

Introduction

Modelling parallel systems

Linear Time Properties

state-based and linear time view

definition of linear time properties

invariants and safety

liveness and fairness



Regular Properties

Linear Temporal Logic

Computation-Tree Logic

Equivalences and Abstraction

Let E be an LT property over AP .

E is called an **invariant** if there exists a propositional formula Φ over AP such that

$$E = \{ A_0 A_1 A_2 \dots \in (2^{AP})^\omega : \forall i \geq 0. A_i \models \Phi \}$$

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ϕ is called the **invariant condition** of E .

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- deadlock freedom: e.g., for dining philosophers
 $\text{never } \bigwedge_{0 \leq i < n} \mathit{wait}_i$

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every red phase is preceded by a yellow phase
- beverage machine:
no drink must be released if the user did not enter a coin before
the total number of entered coins is never less than the total number of released drinks

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bad prefix, e.g., {pay} {drink} {drink}

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$$\sigma = A_0 A_1 A_2 \dots \in (2^{AP})^\omega \setminus E$$

there exists a finite prefix $A_0 A_1 \dots A_n$ of σ such that *none* of the words $A_0 A_1 \dots A_n B_{n+1} B_{n+2} B_{n+3} \dots$ belongs to E

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IS2.5-11

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$E =$ set of all infinite words that
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briefly: $BadPref$

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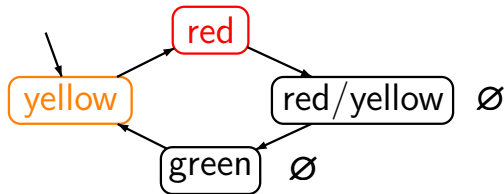
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minimal bad prefixes: any word $A_0 \dots A_i \dots A_n \in \mathbf{BadPref}$
s.t. no proper prefix $A_0 \dots A_i$ is a bad prefix for E

Safety property for a traffic light

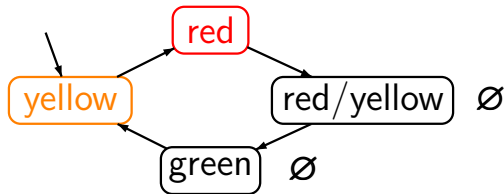
IS2.5-12



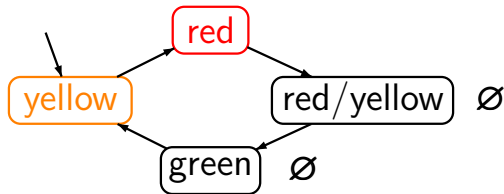
$$AP = \{ \text{red}, \text{yellow} \}$$

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IS2.5-12



“every red phase is preceded by a yellow phase”



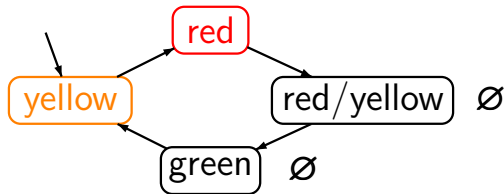
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E = set of all infinite words $A_0 A_1 A_2 \dots$
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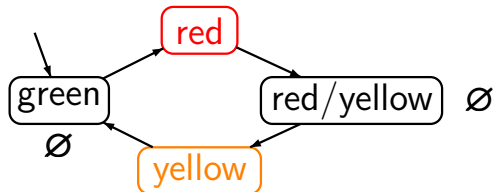
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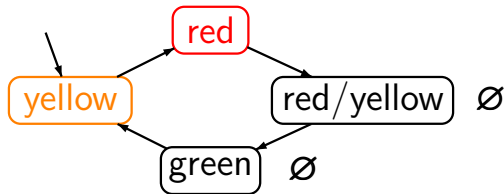
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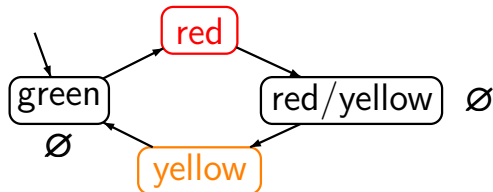
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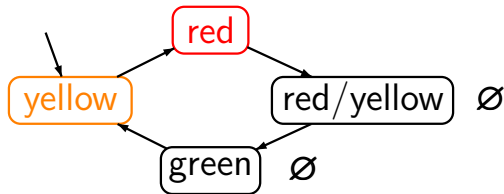
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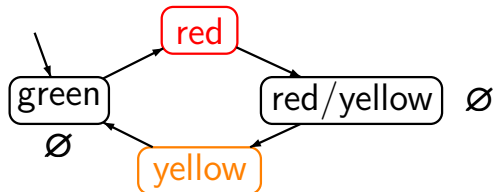
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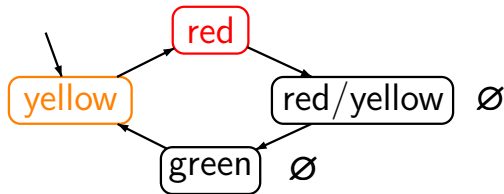


