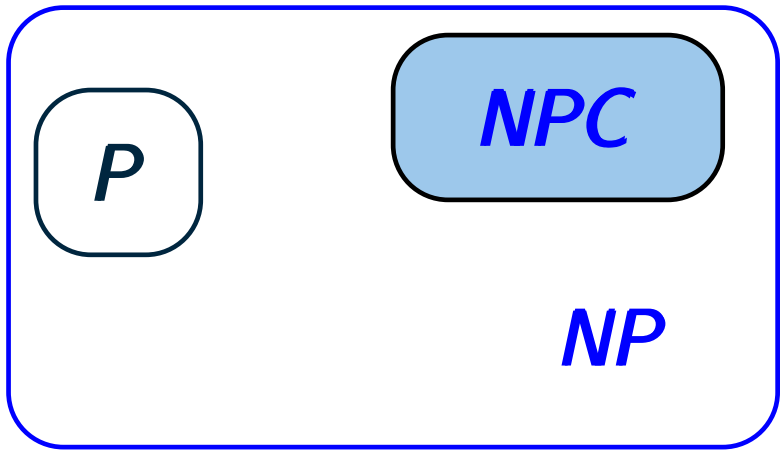
Recall: complexity classes

Complexity classes *P*, *NP*

P = class of decision problem solvable in deterministic polynomial time

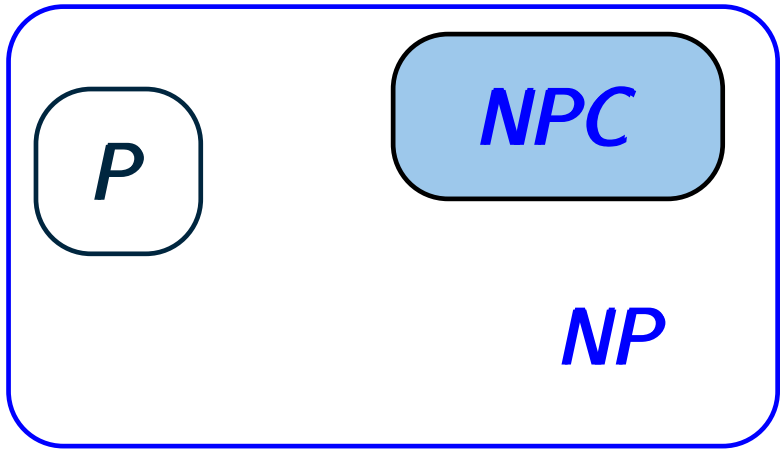
NP = class of decision problem solvable in nondeterministic polynomial time

Complexity classes *P*, *NP*



NPC = class of ***NP***-complete problems

Complexity classes *P*, *NP*



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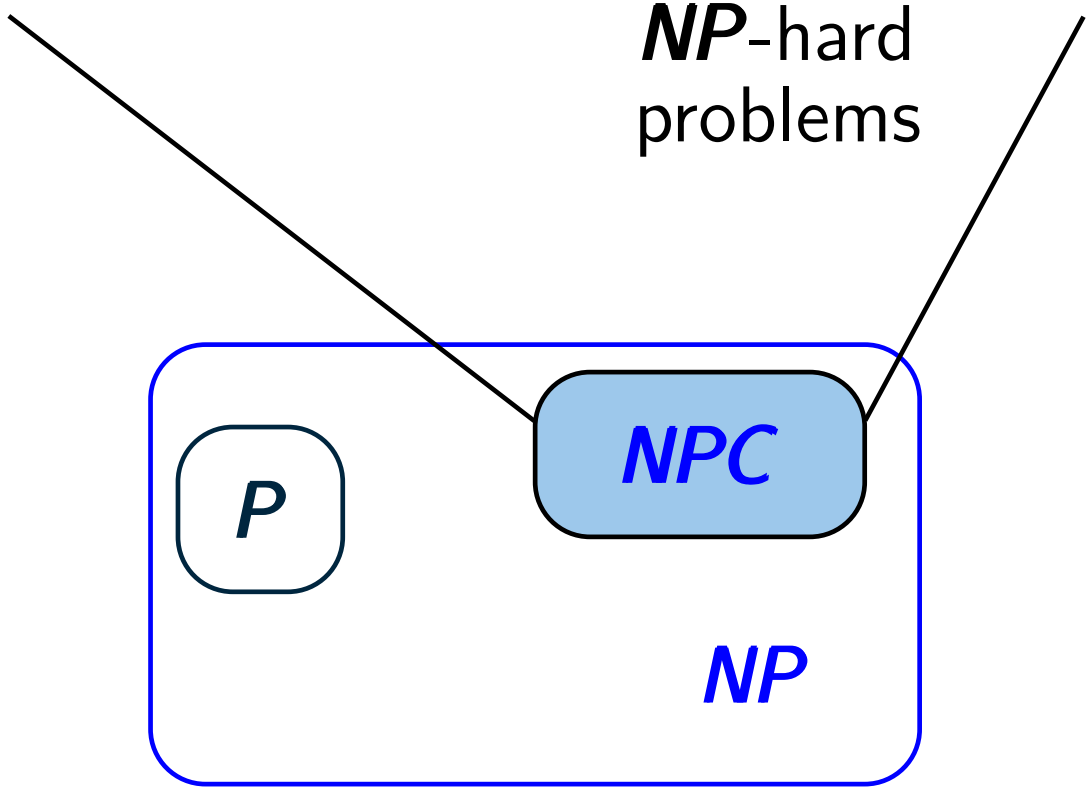

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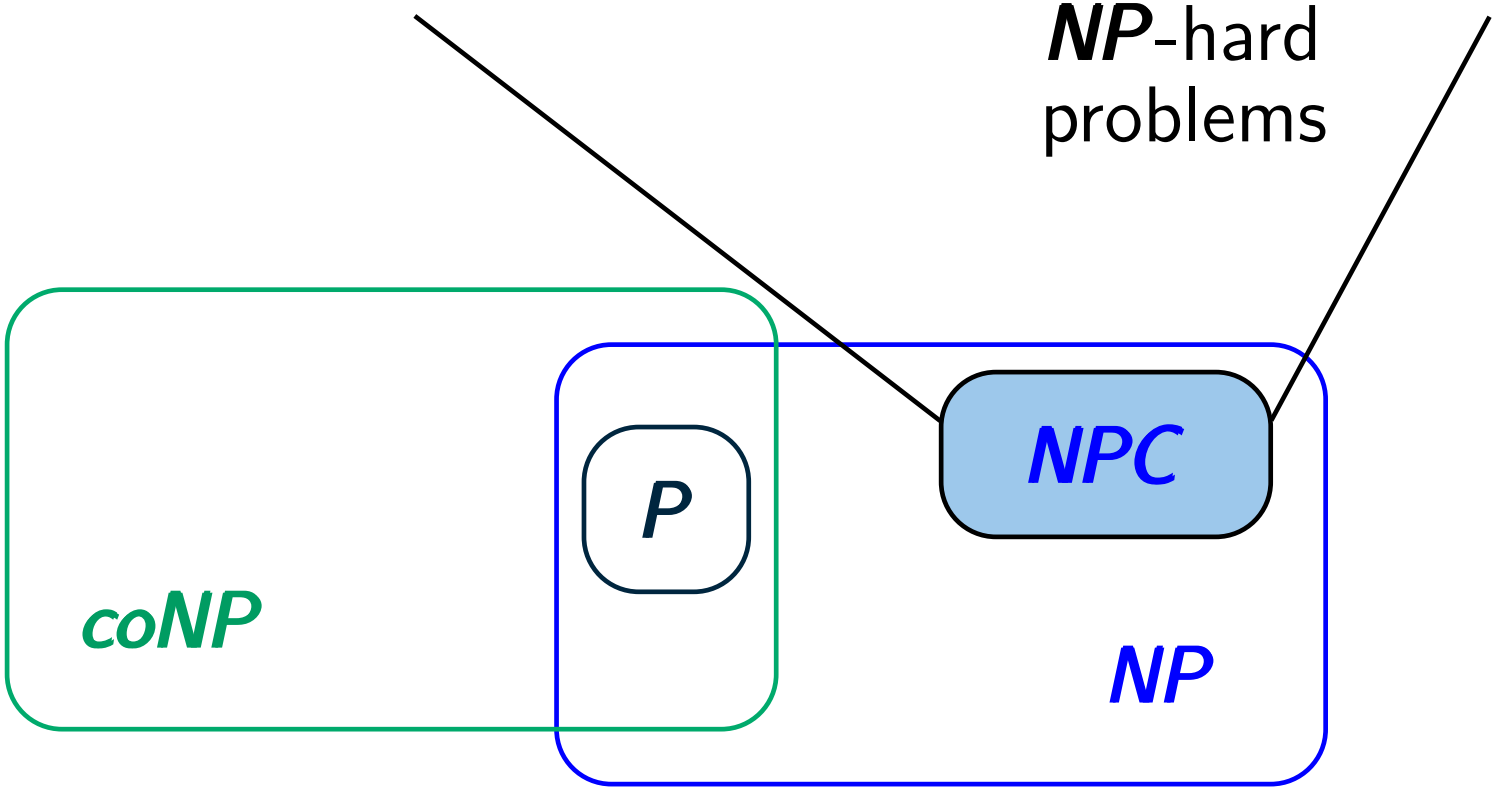
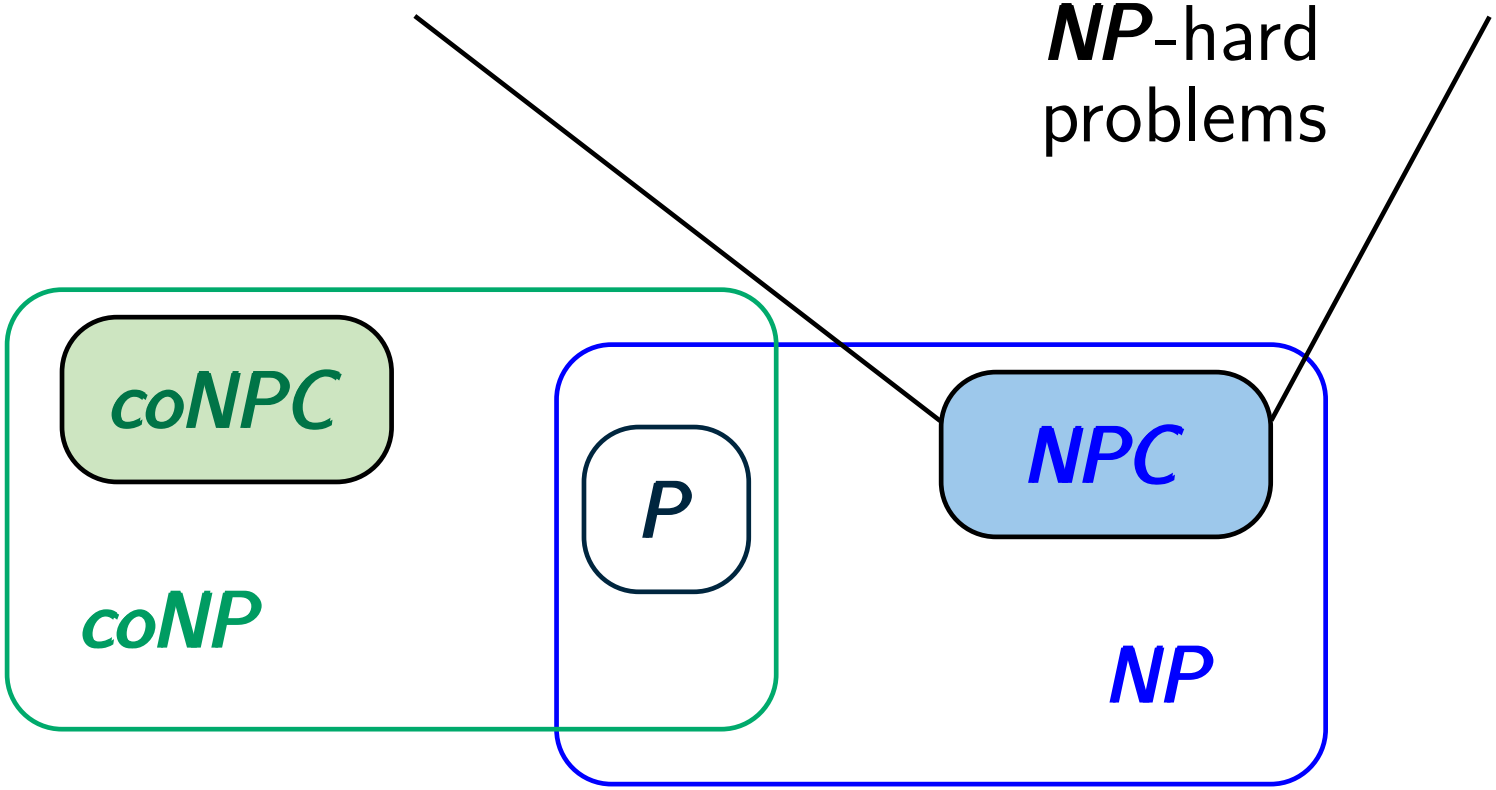


$$coNP = \{ \overline{L} : L \in NP \}$$

$\overline{L}$: complement of L

Complexity classes *P*, *NP*, *coNP*



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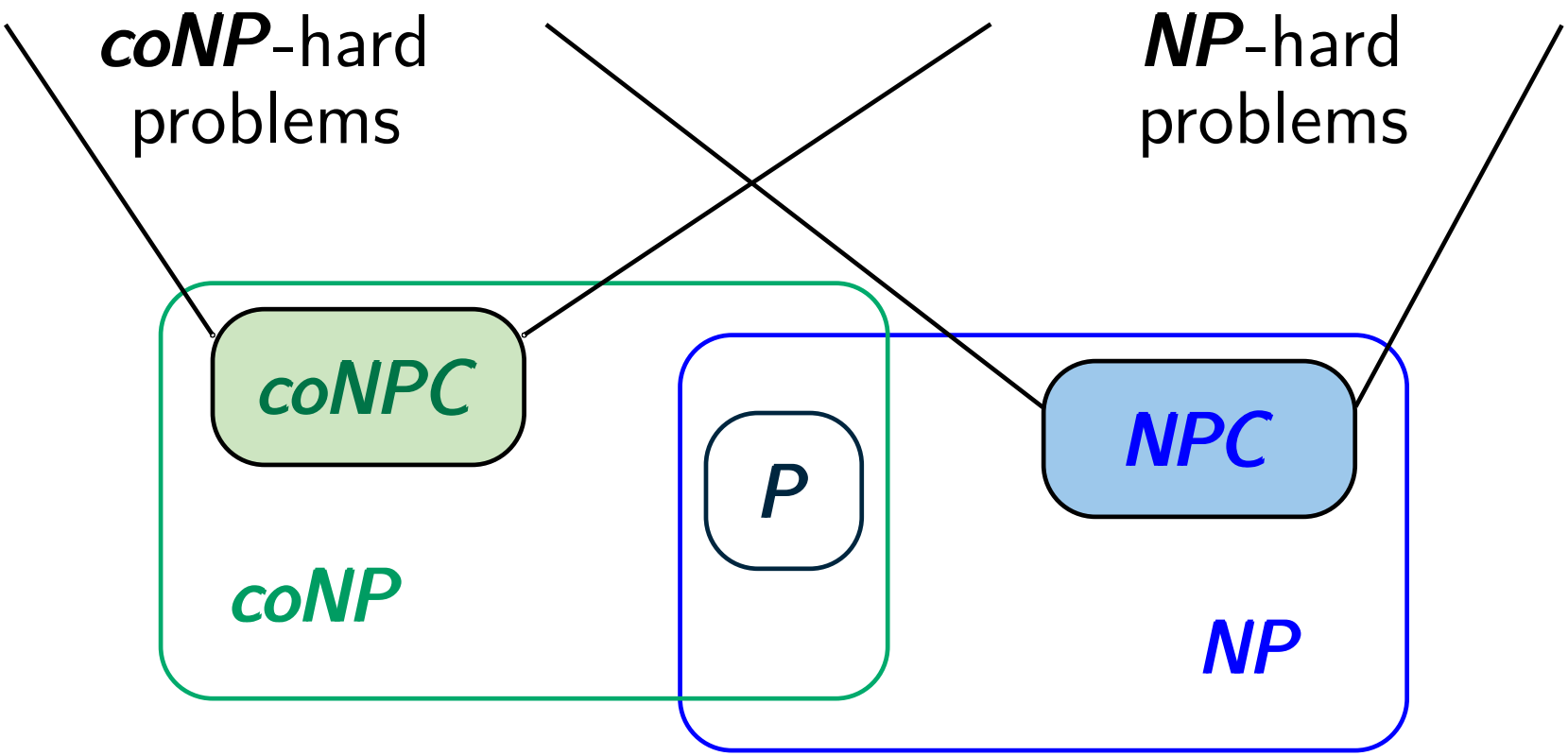

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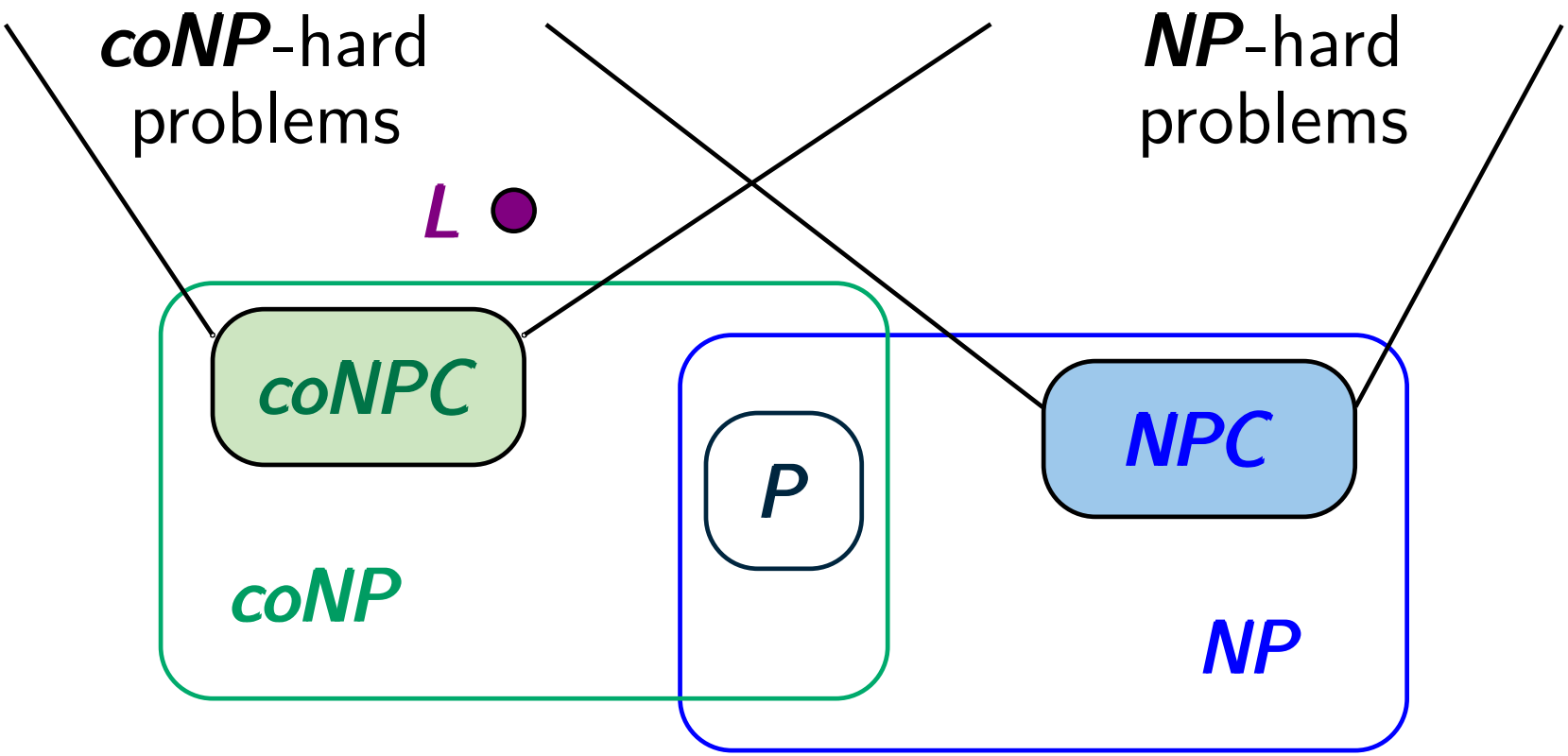

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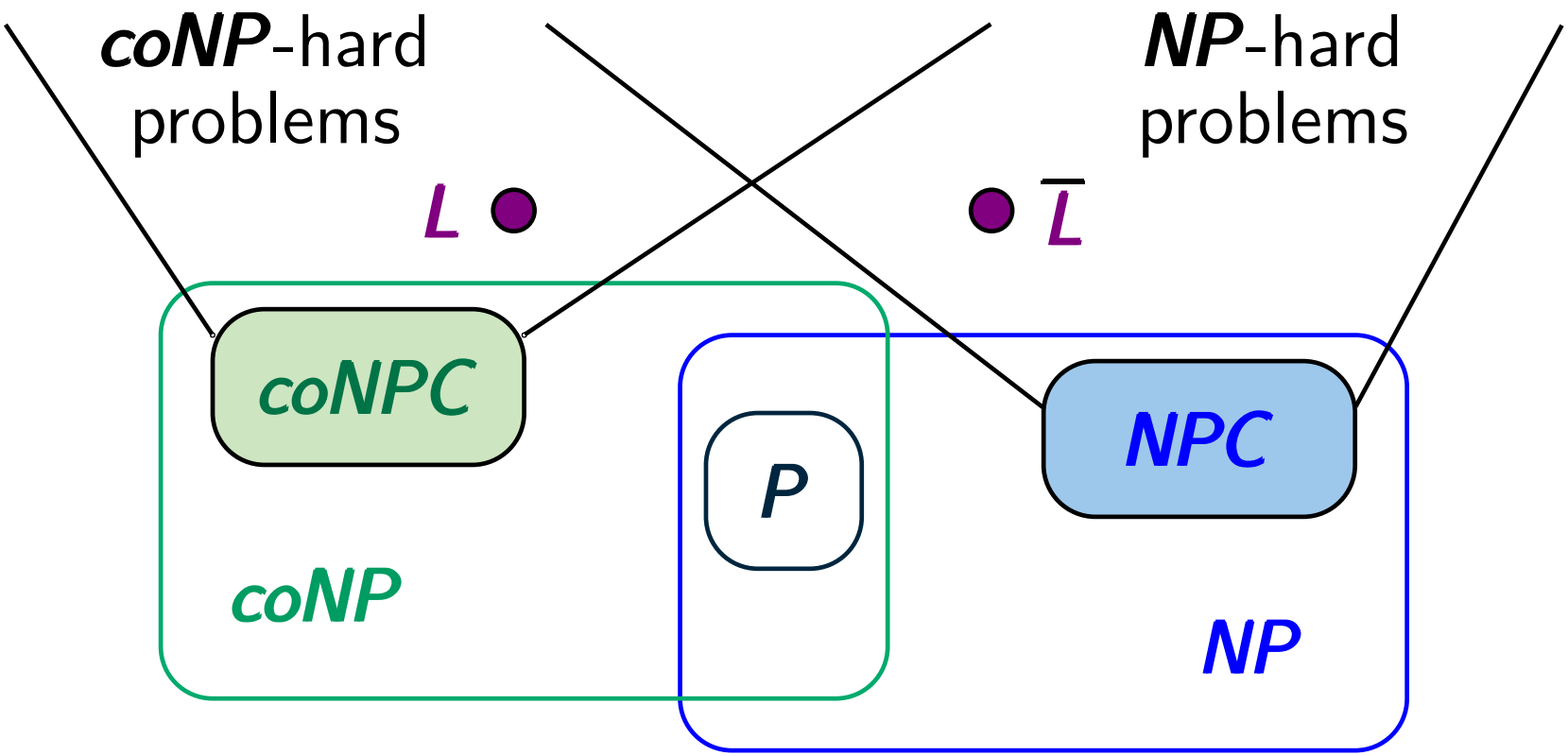

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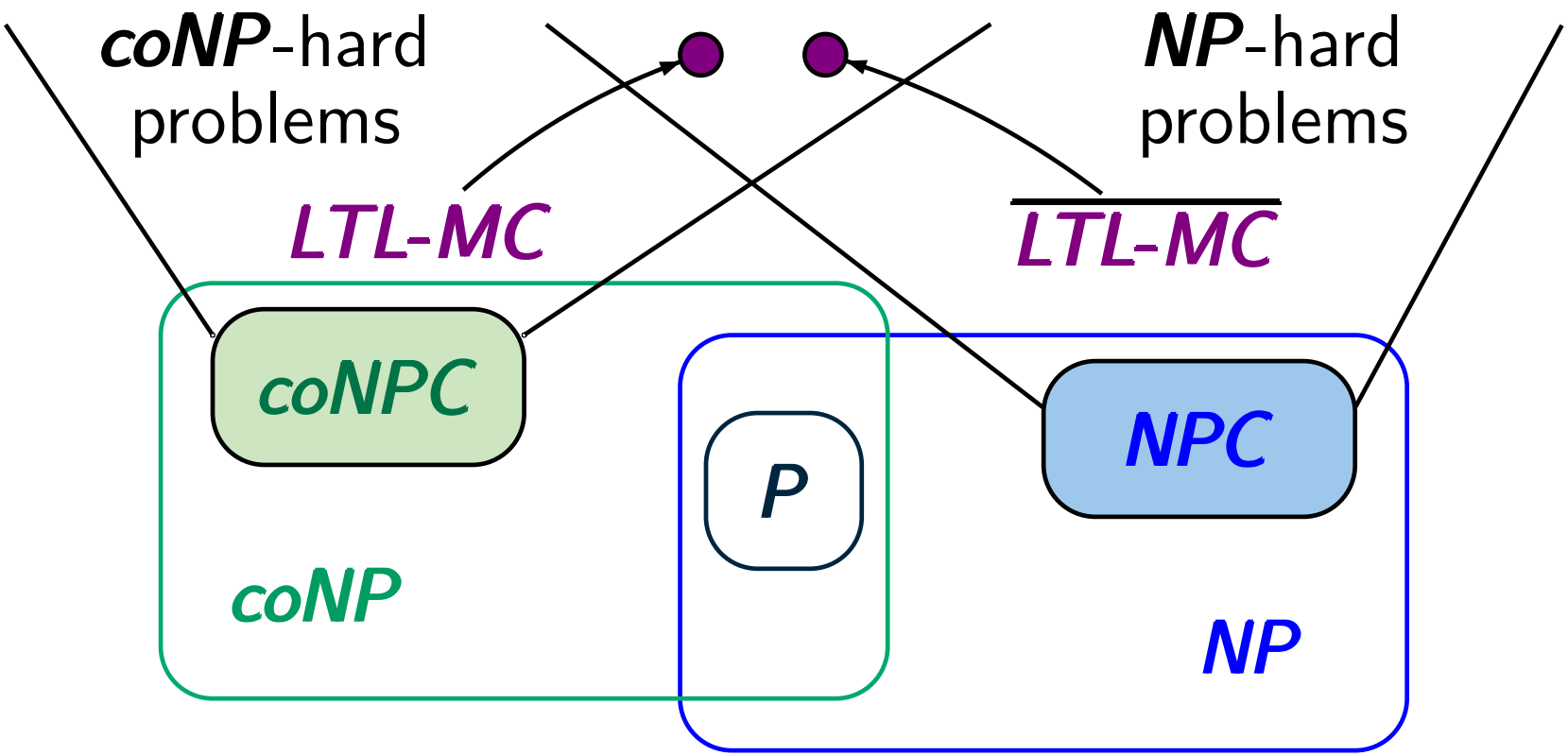

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coNP-hardness

The **LTL** model checking problem is ***coNP***-hard

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proof by a polynomial reduction

$$HP \leq_{poly} \overline{LTL\text{-}MC}$$

Hamilton path problem

complement of the **LTL** model checking problem

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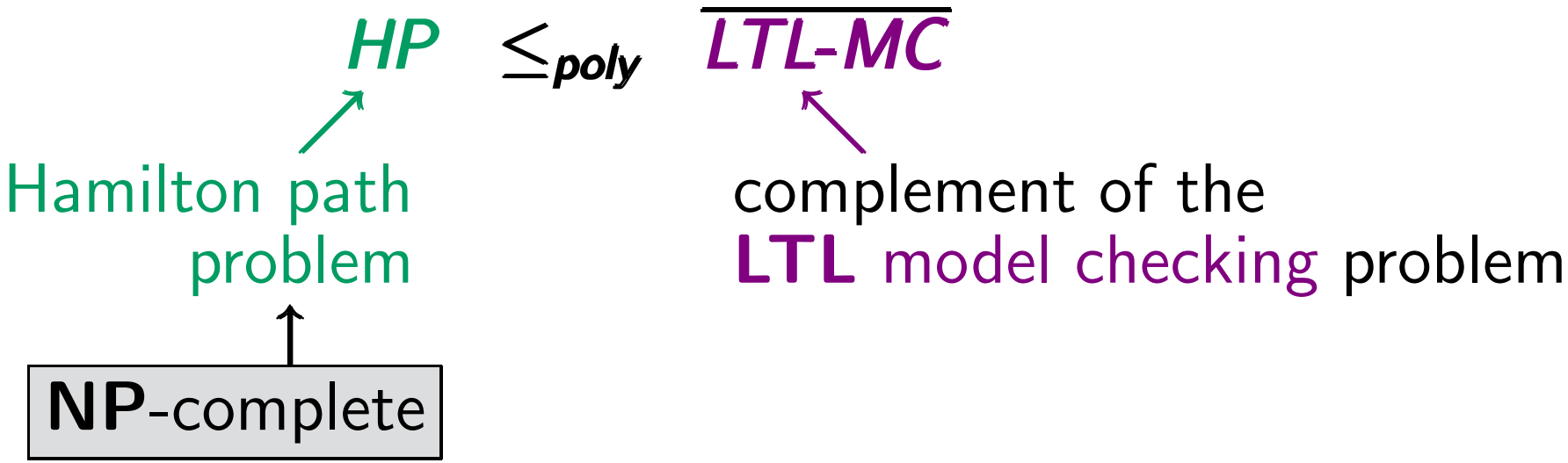

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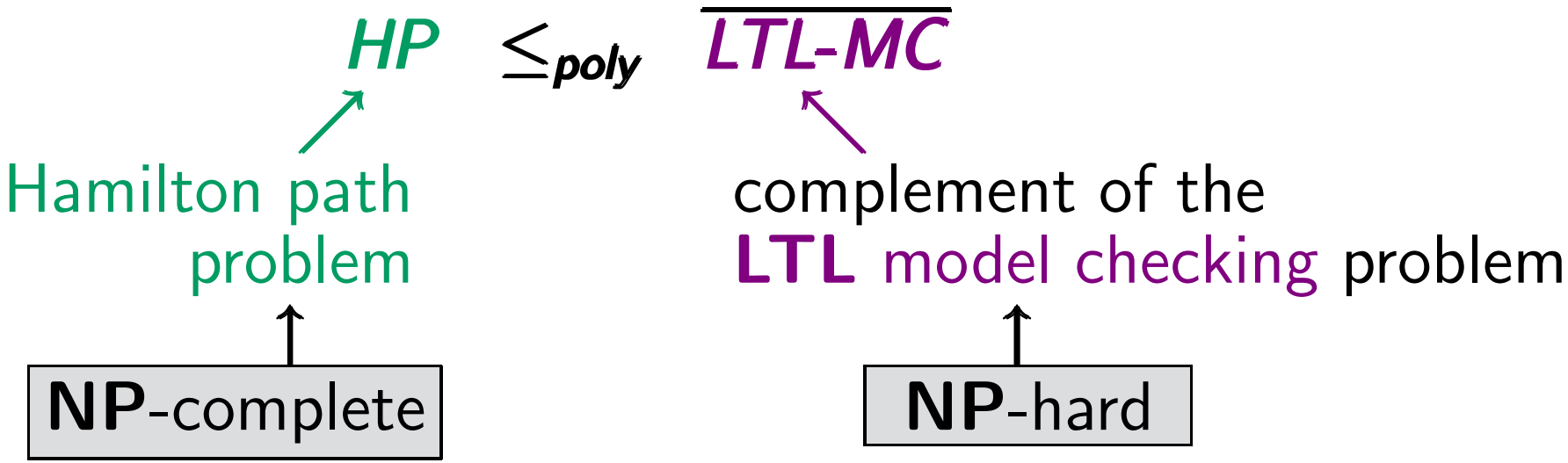

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Hamilton path problem *HP*

LTLMC3.2-72B

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given:	finite directed graph G
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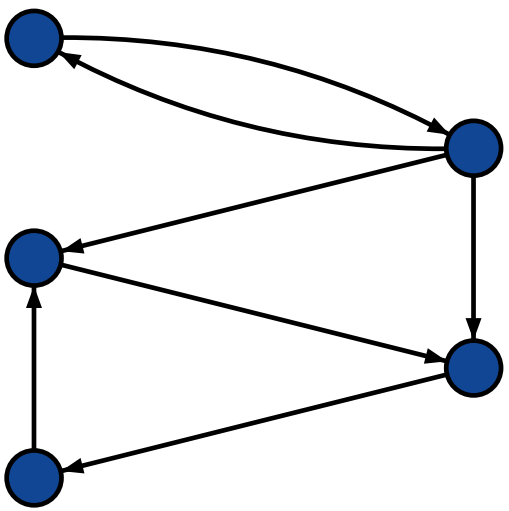

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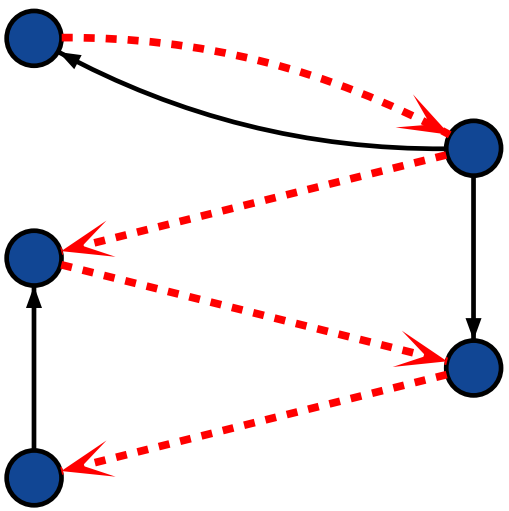

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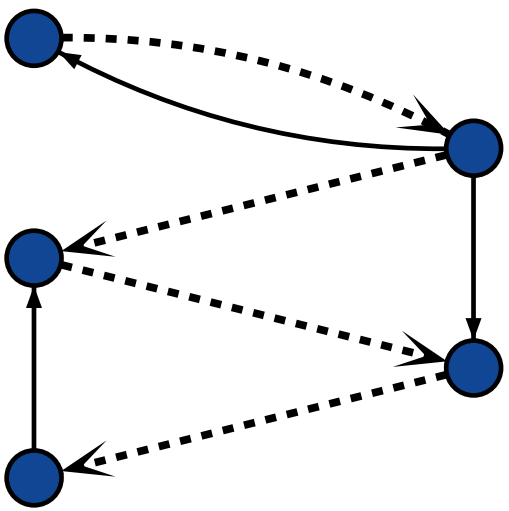

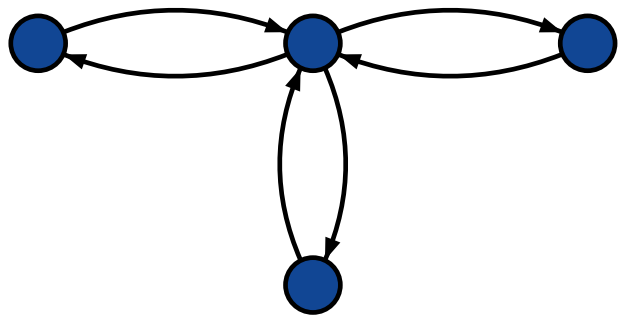

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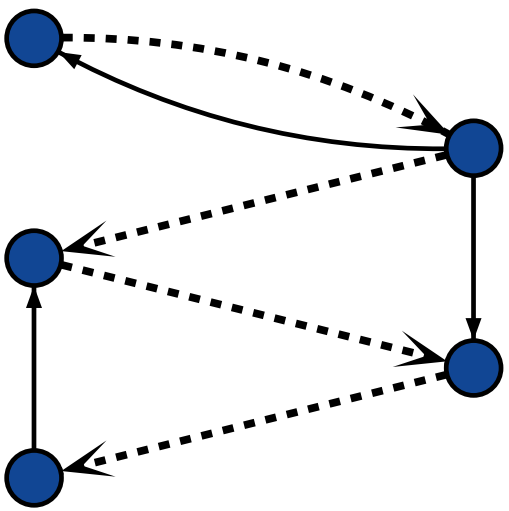

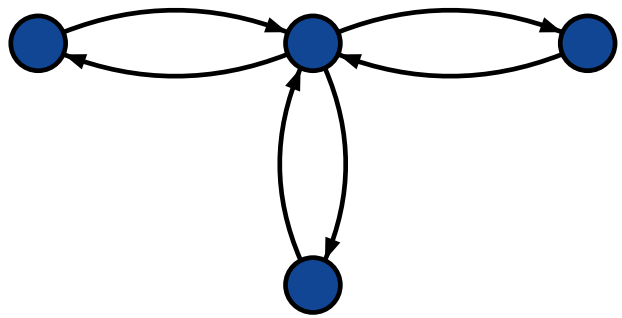

has no Hamilton path

HP is known to be ***NP***-complete

Polynomial reduction

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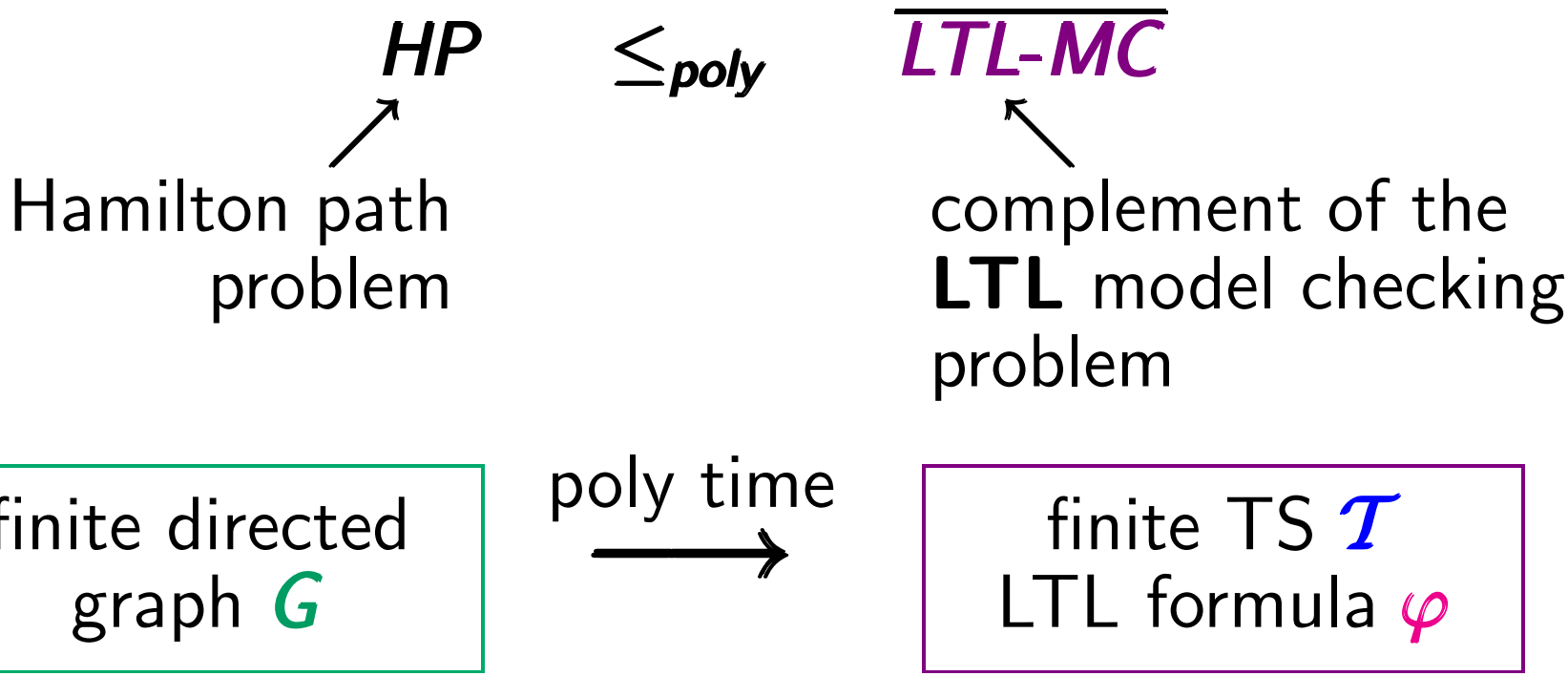

Polynomial reduction

$HP \leq_{poly} \overline{\text{LTL-MC}}$

Hamilton path problem

complement of the **LTL** model checking problem

finite directed graph G

poly time $\longrightarrow$

finite TS $\mathcal{T}$
LTL formula φ

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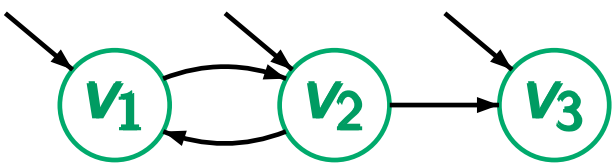
node-set V of G $\quad \widehat{=} \quad$ states of $\mathcal{T}$



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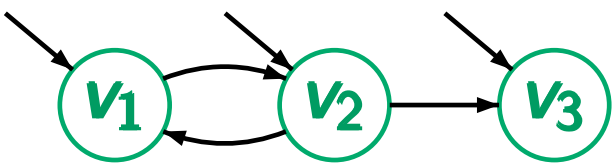

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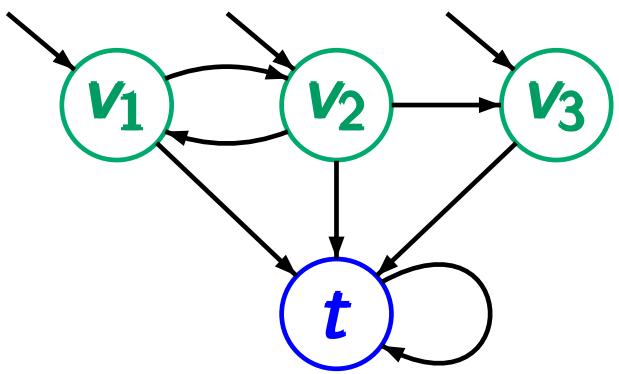
additional trap state t



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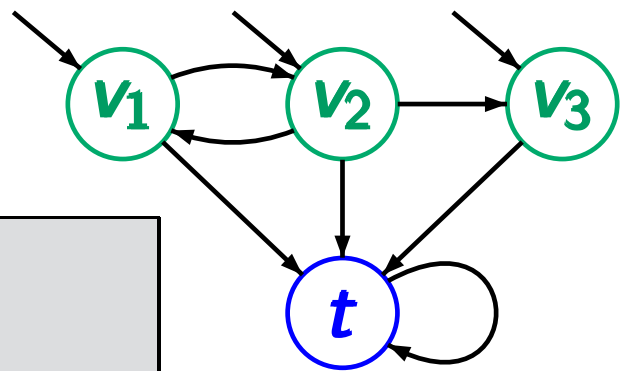

$\varphi = ?$

Polynomial reduction

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finite directed graph G — poly time $\longrightarrow$ finite TS $\mathcal{T}$, LTL formula φ

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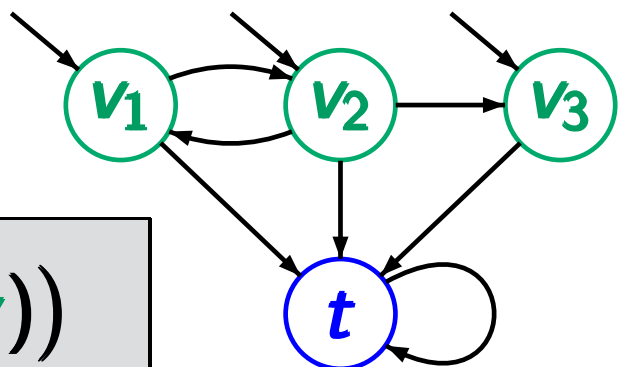
node-set V of G $\;\widehat{=}\;$ states of $\mathcal{T}$, $AP = V$, additional trap state t



$$\varphi = \bigwedge_{v \in V} \left(\Diamond v \wedge \Box(v \rightarrow \bigcirc\Box\neg v) \right)$$

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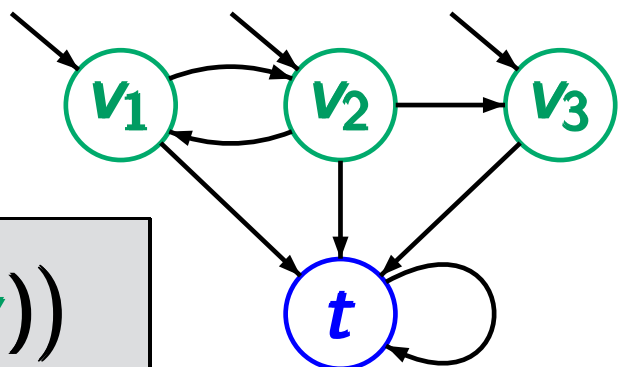

$$\varphi = \neg \bigwedge_{v \in V} \left(\Diamond v \wedge \Box(v \rightarrow \bigcirc\Box\neg v) \right)$$

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We just saw:

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We now prove:

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  DFS-based analysis of the computation tree of an ***NP***-algorithm

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  $\uparrow$

  DFS-based analysis of the computation tree of an ***NP***-algorithm

  space requirements:

  recursion depth $\widehat{=}$ height of computation tree

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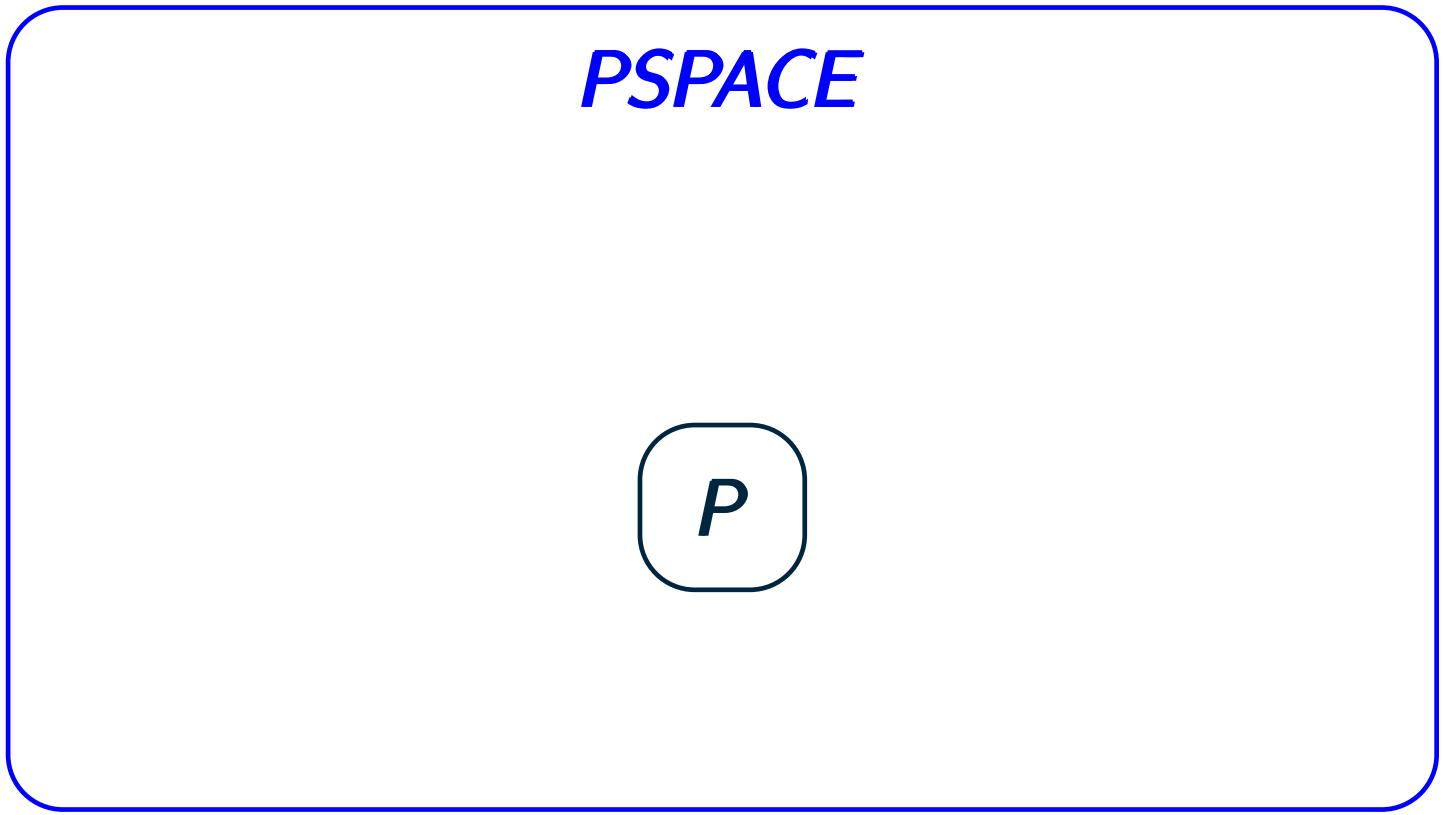
To prove $L \in$ ***PSPACE*** it suffices to provide a nondeterministic polynomially space-bounded algorithm for the complement $\overline{L}$ of L