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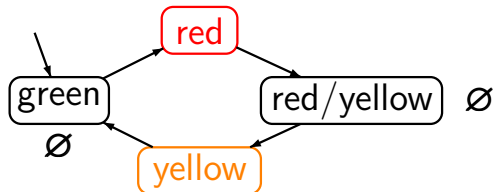
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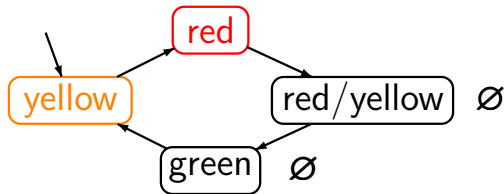


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bad prefix, e.g.,
 $\emptyset \{red\} \emptyset \{yellow\}$

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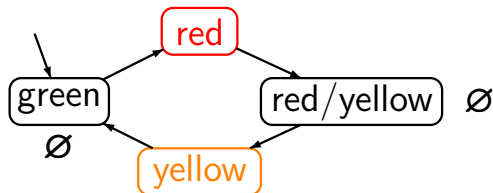
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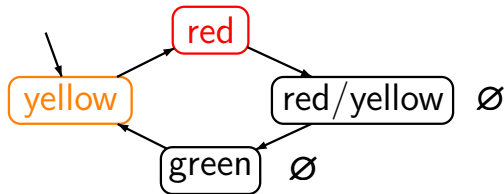
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minimal bad prefix:

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Safety property for a traffic light

IS2.5-12A



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is a safety property over $AP = \{red, yellow\}$ with

$BadPref$ = set of all finite words $A_0 A_1 \dots A_n$
over 2^{AP} s.t. for some $i \in \{0, \dots, n\}$:
 $red \in A_i \wedge (i=0 \vee yellow \notin A_{i-1})$

Let $E \subseteq (2^{AP})^\omega$ be a safety property, \mathcal{T} a TS over AP .

$$\mathcal{T} \models E \text{ iff } \text{Traces}(\mathcal{T}) \subseteq E$$

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Correct or wrong?

IS2.5-36

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“For all words $\in \underbrace{(2^{AP})^\omega \setminus (2^{AP})^\omega}_{= \emptyset} \dots$ ”

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Given an LT property E , the prefix closure of E is:

$$\mathit{cl}(E) \stackrel{\text{def}}{=} \{\sigma \in (2^{AP})^\omega : \mathit{pref}(\sigma) \subseteq \mathit{pref}(E)\}$$

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Theorem:

E is a safety property iff $\mathit{cl}(E) = E$

remind: LT properties and trace inclusion:

If \mathcal{T}_1 and \mathcal{T}_2 are TS over AP then:

$$\text{Traces}(\mathcal{T}_1) \subseteq \text{Traces}(\mathcal{T}_2)$$

iff for all LT properties E : $\mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$

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safety properties and finite trace inclusion:

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Proof " \implies ": obvious, as for safety property E :

$$\mathcal{T} \models E \quad \text{iff} \quad \text{Traces}_{fin}(\mathcal{T}) \cap \text{BadPref} = \emptyset$$

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$$\begin{aligned} & \text{Traces}_{fin}(\mathcal{T}_1) \cap \text{BadPref} \\ & \subseteq \text{Traces}_{fin}(\mathcal{T}_2) \cap \text{BadPref} = \emptyset \end{aligned}$$

$$\text{Traces}_{fin}(\mathcal{T}_1) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)$$

iff for all safety properties E : $\mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$