Complexity classes *P*, *NP*, *coNP*, *PSPACE*



PSPACE = class of decision problems that are decidable in deterministic polynomial space

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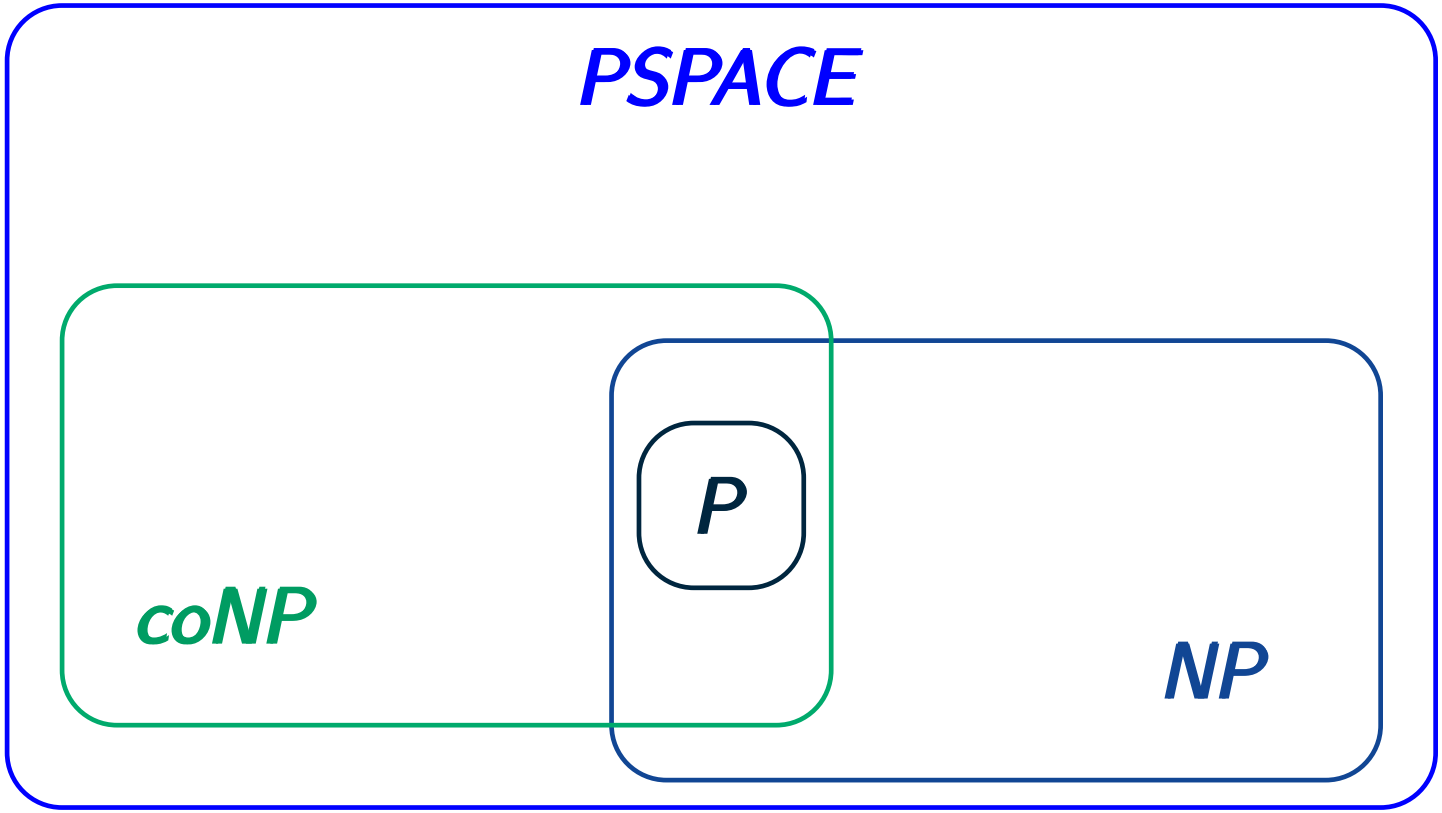

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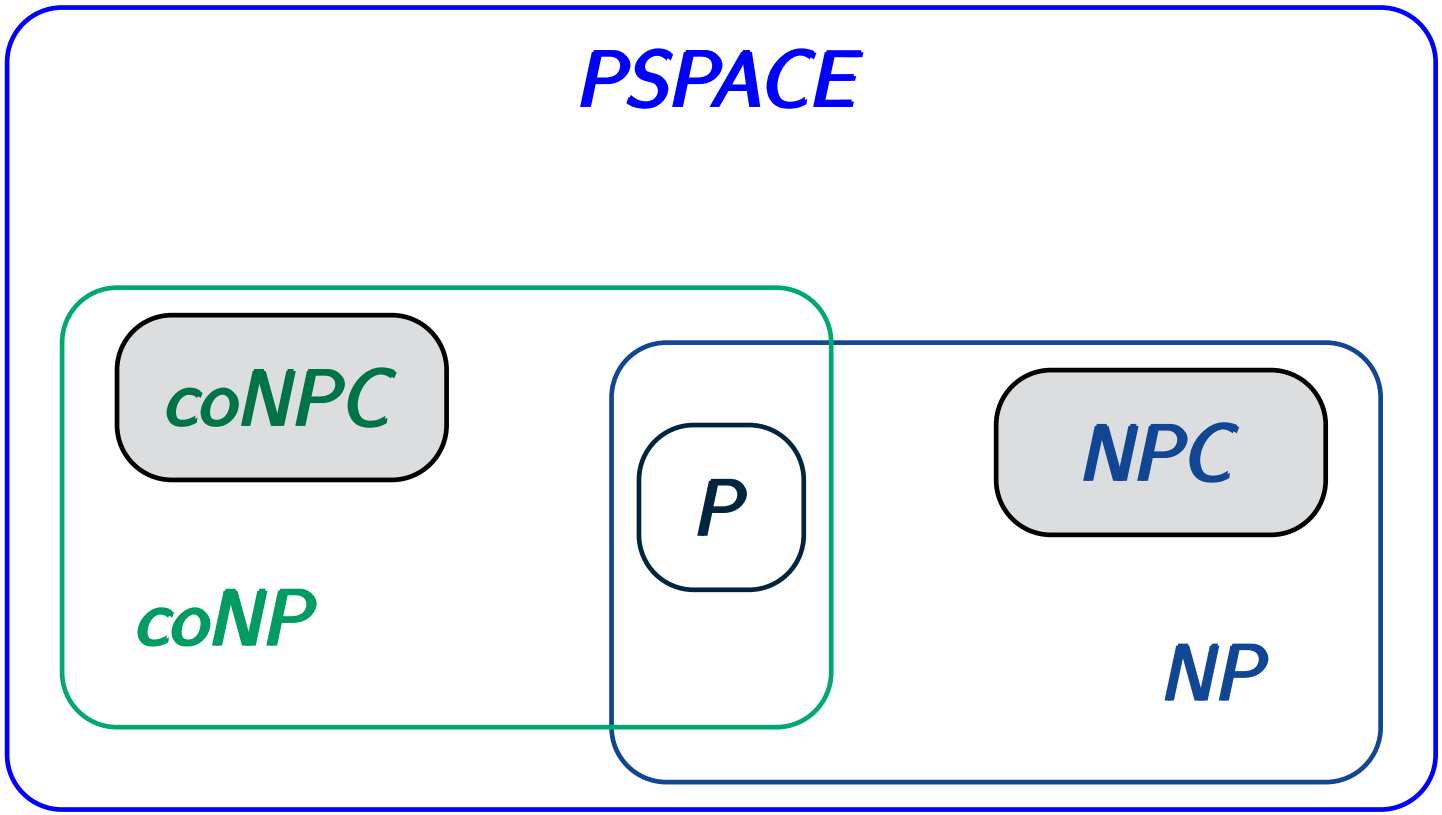

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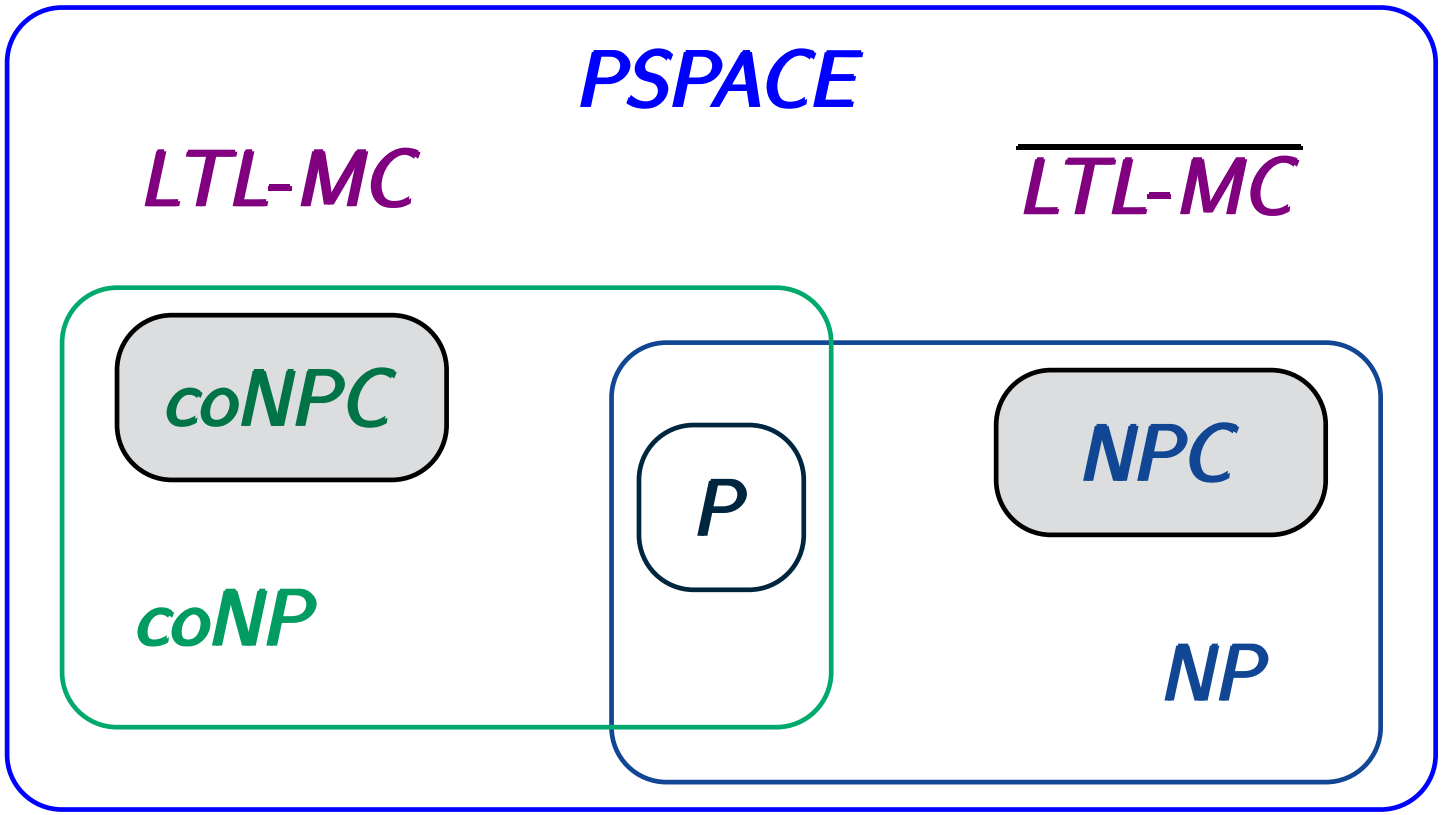

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PSPACE-completeness

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decision problem L is *PSPACE*-complete iff

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$L \in$ *PSPACE* $\iff$ $\overline{L} \in$ *NPSPACE*

PSPACE-completeness of *LTL-MC*

LTL-MC LTL model checking problem

"does $\pi \models \varphi$ hold for all paths π of $\mathcal{T}$ **?**"

$\overline{\textit{LTL-MC}}$ = complement of ***LTL-MC***

"does $\pi \not\models \varphi$ hold for some path π of $\mathcal{T}$ **?**"

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∃LTL-MC existential LTL model checking problem

for $\mathcal{T}$ and LTL formula $\psi = \neg\varphi$

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show: ***∃LTL-MC*** $\in$ ***NPSPACE***

∃LTL-MC is ***PSPACE***-hard

PSPACE-completeness of *LTL-MC*

LTL-MC LTL model checking problem

"does $\pi \models \varphi$ hold for all paths π of $\mathcal{T}$?"

$\overline{\textit{LTL-MC}}$ = complement of ***LTL-MC***

"does $\pi \not\models \varphi$ hold for some path π of $\mathcal{T}$?"

↑

∃LTL-MC existential LTL model checking problem for $\mathcal{T}$ and LTL formula $\psi = \neg\varphi$

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show: ***∃LTL-MC*** $\in$ ***NPSPACE*** $\Longrightarrow$ ***LTL-MC*** $\in$ ***PSPACE***

∃LTL-MC is ***PSPACE***-hard

PSPACE-completeness of *LTL-MC*

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Existential LTL model checking problem

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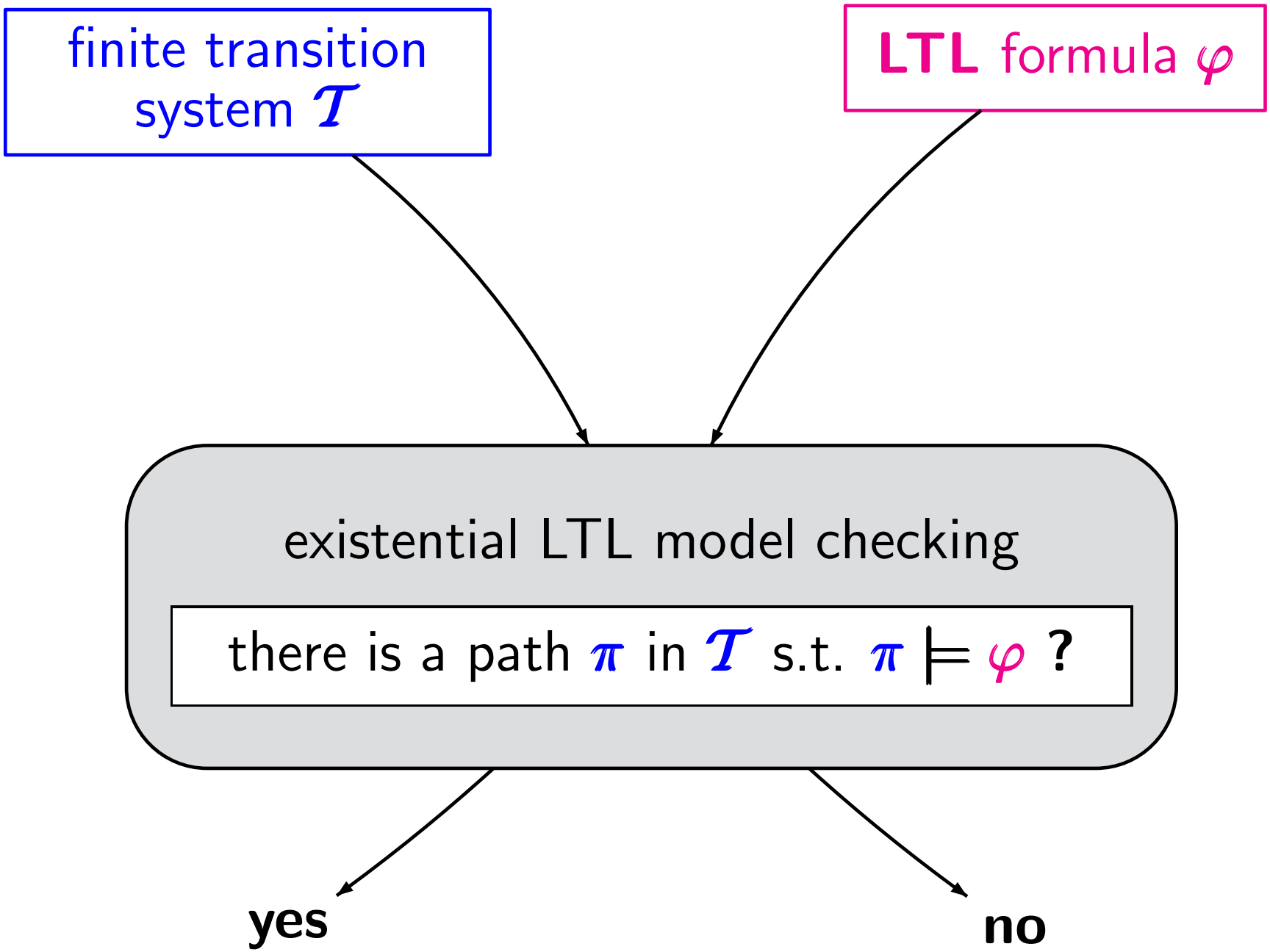
idea: use the **GNBA $\mathcal{G}$** for φ (constructed by our LTL-2-GNBA algorithm)

Existential LTL model checking



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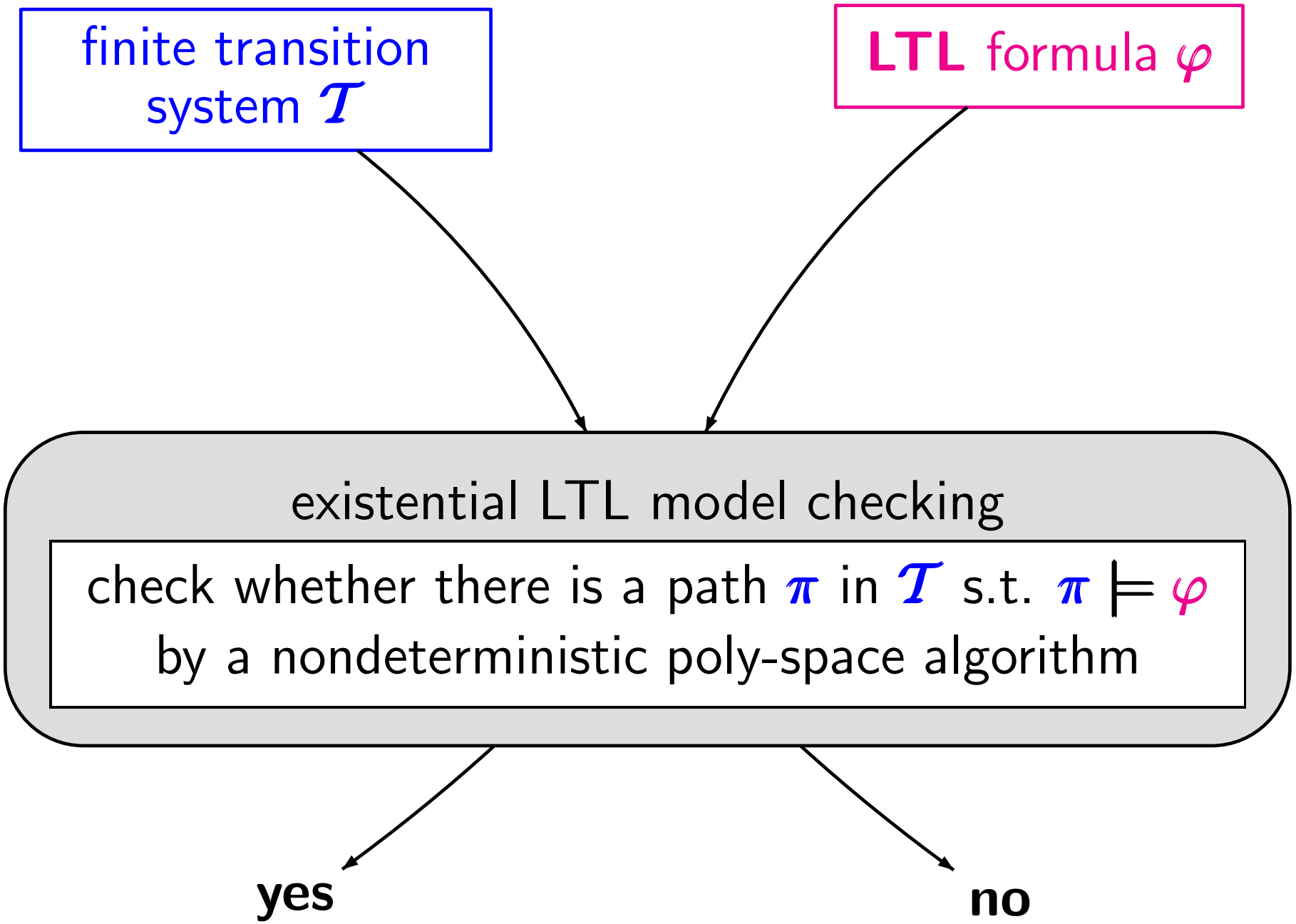

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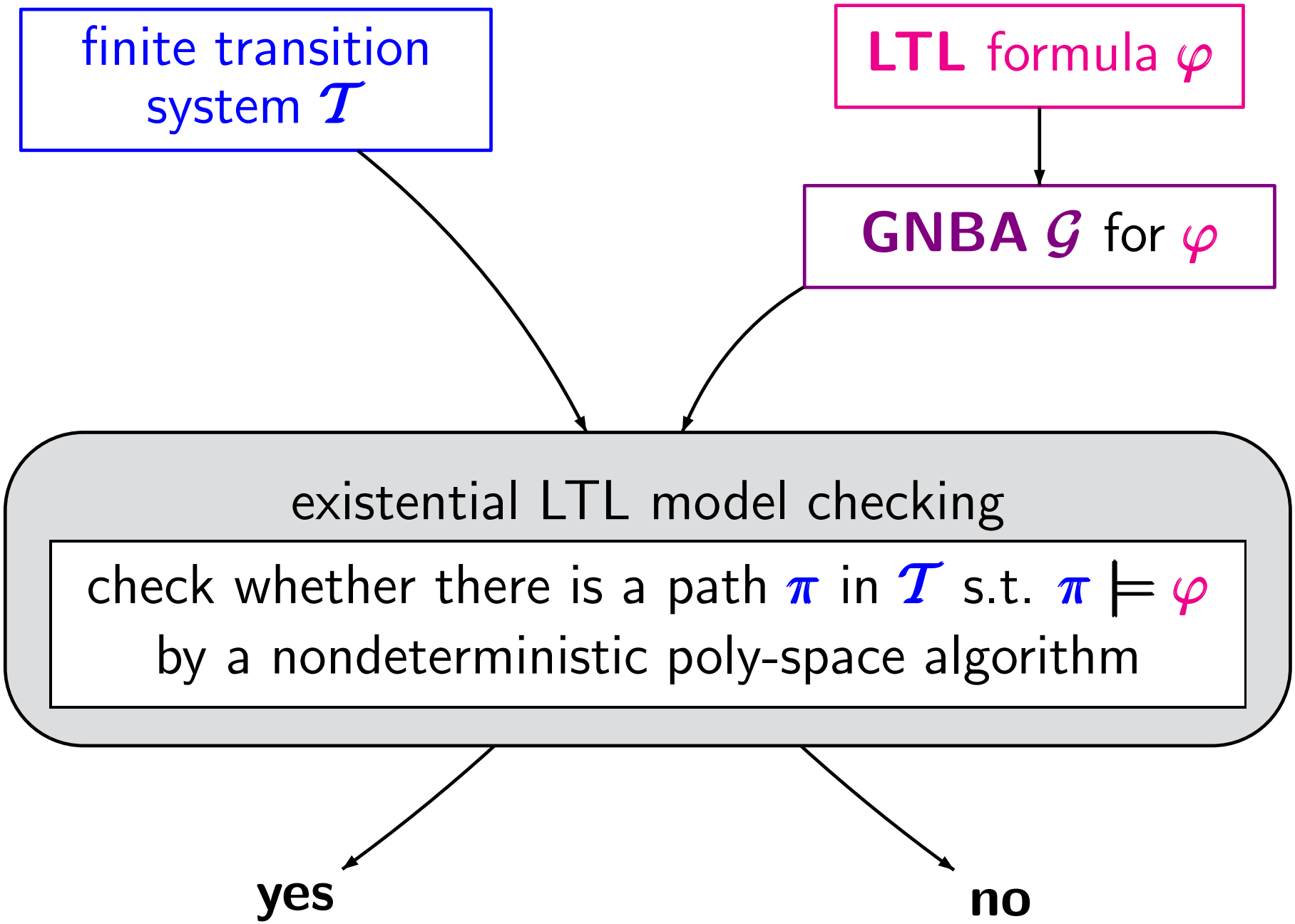

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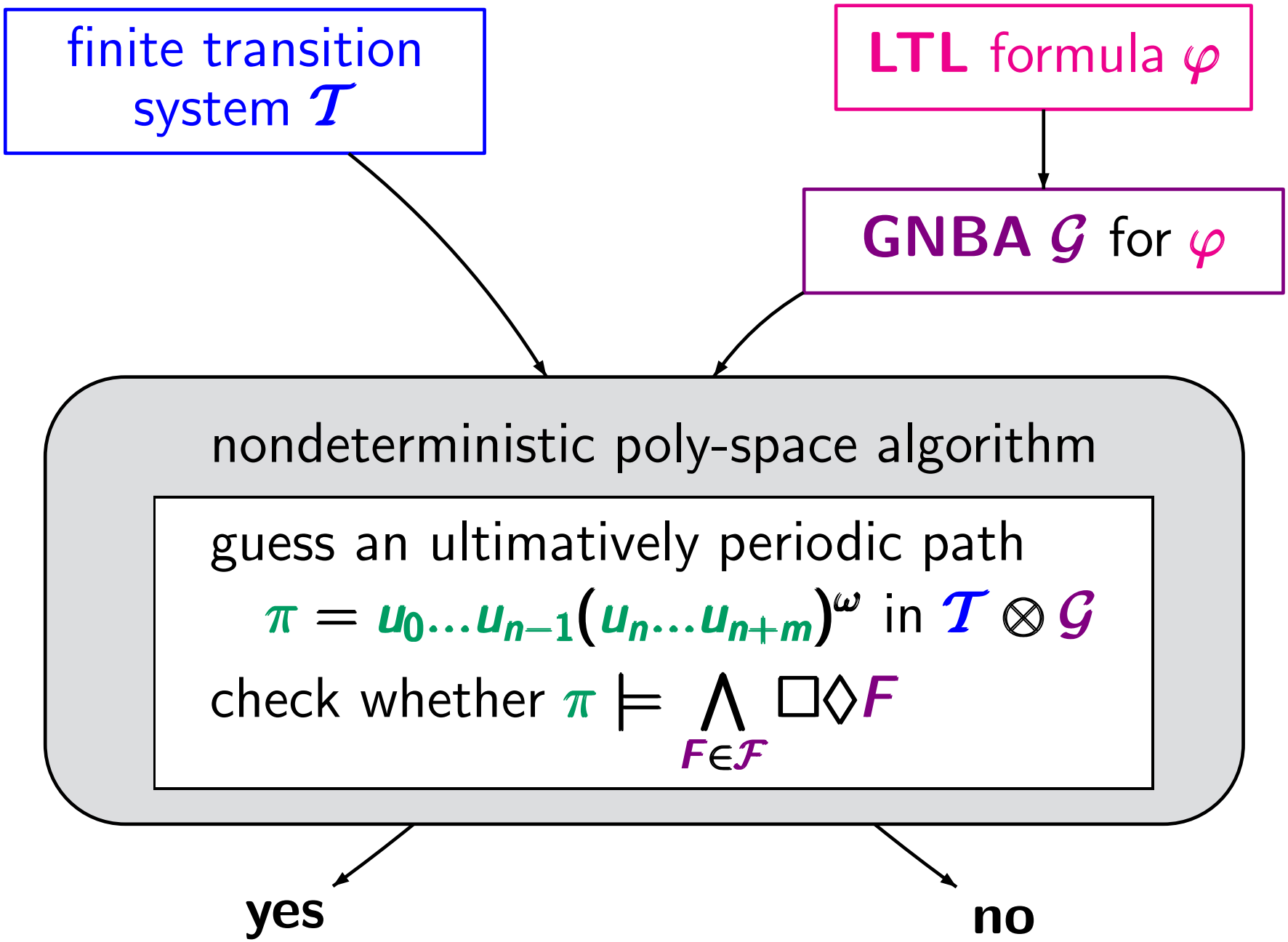

Recall: elementary formula-sets

closure $cl(\varphi)$:

- set of all subformulas of φ and their negations
- ψ and $\neg\neg\psi$ are identified

elementary formula-sets: subsets B of $cl(\varphi)$

- maximal consistent w.r.t. propositional logic
- locally consistent w.r.t. U

For $\varphi = a\,\mathsf{U}(\neg a \wedge b)$, the elementary sets are:

$$\{\ a,\ b, \neg(\neg a \wedge b), \varphi\} \qquad \{\ a,\ b, \neg(\neg a \wedge b), \neg\varphi\}$$
$$\{\ a, \neg b, \neg(\neg a \wedge b), \varphi\} \qquad \{\ a, \neg b, \neg(\neg a \wedge b), \neg\varphi\}$$
$$\{\neg a,\ b,\ \neg a \wedge b\ , \varphi\} \qquad \{\neg a, \neg b, \neg(\neg a \wedge b), \neg\varphi\}$$

Recall: GNBA for LTL-formula φ

$\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$

state space: $Q = \{B \subseteq cl(\varphi) : B \text{ is elementary}\}$

initial states: $Q_0 = \{B \in Q : \varphi \in B\}$

transition relation: for $B \in Q$ and $A \in 2^{AP}$:

if $A \neq B \cap AP$ then $\delta(B, A) = \varnothing$

if $A = B \cap AP$ then $\delta(B, A) =$ set of all $B' \in Q$ s.t.

$$\bigcirc\psi \in B \quad \text{iff} \quad \psi \in B'$$

$$\psi_1 \,\mathsf{U}\, \psi_2 \in B \quad \text{iff} \quad (\psi_2 \in B) \vee (\psi_1 \in B \wedge \psi_1 \,\mathsf{U}\, \psi_2 \in B')$$

acceptance set $\mathcal{F} = \{F_{\psi_1 \,\mathsf{U}\, \psi_2} : \psi_1 \,\mathsf{U}\, \psi_2 \in cl(\varphi)\}$

where $F_{\psi_1 \,\mathsf{U}\, \psi_2} = \{B \in Q : \psi_1 \,\mathsf{U}\, \psi_2 \notin B \vee \psi_2 \in B\}$

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NPSPACE-algorithm for ∃*LTL-MC*

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given: finite TS $\mathcal{T}$, LTL formula φ

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- guess nondeterministically an ultimatively periodic path $\pi = u_0\, u_1 \ldots u_{n-1} (u_n \ldots u_{n+m})^\omega$ of $\mathcal{T} \otimes \mathcal{G}$

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- check whether the guessed path meets the acceptance condition of $\mathcal{G}$

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If so then return "**yes**". Otherwise return "**no**".

PSPACE-completeness of ∃*LTL-MC*

We saw that:

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It remains to prove the ***PSPACE***-hardness

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we show that for all problems $K \in \mathit{PSPACE}$:

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Let

- $\mathcal{M}$ be a deterministic Turing machine (DTM) that decides K,
- P a polynomial

such that $\mathcal{M}$ started with an input word w visits at most $P(|w|)$ tape cells

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Given DTM $\mathcal{M}$ that decides K with polynomial space bound $P(n)$, provide a polynomial reduction:

input word w for $\mathcal{M}$ —poly time→ TS $\mathcal{T}$, LTL-formula φ

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input word w for $\mathcal{M}$	poly time $\longrightarrow$	TS $\mathcal{T}$ LTL-formula φ
$\mathcal{M}$ accepts w, i.e., $w \in K$	iff	there is path π of $\mathcal{T}$ with $\pi \models \varphi$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$

DTM $\mathcal{M}$ visits at the most the tape cells $1, 2, \ldots, P(n)$ for inputs of length n (where P is a polynomial)

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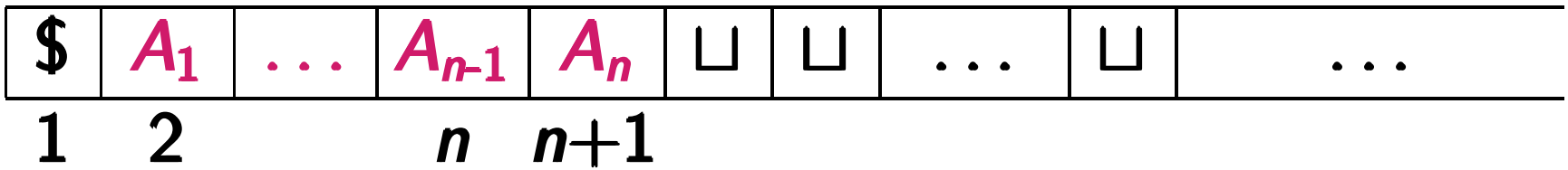

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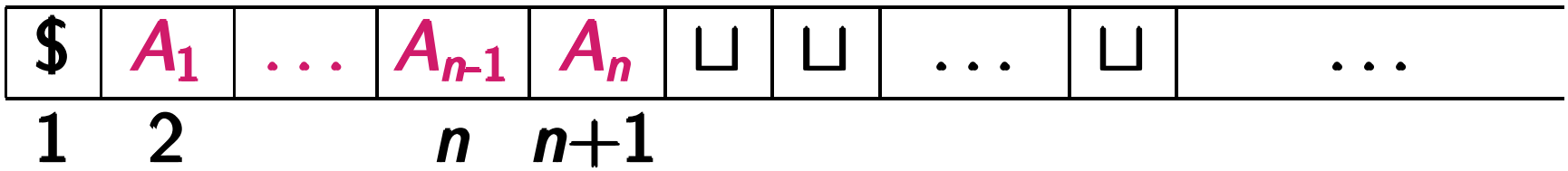

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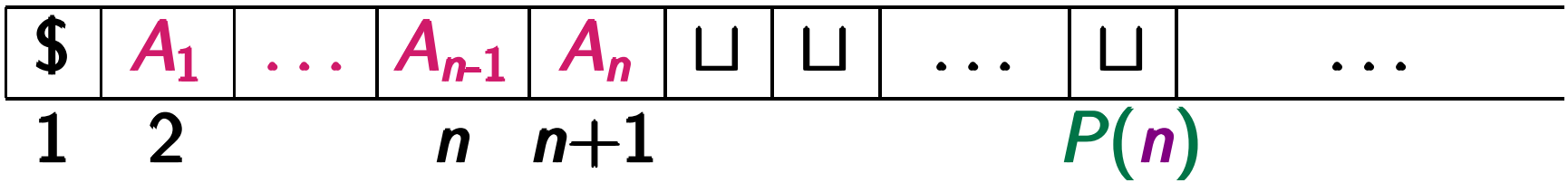

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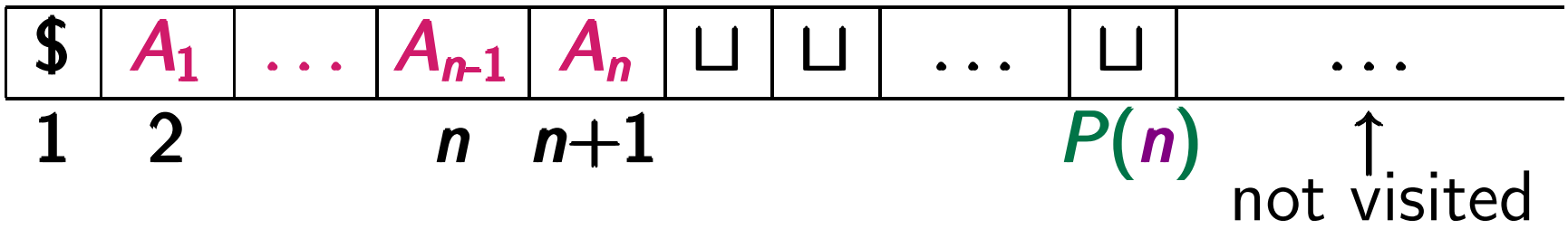

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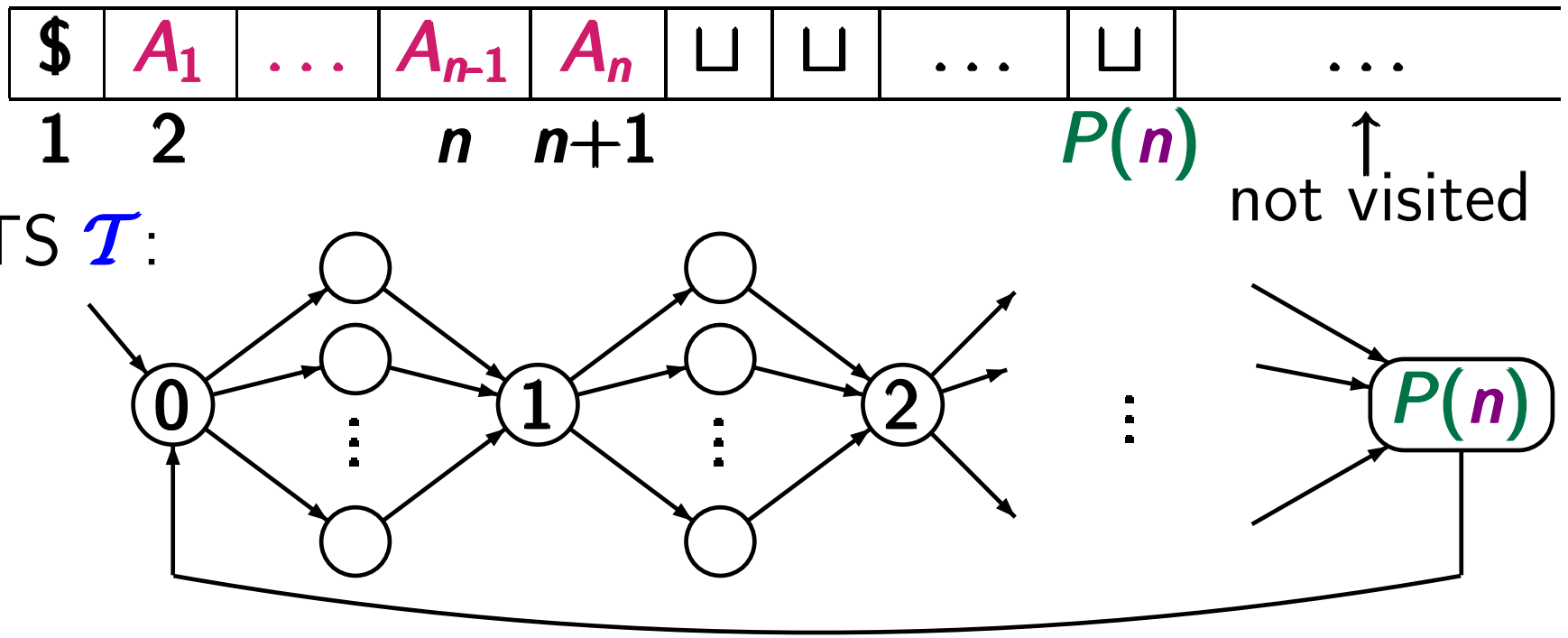

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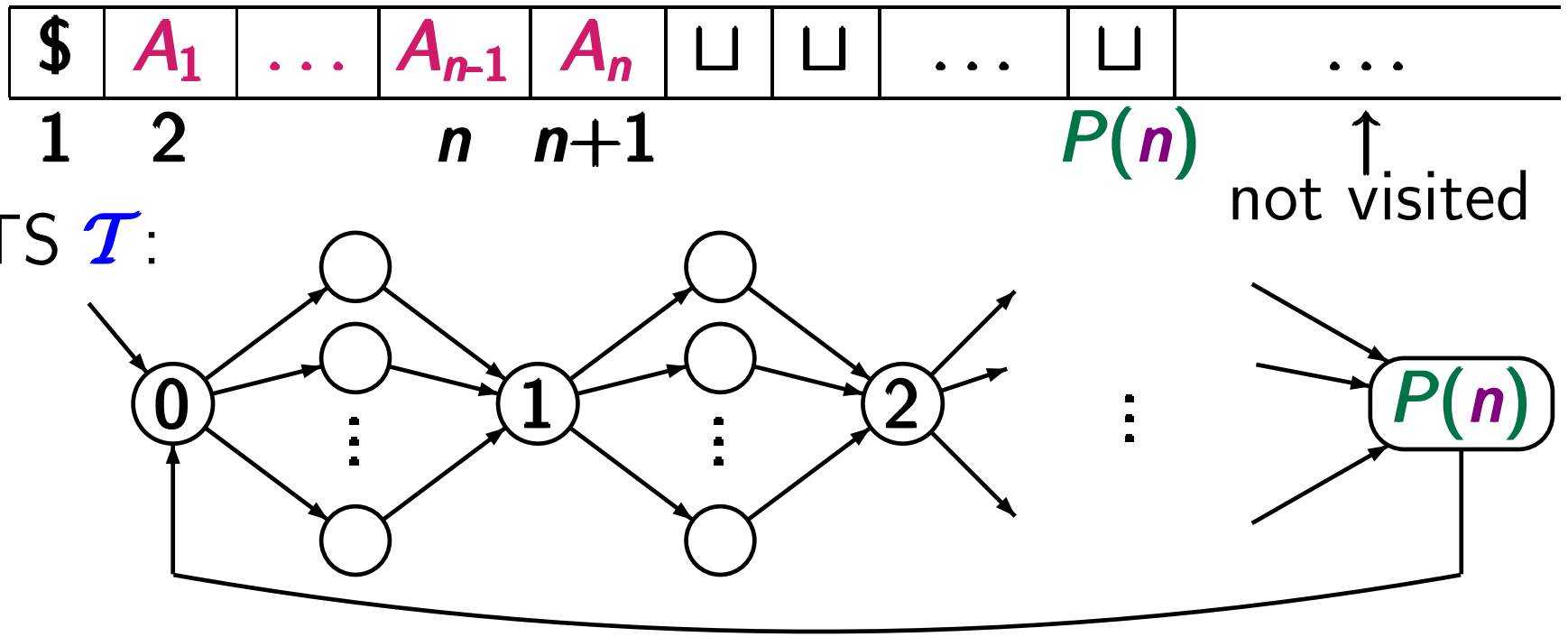

states of $\mathcal{T}$: $0, 1, \ldots, P(n)$,

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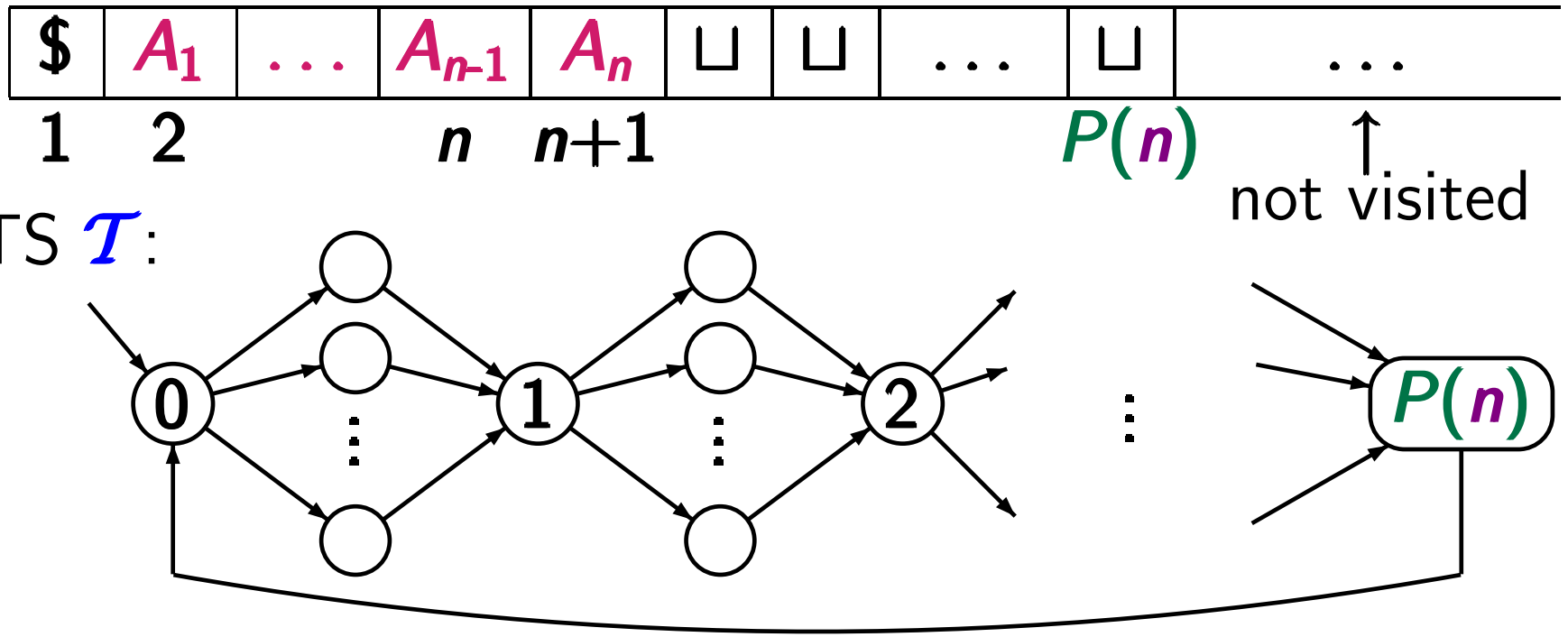

states of $\mathcal{T}$: $0, 1, \ldots, P(n)$, $\langle q, A, i \rangle$, $\langle *, A, i \rangle$

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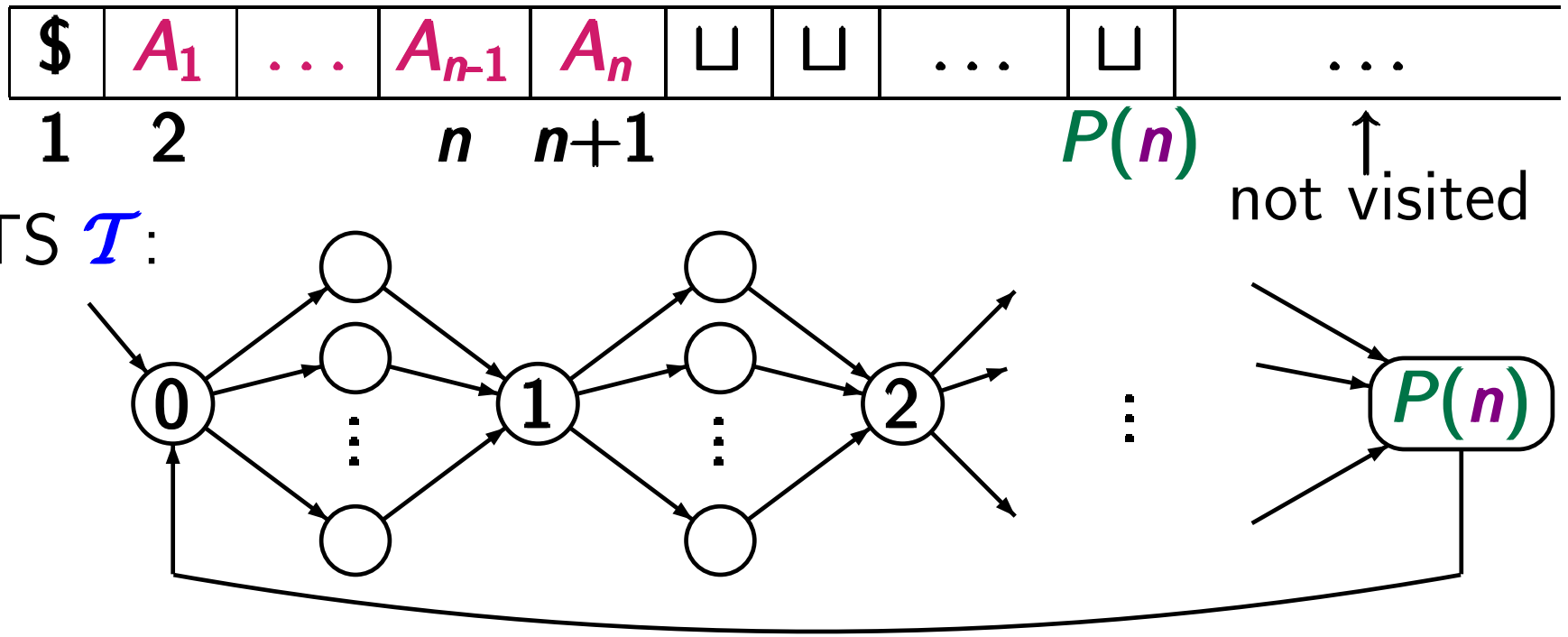

states of $\mathcal{T}$: $0, 1, \ldots, P(n)$, $\langle q, A, i \rangle$, $\langle *, A, i \rangle$

where q is a state of $\mathcal{M}$, A a tape symbol, $1 \leq i \leq P(n)$

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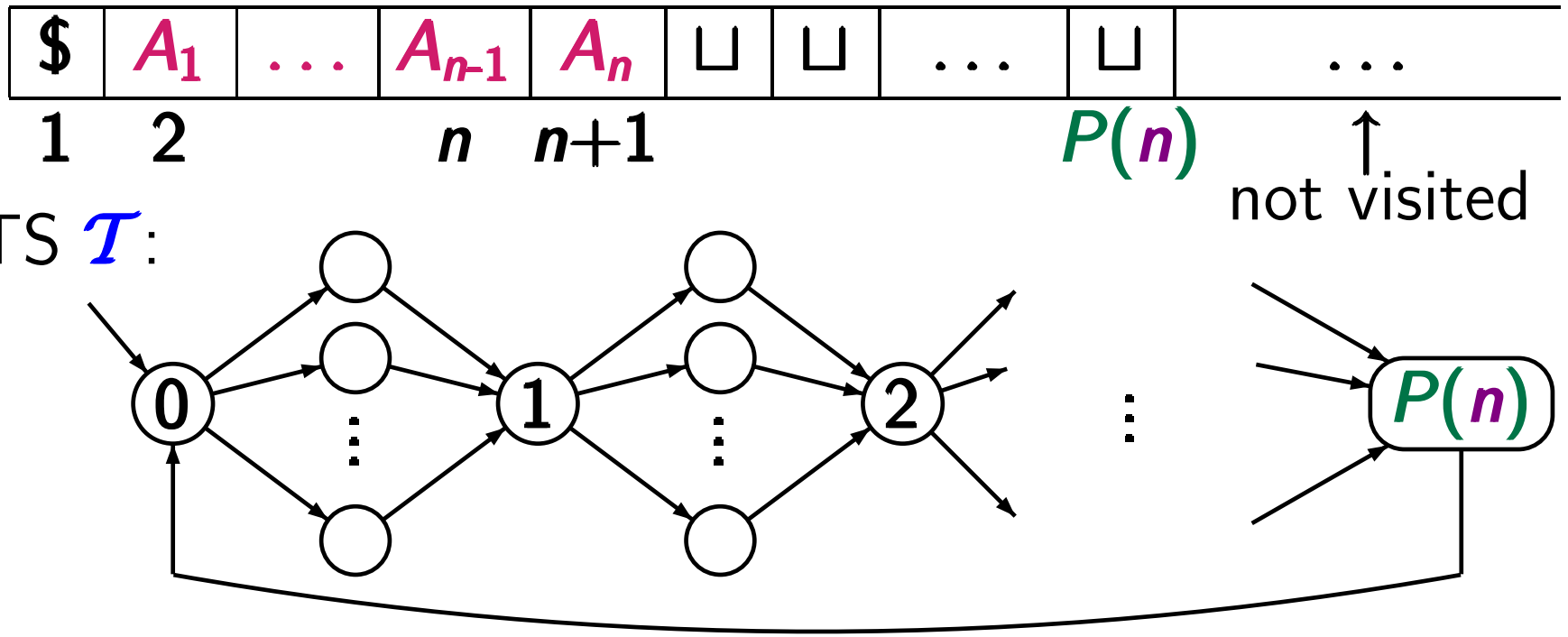

idea: TS $\mathcal{T}$ encodes each configuration of $\mathcal{M}$ by a path fragment from state **0** to state $P(n)$

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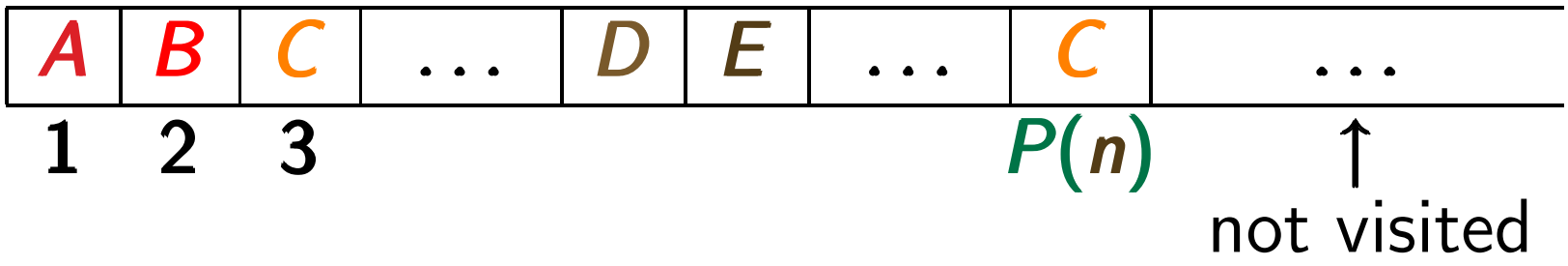

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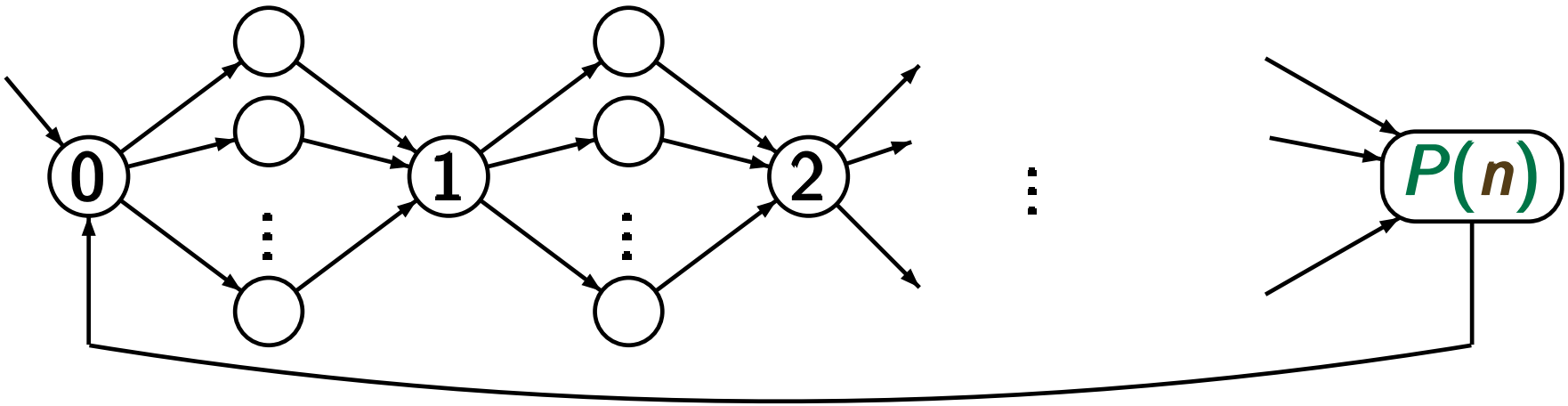
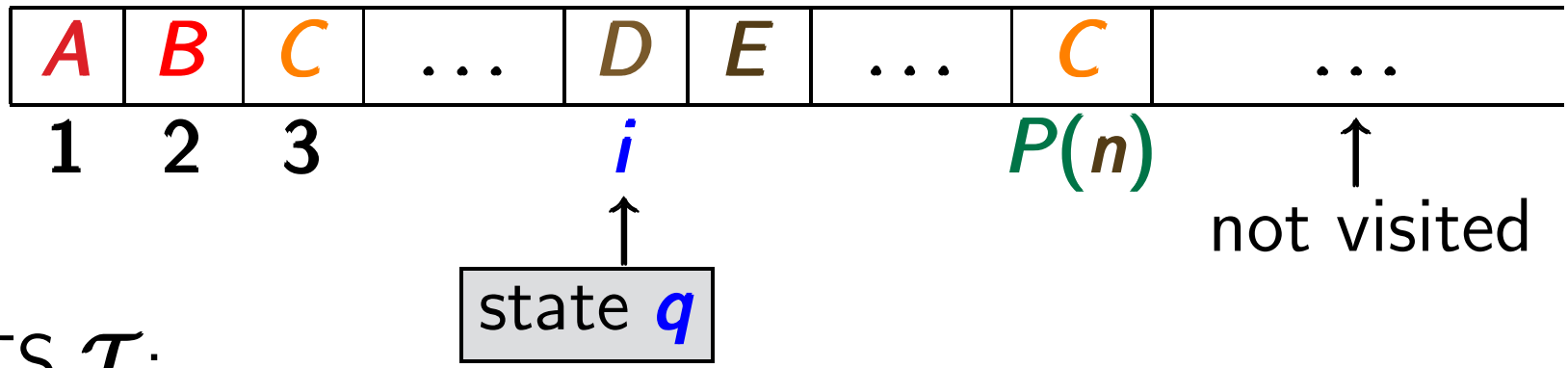
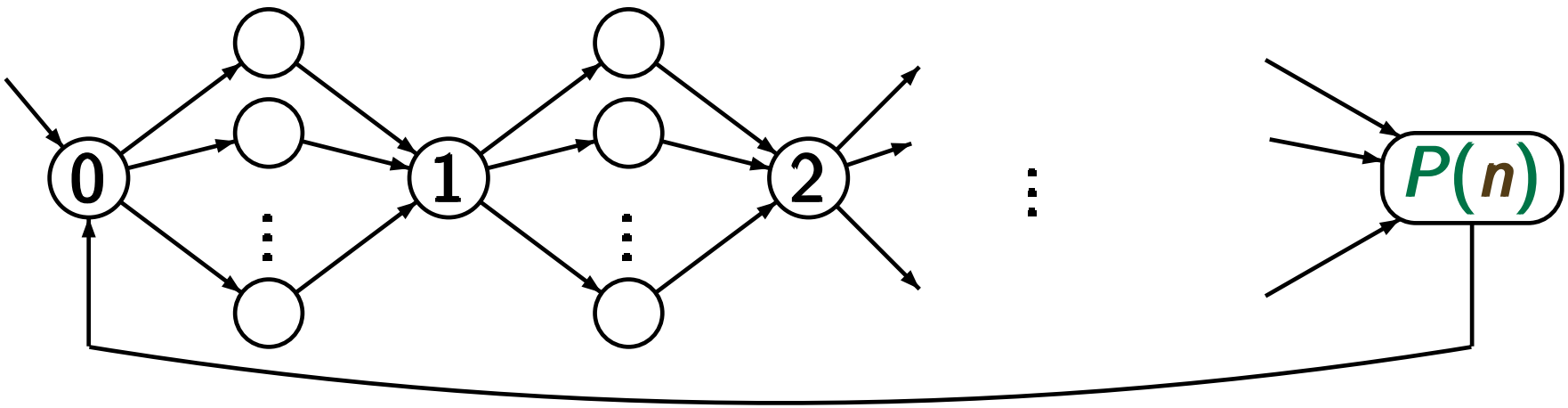


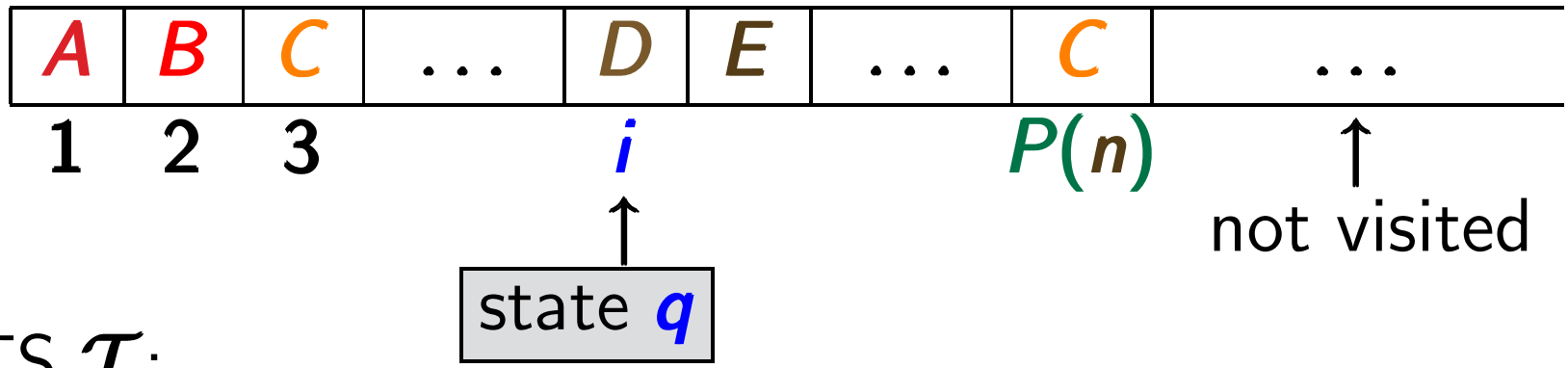
Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



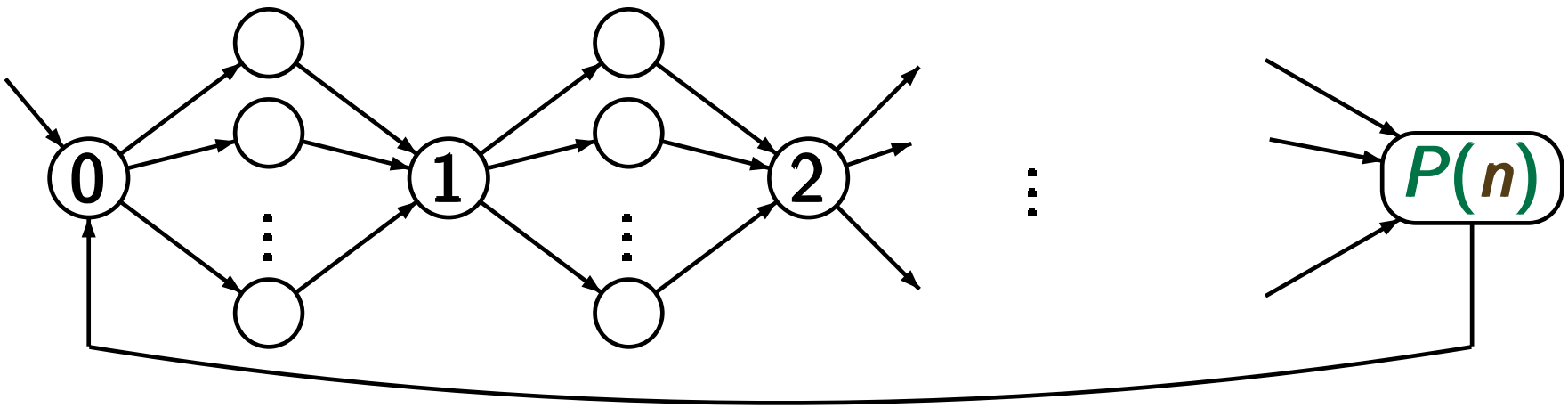
TS $\mathcal{T}$:



Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$

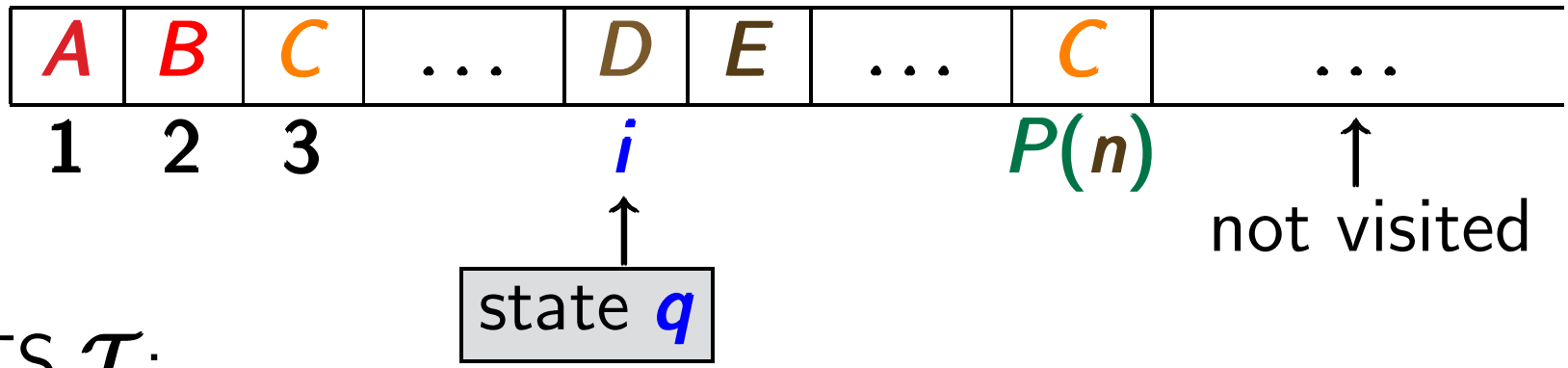


TS $\mathcal{T}$:

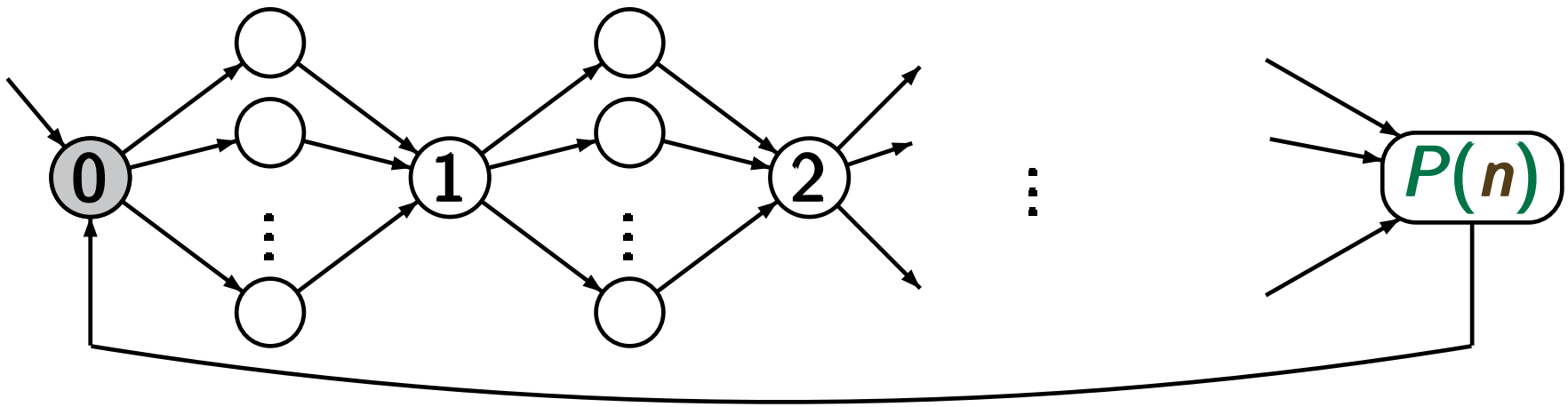


suppose $\delta(q, D) = (p, B, +1)$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



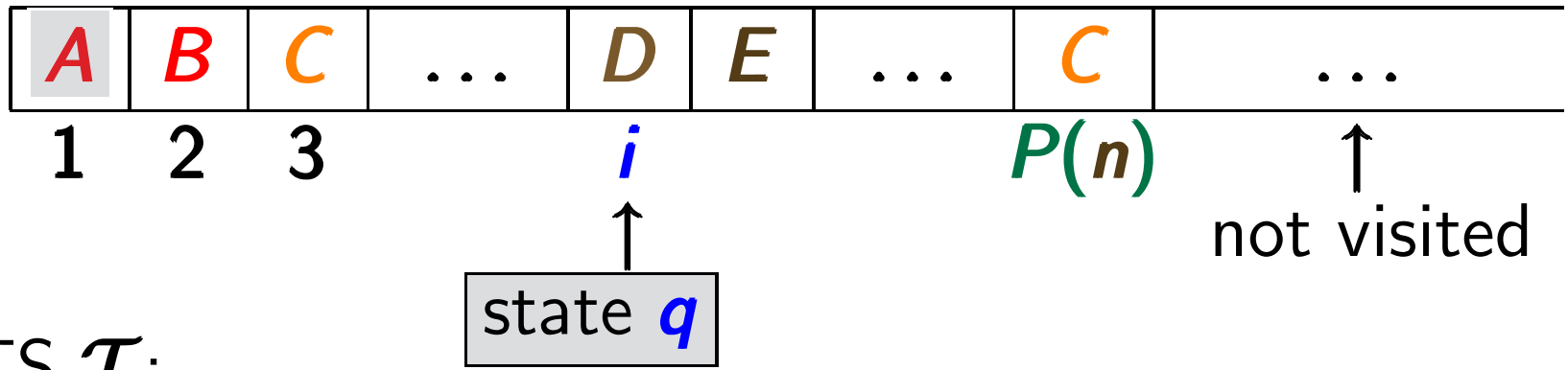
TS $\mathcal{T}$:



0

path fragment for the configuration $ABC\ldots\ q\ D\ldots C$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



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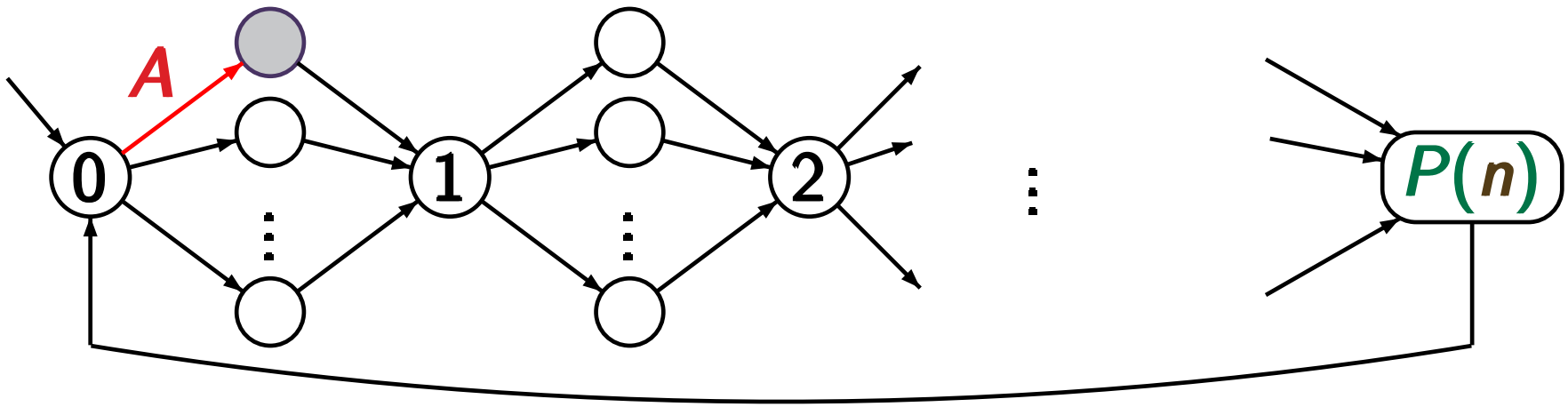
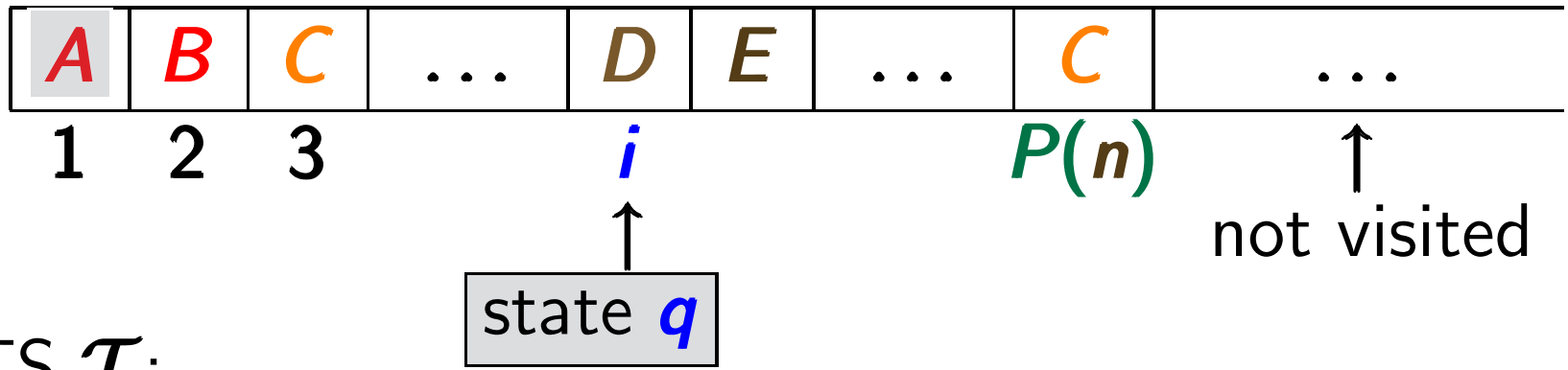


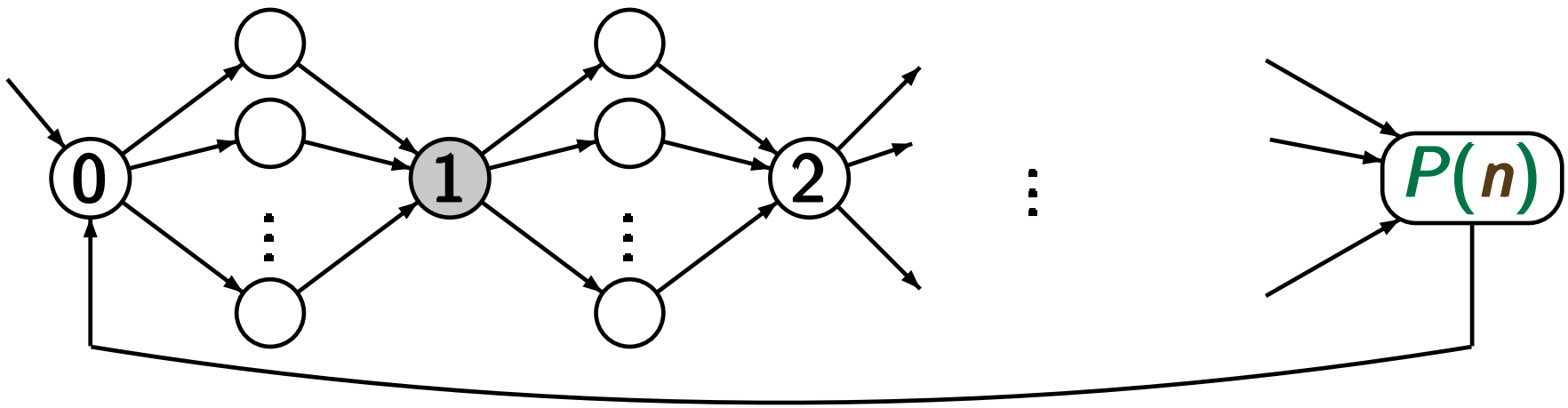
$0\ \langle *, A, 1 \rangle$

path fragment for the configuration $ABC\ldots\ q\ D\ldots C$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



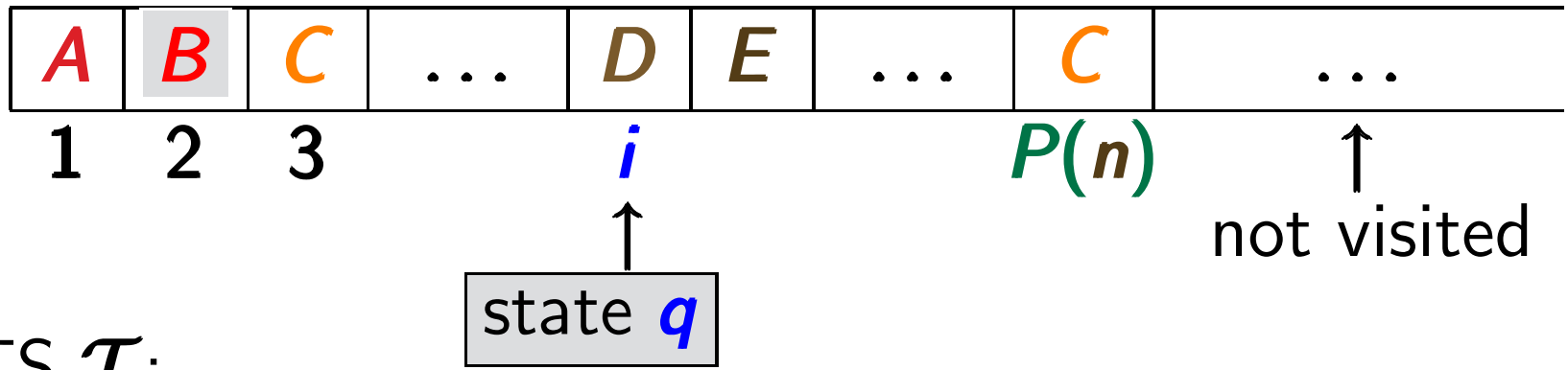
TS $\mathcal{T}$:



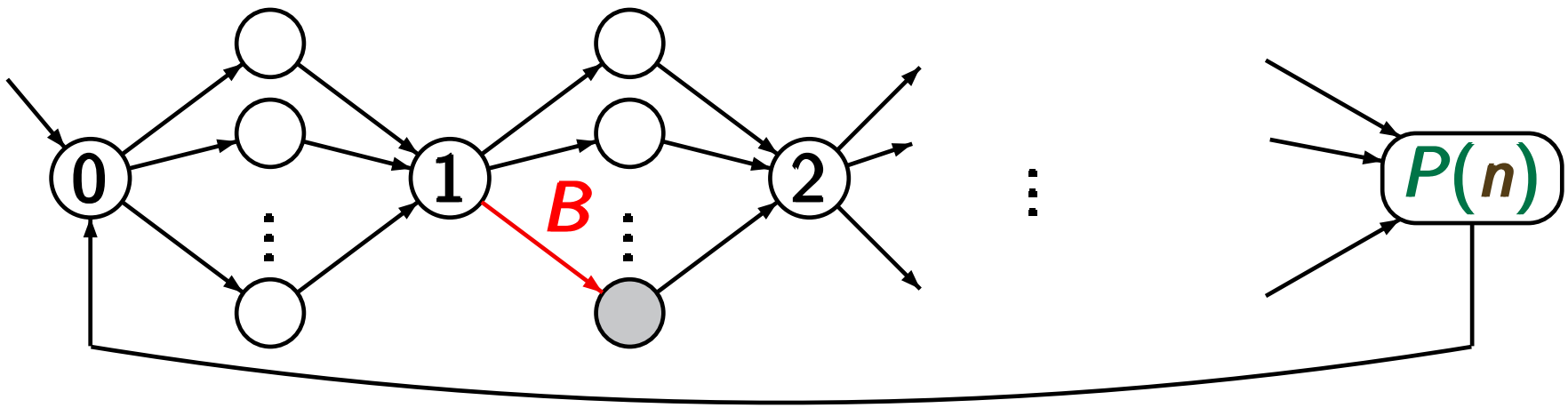
$0\ \langle *, A, 1\rangle\ 1$

path fragment for the configuration $ABC\ldots\ q\ D\ldots C$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



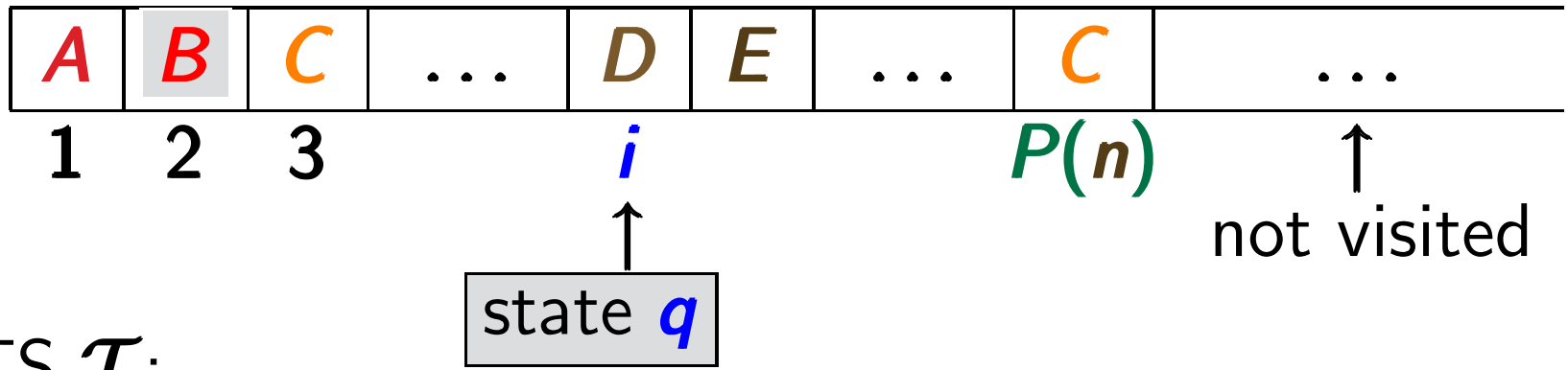
TS $\mathcal{T}$:



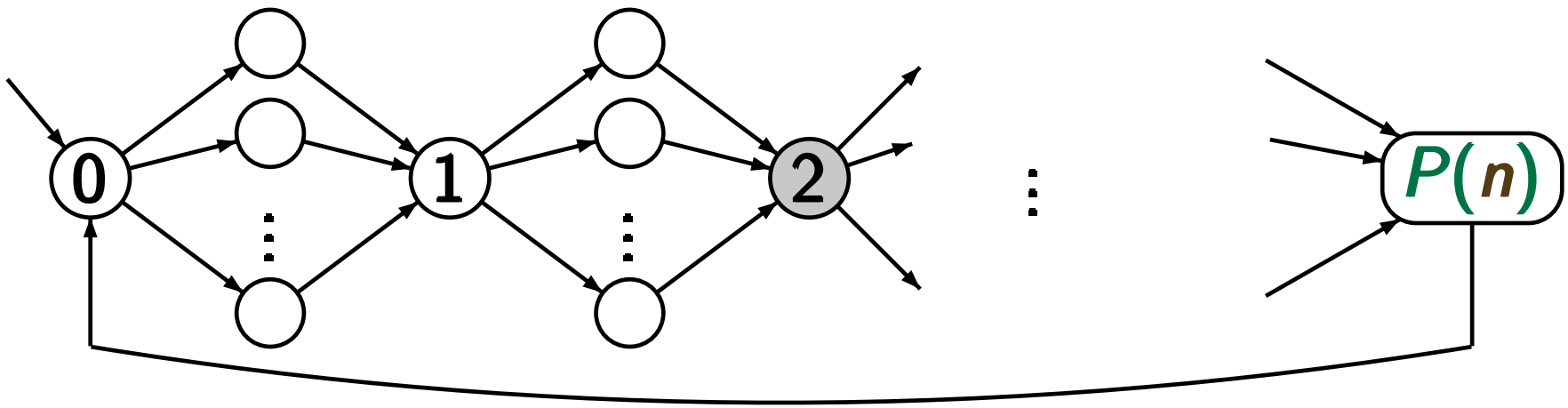
$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle$

path fragment for the configuration $ABC\ldots\ q\ D\ldots C$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



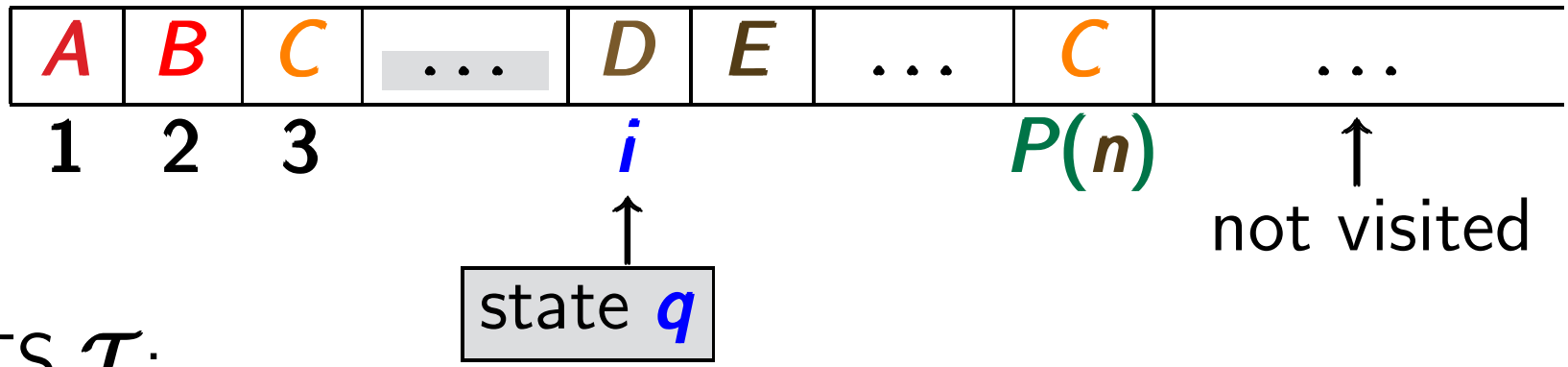
TS $\mathcal{T}$:



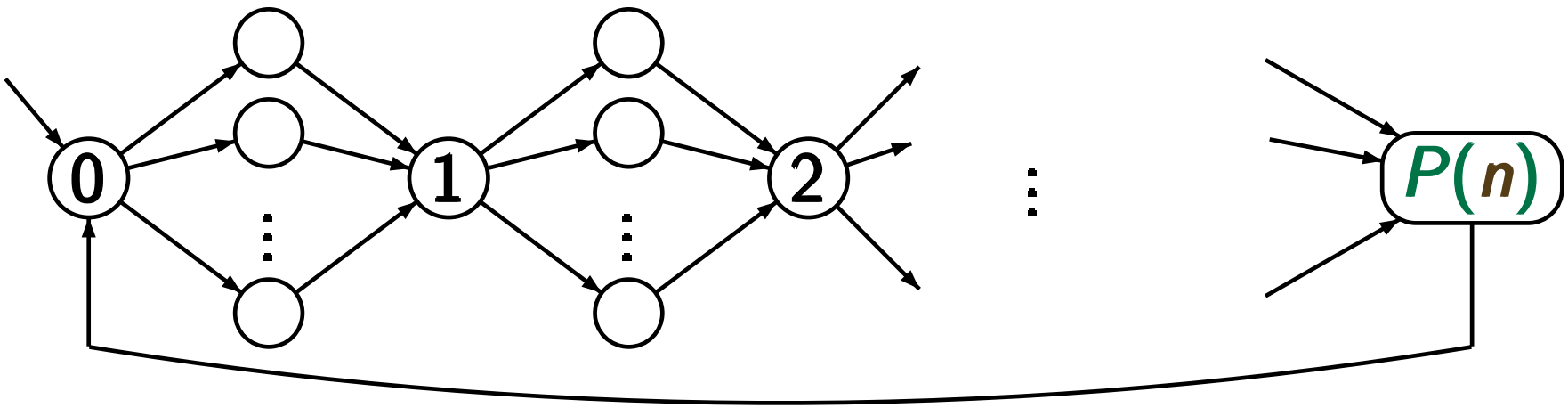
$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2$

path fragment for the configuration $ABC\ldots\ q\ D\ldots C$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



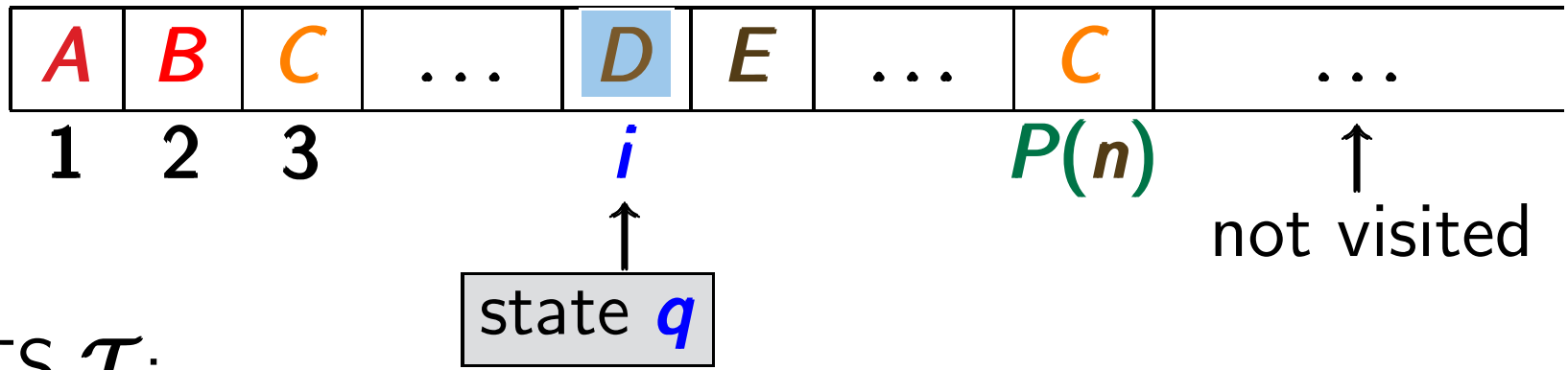
TS $\mathcal{T}$:



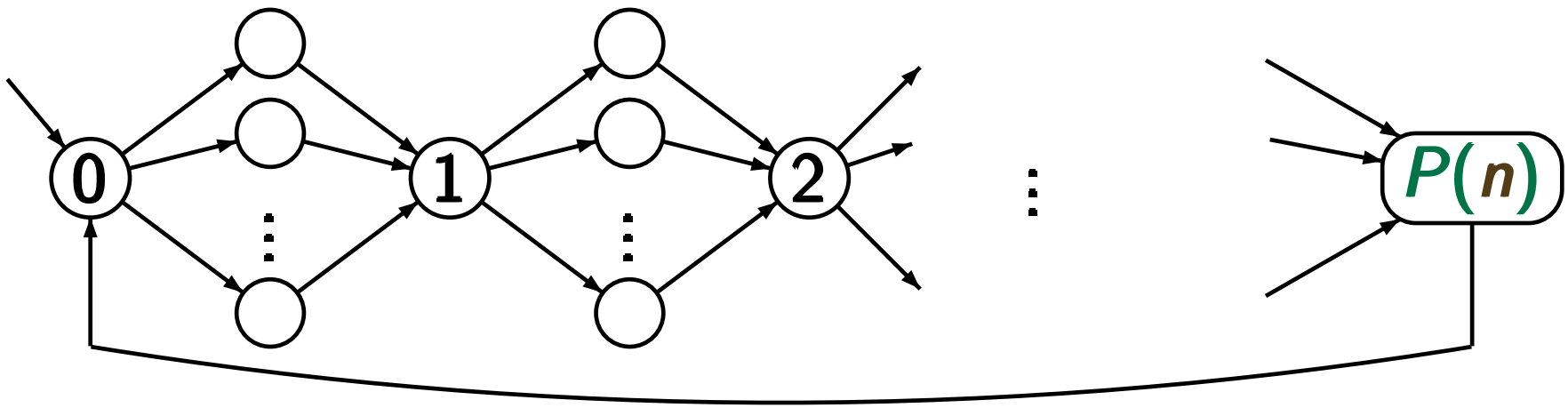
$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2\ \ldots\ (i-1)$

path fragment for the configuration $ABC\ldots\ q\ D\ldots C$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



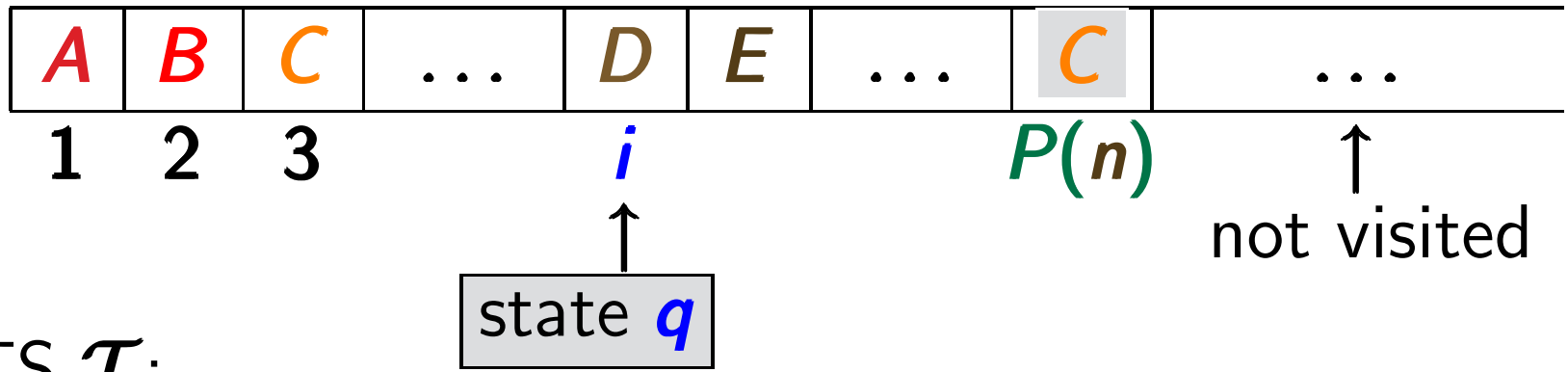
TS $\mathcal{T}$:



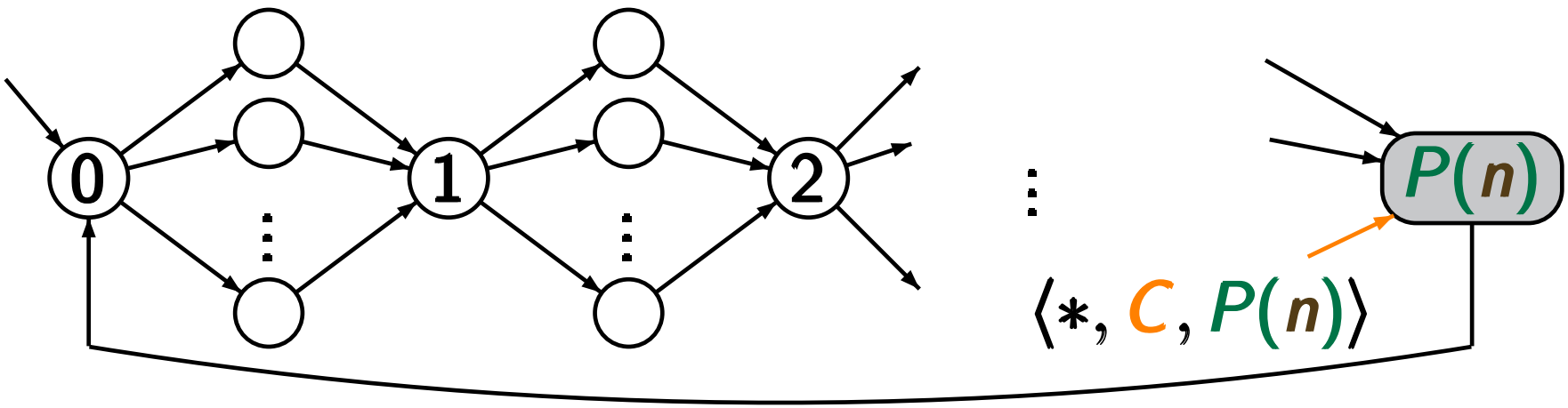
$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2\ \ldots\ (i{-}1)\langle q, D, i\rangle$

path fragment for the configuration $ABC\ldots\ q\ D\ldots C$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



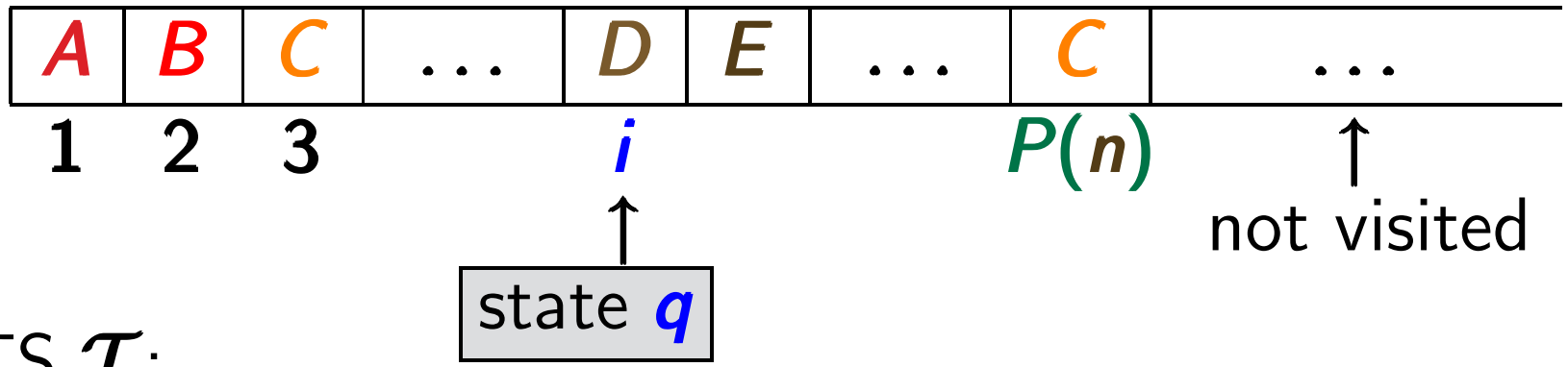
TS $\mathcal{T}$:



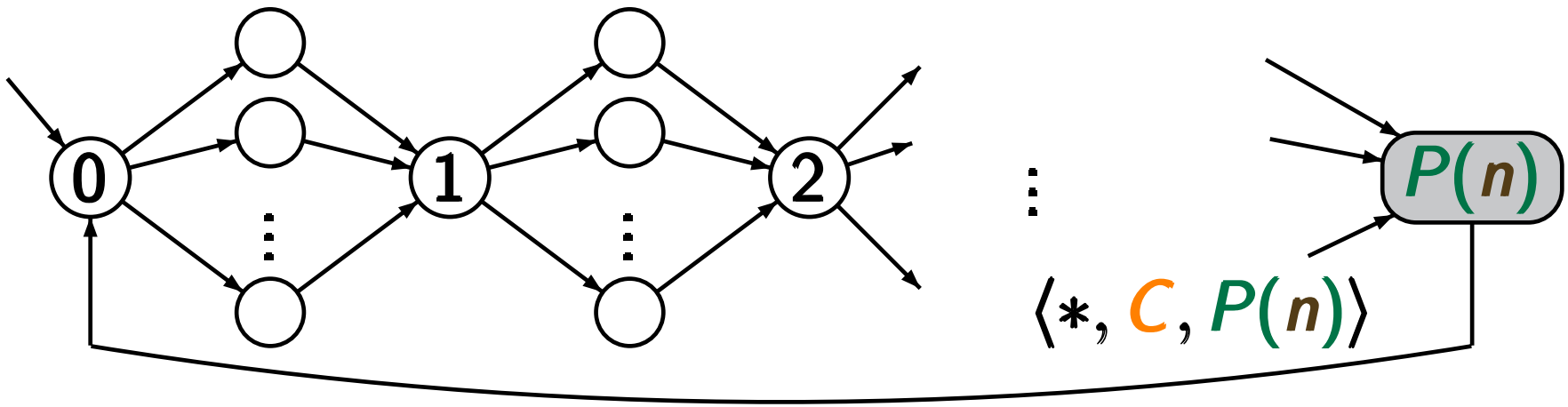
$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2\ \ldots\ (i{-}1)\langle q, D, i\rangle\ i\ \ldots\ P(n)$

path fragment for the configuration $ABC\ldots\ q\ D\ldots C$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



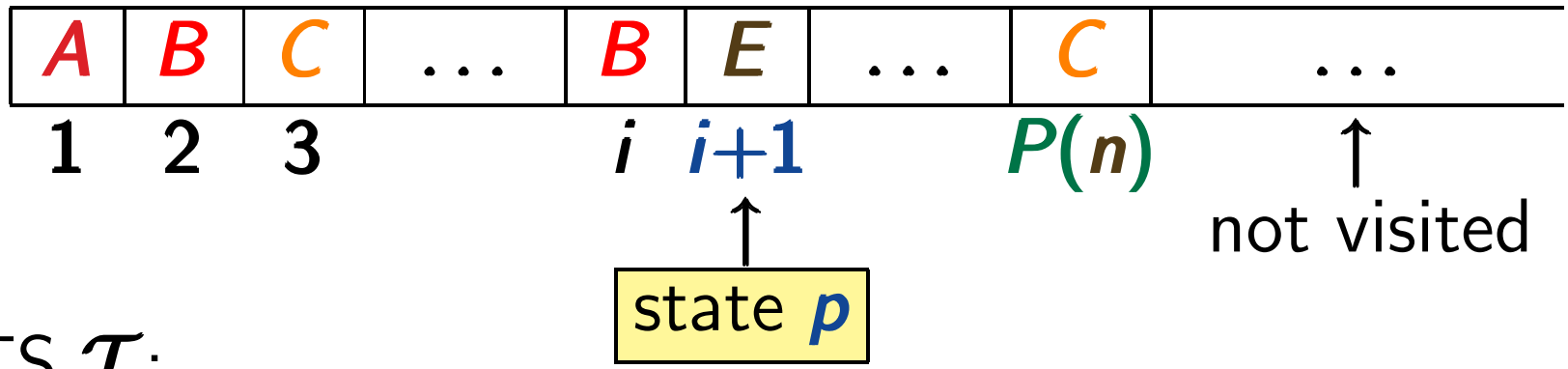
TS $\mathcal{T}$:



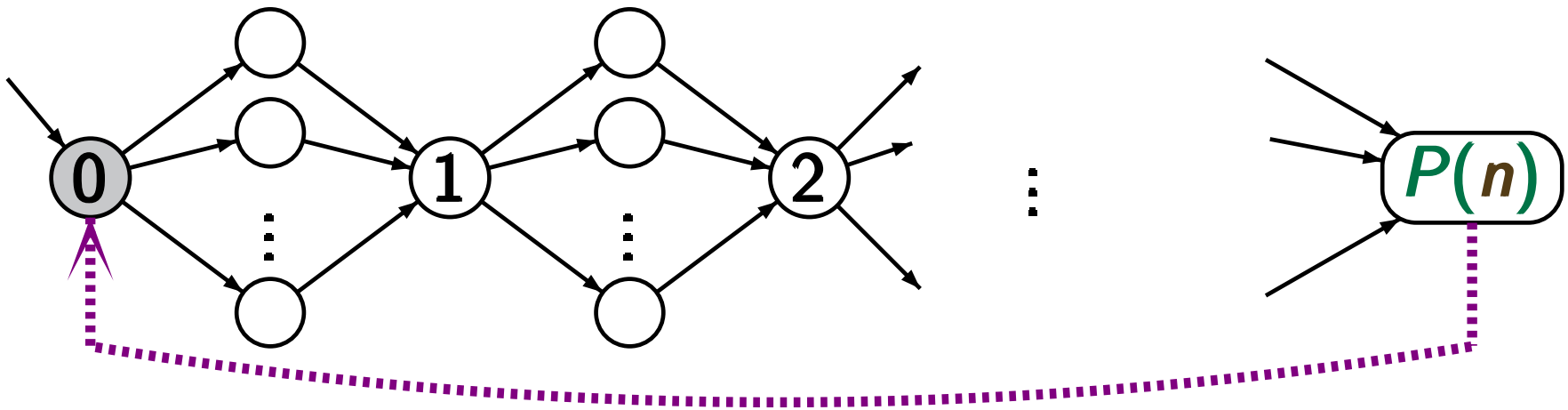
$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2\ \ldots\ (i-1)\langle q, D, i\rangle\ i\ \ldots\ P(n)$

suppose $\delta(q, D) = (p, B, +1)$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



TS $\mathcal{T}$:

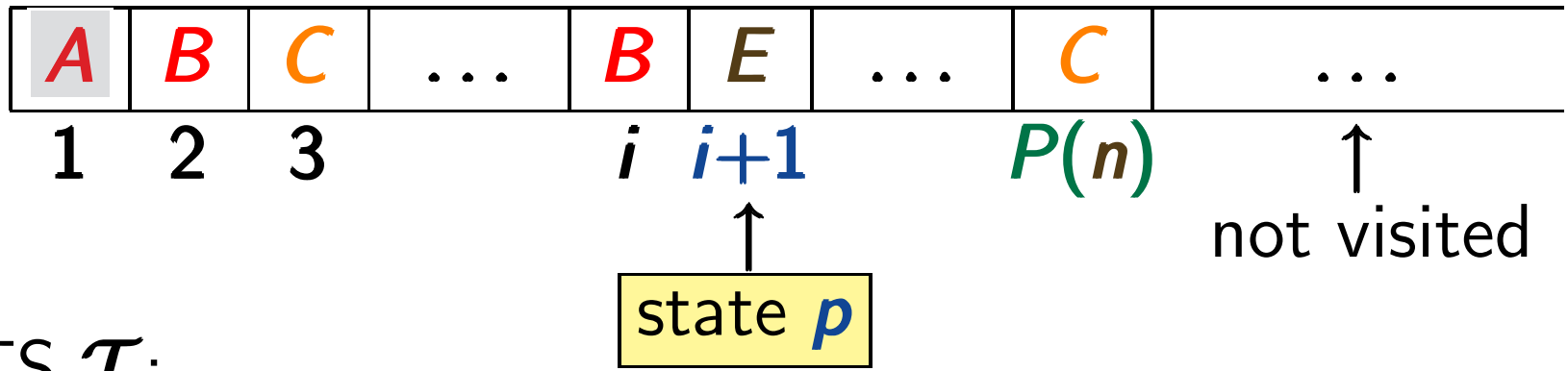


$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2\ \ldots\ \langle q, D, i\rangle\ i\ \langle *, E, i{+}1\rangle\ \ldots\ P(n)$

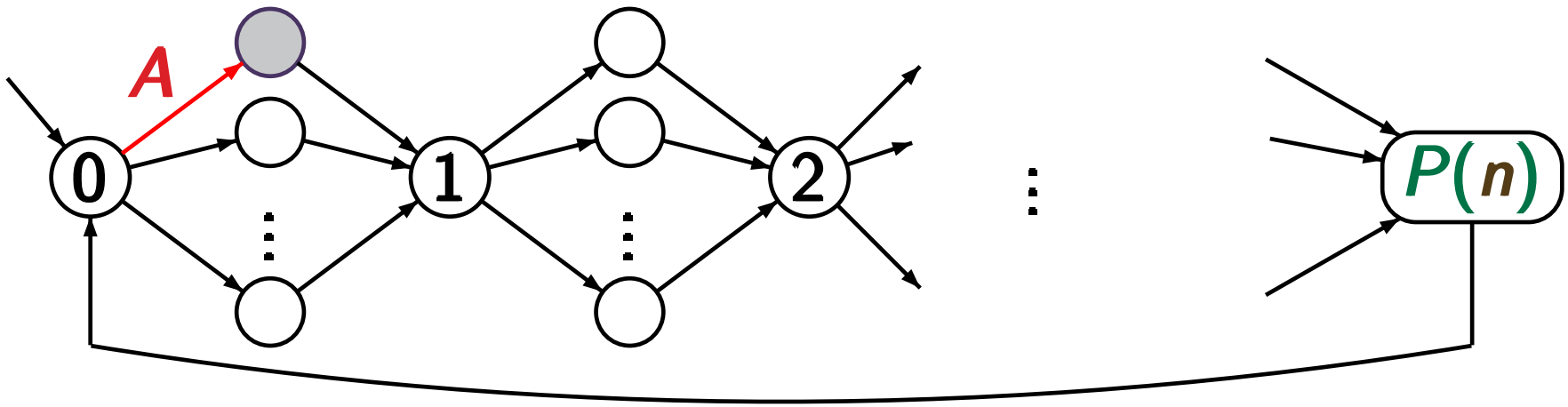
0

suppose $\delta(q, D) = (p, B, +1)$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



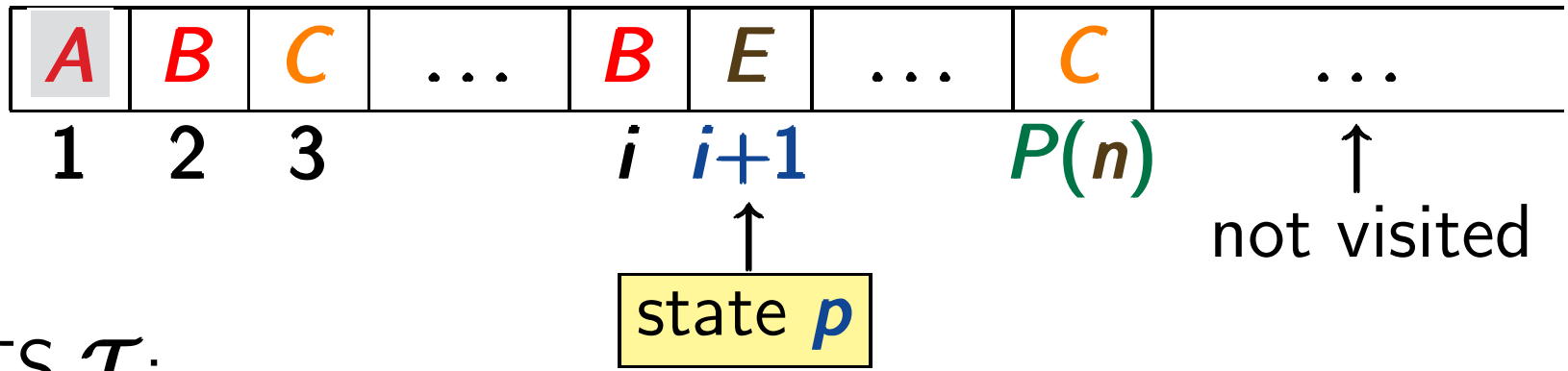
TS $\mathcal{T}$:



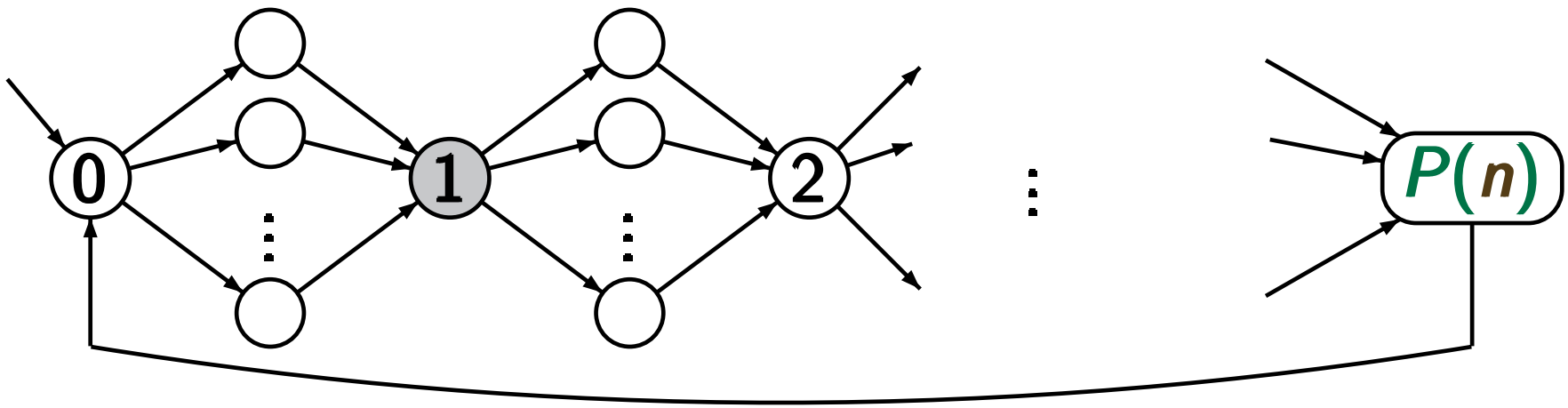
$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2\ \ldots\ \langle q, D, i\rangle\ i\ \langle *, E, i{+}1\rangle\ \ldots\ P(n)$

$0\ \langle *, A, 1\rangle$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



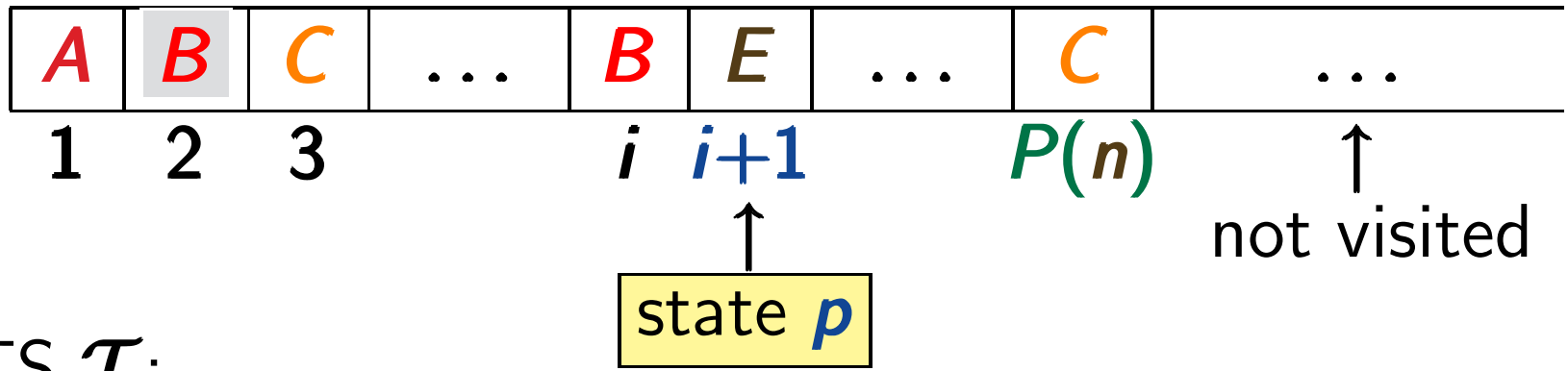
TS $\mathcal{T}$:



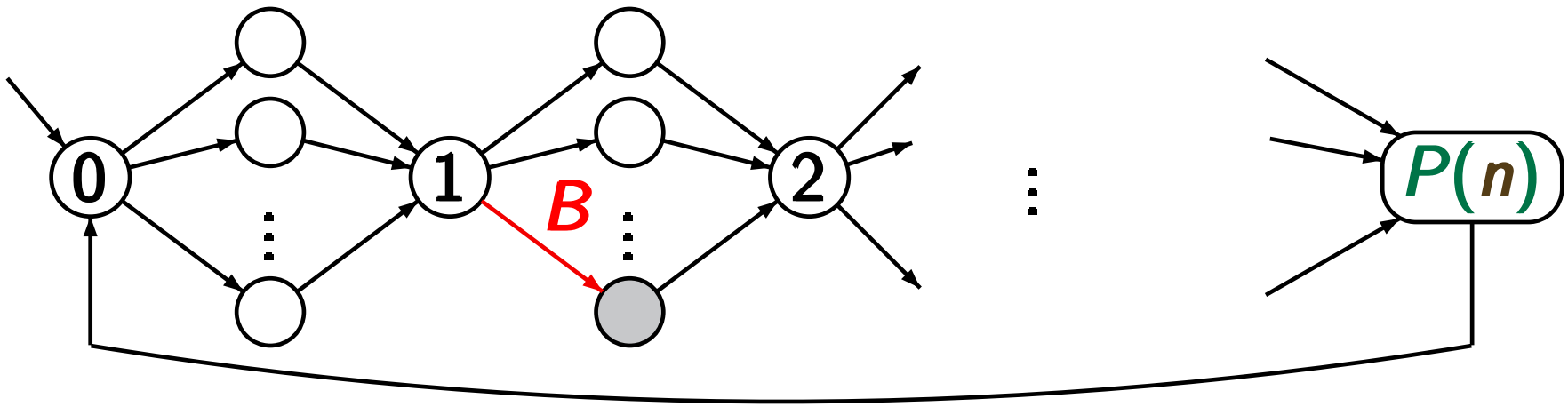
$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2\ \ldots\ \langle q, D, i\rangle\ i\ \langle *, E, i{+}1\rangle\ \ldots\ P(n)$

$0\ \langle *, A, 1\rangle\ 1$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



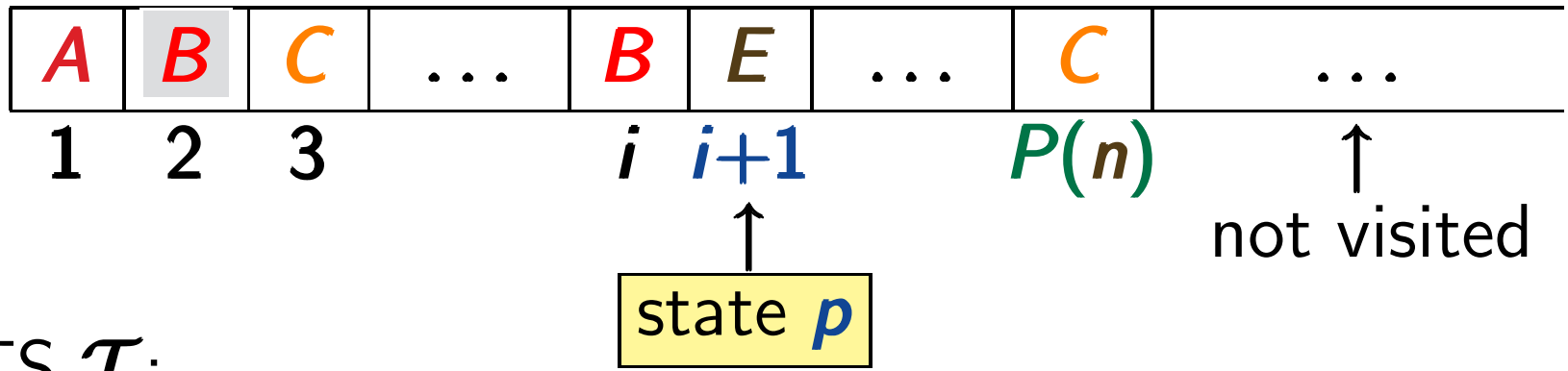
TS $\mathcal{T}$:



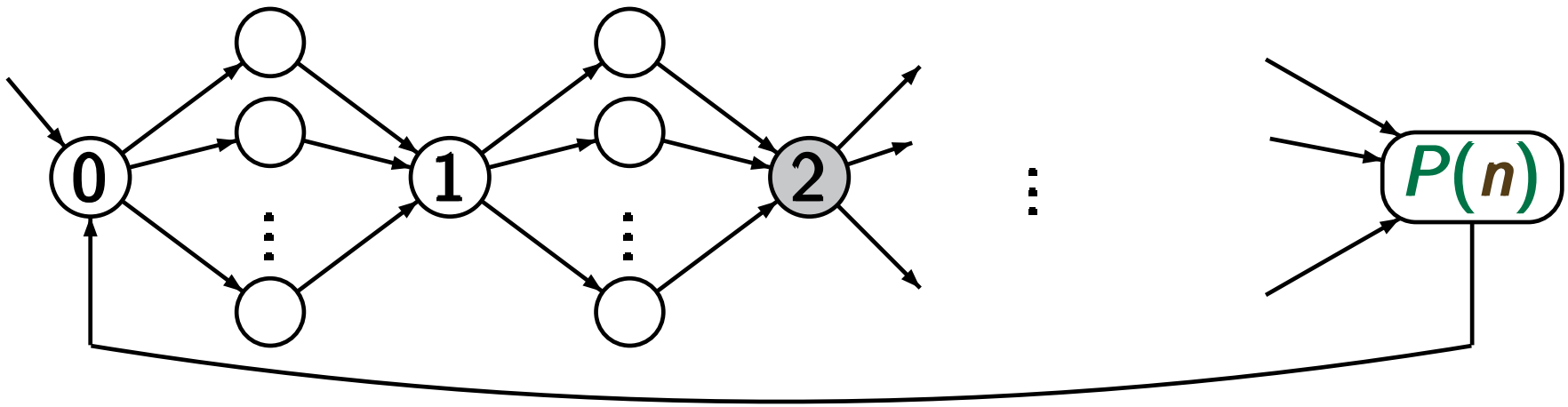
$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2\ \ldots\ \langle q, D, i\rangle\ i\ \langle *, E, i{+}1\rangle\ \ldots\ P(n)$

$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



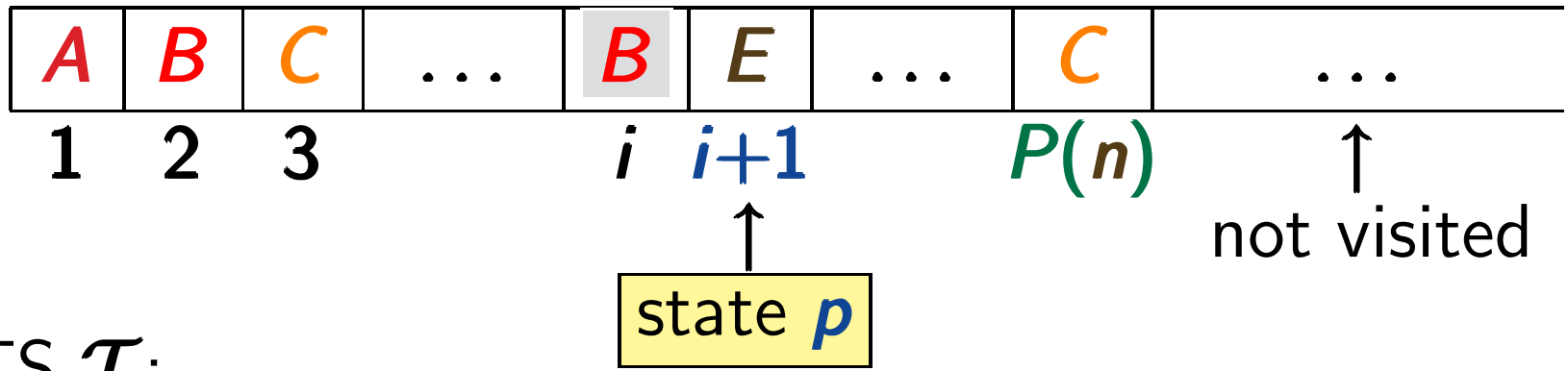
TS $\mathcal{T}$:



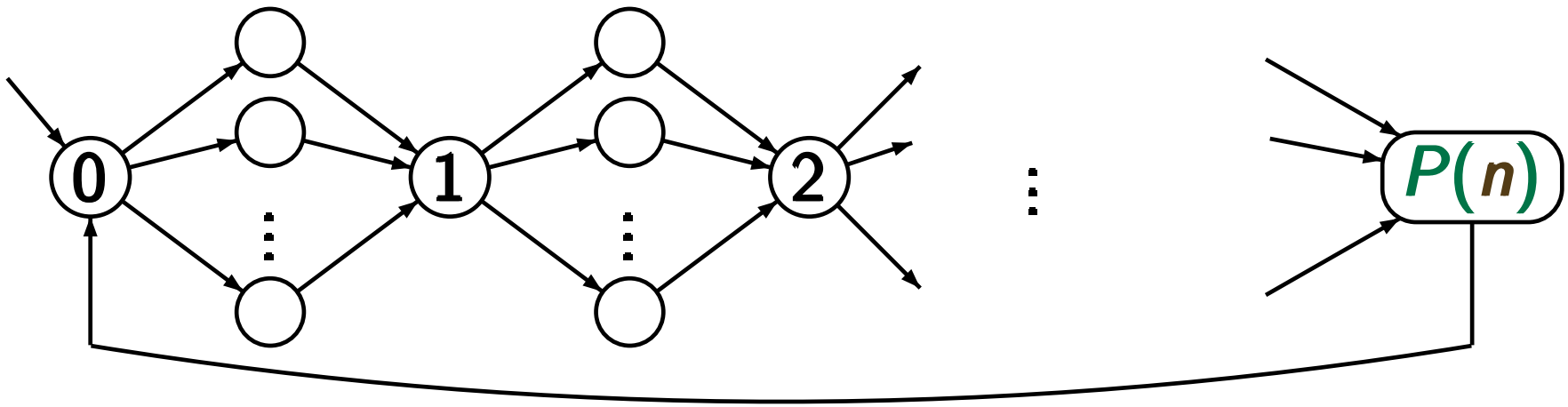
$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2\ \ldots\ \langle q, D, i\rangle\ i\ \langle *, E, i{+}1\rangle\ \ldots\ P(n)$

$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



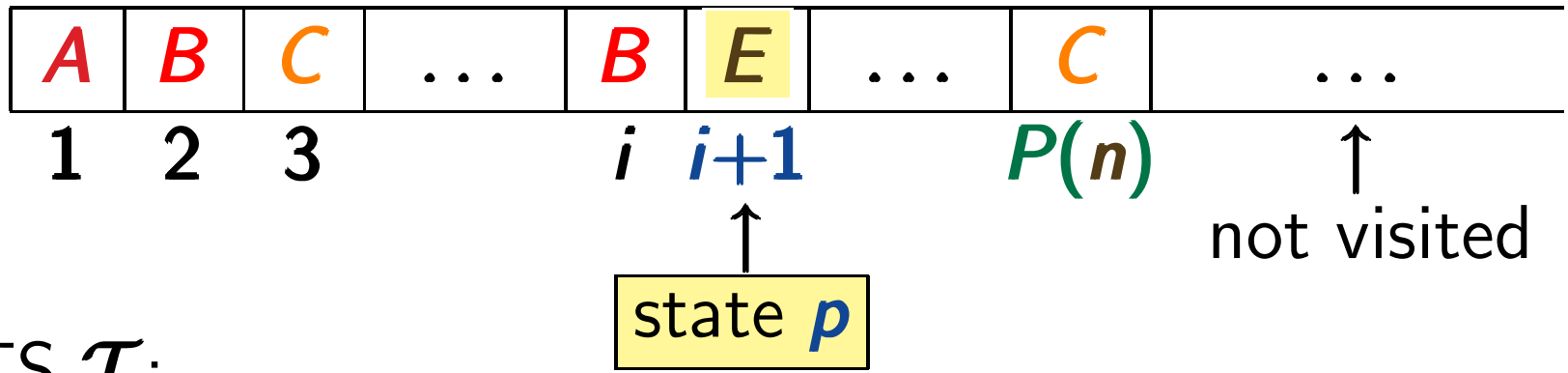
TS $\mathcal{T}$:



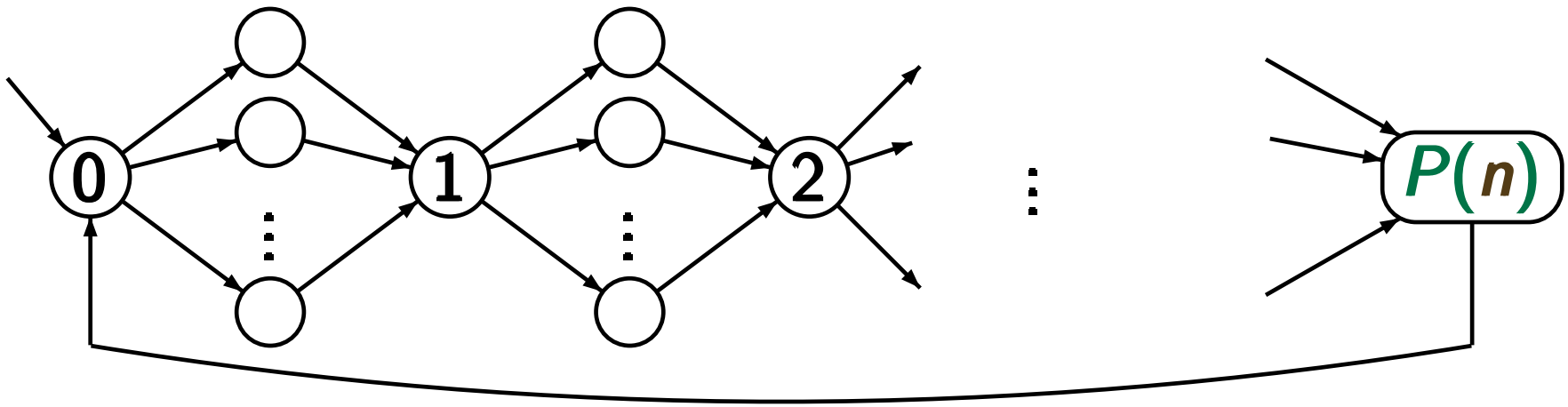
$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2\ \ldots\ \langle q, D, i\rangle\ i\ \langle *, E, i{+}1\rangle\ \ldots\ P(n)$

$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2\ \ldots\ \langle *, B, i\rangle\ i$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



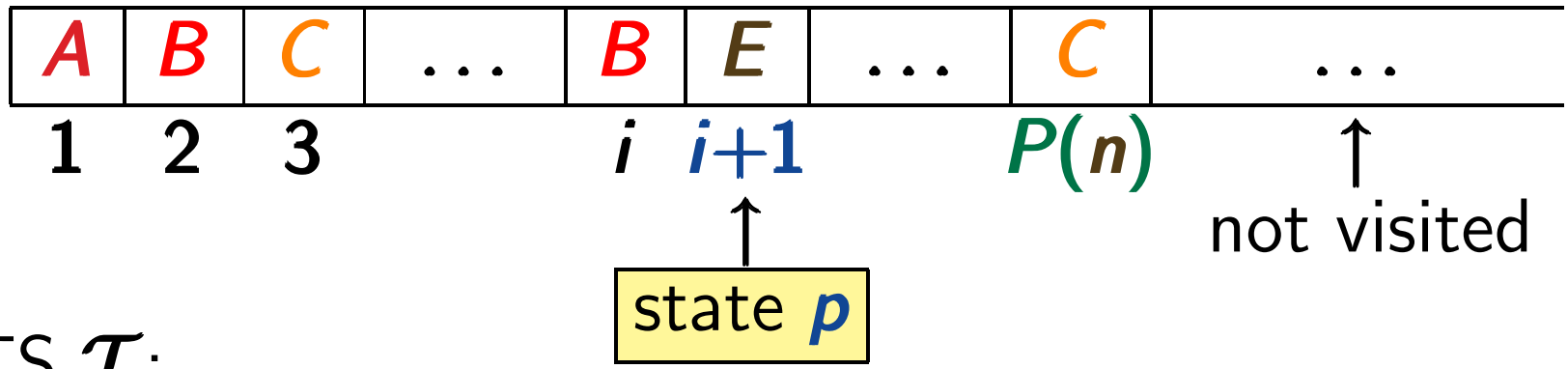
TS $\mathcal{T}$:



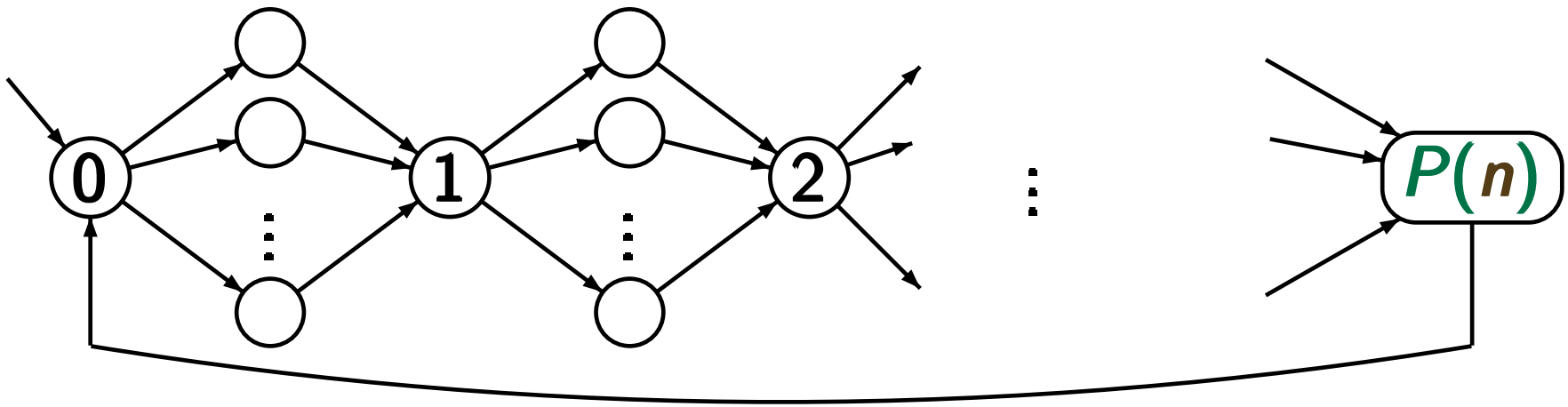
$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2\ \ldots\ \langle q, D, i\rangle\ i\ \langle *, E, i{+}1\rangle\ \ldots\ P(n)$

$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2\ \ldots\ \langle *, B, i\rangle\ i\ \langle p, E, i{+}1\rangle$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



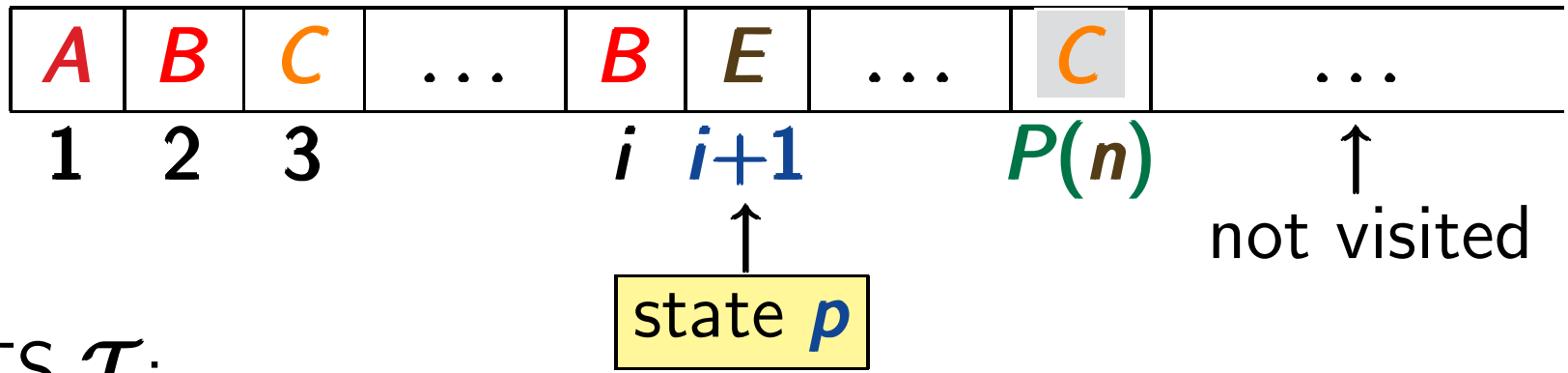
TS $\mathcal{T}$:



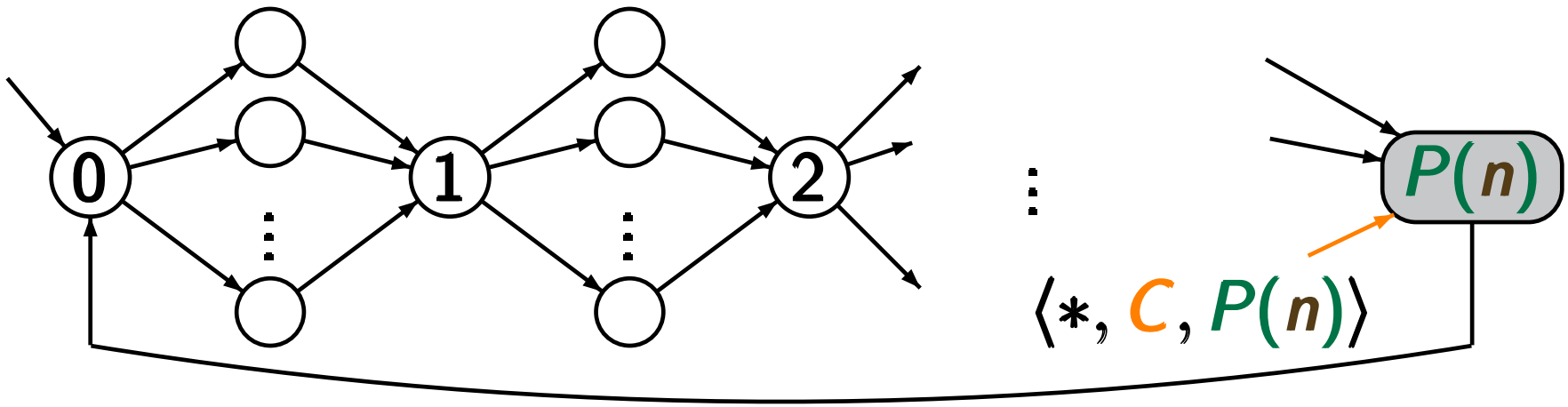
$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2\ \ldots\ \langle q, D, i\rangle\ i\ \langle *, E, i{+}1\rangle\ \ldots\ P(n)$

$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2\ \ldots\ \langle *, B, i\rangle\ i\ \langle p, E, i{+}1\rangle\ \ldots$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$



TS $\mathcal{T}$:



$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2\ \ldots\ \langle q, D, i\rangle\ i\ \langle *, E, i{+}1\rangle\ \ldots\ P(n)$

$0\ \langle *, A, 1\rangle\ 1\ \langle *, B, 2\rangle\ 2\ \ldots\ \langle *, B, i\rangle\ i\ \langle p, E, i{+}1\rangle\ \ldots\ P(n)$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$

Let $\mathcal{M}$ be a DTM with polynomial space bound $P(n)$

- state space Q
- initial state q_0
- set of accept states F
- tape alphabet Γ
- input alphabet $\Sigma \subseteq \Gamma$
- blank symbol $\sqcup$

transition function $\delta : Q \times \Gamma \to Q \times \Gamma \times \{-1, 0, +1\}$

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input word w for $\mathcal{M}$	poly time $\longrightarrow$	TS $\mathcal{T}$ LTL-formula φ
$\mathcal{M}$ accepts w, i.e., $w \in K$	iff	there is path π of $\mathcal{T}$ with $\pi \models \varphi$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$

Let $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, \sqcup, F)$ be a DTM with polynomial space bound $P(n)$, and $w \in \Sigma^*$, $|w|=n$.

Transition system $\mathcal{T} \stackrel{\text{def}}{=} (S, Act, \rightarrow, S_0, AP, L)$ where

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$

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Transition system $\mathcal{T} \stackrel{\text{def}}{=} (S, Act, \rightarrow, S_0, AP, L)$ where

$$S = \{0, 1, \ldots, P(n)\} \cup \{\langle q, A, i\rangle, \langle *, A, i\rangle : q \in Q, A \in \Gamma, 1 \leq i \leq P(n)\}$$

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$

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$$S_0 = \{0\}$$

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$$S_0 = \{0\}$$

$AP = S$ with obvious labeling function

Polynomial reduction $w \mapsto (\mathcal{T}, \varphi)$

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Transition system $\mathcal{T} \stackrel{\text{def}}{=} (S, Act, \rightarrow, S_0, AP, L)$ where

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We saw that:

The existential **LTL** model checking problem

given: finite TS $\mathcal{T}$, LTL formula φ

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As ***PSPACE*** = ***coPSPACE*** we get:

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additionally we proved *coNP*-hardness

using an LTL-encoding of the Hamilton-path problem

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There is **no** LTL formula φ over $AP = \{a\}$ s.t.

$$Words(\varphi) = \text{set of words } A_0 A_1 A_2 \ldots \in (2^{AP})^\omega \text{ s.t.}$$
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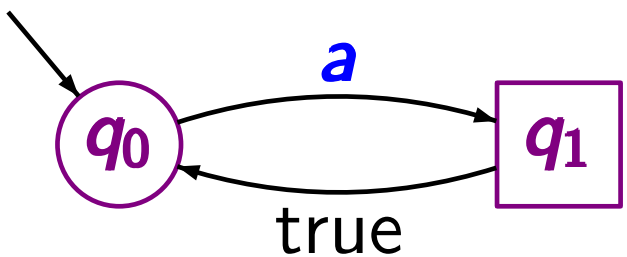

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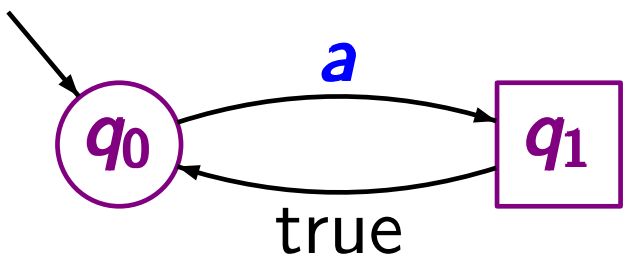

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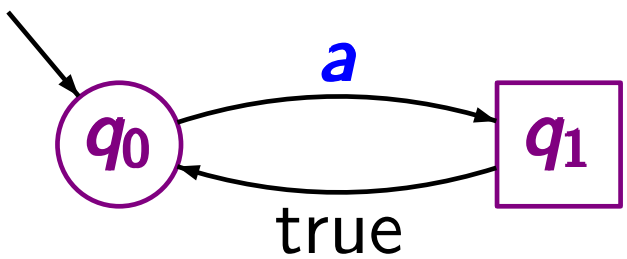

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$\sigma = \{a\}\{a\}\{a\}\varnothing\{a\}^\omega \not\models \varphi$, but $\sigma \in \mathcal{L}_\omega(\mathcal{A})$

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examples:

$\Diamond\Box a \wedge \Box\Diamond\neg a$	unsatisfiable
$a \,\mathsf{U}\, b \wedge \Box\neg b$	unsatisfiable
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