Proof " \implies ": obvious, as for safety property E :

$$\mathcal{T} \models E \quad \text{iff} \quad \text{Traces}_{fin}(\mathcal{T}) \cap \text{BadPref} = \emptyset$$

Hence:

If $\mathcal{T}_2 \models E$ and $\text{Traces}_{fin}(\mathcal{T}_1) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)$ then:

$$\begin{aligned} & \text{Traces}_{fin}(\mathcal{T}_1) \cap \text{BadPref} \\ & \subseteq \text{Traces}_{fin}(\mathcal{T}_2) \cap \text{BadPref} = \emptyset \end{aligned}$$

and therefore $\mathcal{T}_1 \models E$

$$\text{Traces}_{fin}(\mathcal{T}_1) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)$$

iff for all safety properties E : $\mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$

Proof “ \Leftarrow ”: consider the LT property

$$E = \text{cl}(\text{Traces}(\mathcal{T}_2))$$

$$\text{Traces}_{fin}(\mathcal{T}_1) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)$$

iff for all safety properties E : $\mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$

Proof “ \Leftarrow ”: consider the LT property

$$E = \text{cl}(\text{Traces}(\mathcal{T}_2)) = \{\sigma : \text{pref}(\sigma) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)\}$$

$$\text{Traces}_{fin}(\mathcal{T}_1) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)$$

iff for all safety properties E : $\mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$

Proof “ \Leftarrow ”: consider the LT property

$$E = \text{cl}(\text{Traces}(\mathcal{T}_2)) = \{\sigma : \text{pref}(\sigma) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)\}$$

for each transition system \mathcal{T} :

$$\text{pref}(\text{Traces}(\mathcal{T})) = \text{Traces}_{fin}(\mathcal{T})$$

$$\text{Traces}_{fin}(\mathcal{T}_1) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)$$

iff for all safety properties E : $\mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$

Proof “ \Leftarrow ”: consider the LT property

$$E = \text{cl}(\text{Traces}(\mathcal{T}_2)) = \{\sigma : \text{pref}(\sigma) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)\}$$

Then, E is a safety property

$$\text{Traces}_{fin}(\mathcal{T}_1) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)$$

iff for all safety properties E : $\mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$

Proof “ \Leftarrow ”: consider the LT property

$$E = \text{cl}(\text{Traces}(\mathcal{T}_2)) = \{\sigma : \text{pref}(\sigma) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)\}$$

Then, E is a safety property


$$\text{as } \text{cl}(E) = E$$

$$\text{Traces}_{fin}(\mathcal{T}_1) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)$$

iff for all safety properties E : $\mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$

Proof " \Leftarrow ": consider the LT property

$$E = \text{cl}(\text{Traces}(\mathcal{T}_2)) = \{\sigma : \text{pref}(\sigma) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)\}$$

Then, E is a safety property

↑

$$\text{as } \text{cl}(E) = E$$

$$\text{set of bad prefixes: } (2^{AP})^+ \setminus \text{Traces}_{fin}(\mathcal{T}_2)$$

$$\text{Traces}_{fin}(\mathcal{T}_1) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)$$

iff for all safety properties E : $\mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$

Proof “ \Leftarrow ”: consider the LT property

$$E = \text{cl}(\text{Traces}(\mathcal{T}_2)) = \{\sigma : \text{pref}(\sigma) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)\}$$

Then, E is a safety property and $\mathcal{T}_2 \models E$.

$$\text{Traces}_{fin}(\mathcal{T}_1) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)$$

iff for all safety properties E : $\mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$

Proof “ \Leftarrow ”: consider the LT property

$$E = \text{cl}(\text{Traces}(\mathcal{T}_2)) = \{\sigma : \text{pref}(\sigma) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)\}$$

Then, E is a safety property and $\mathcal{T}_2 \models E$.

By assumption: $\mathcal{T}_1 \models E$

$$\text{Traces}_{fin}(\mathcal{T}_1) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)$$

iff for all safety properties E : $\mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$

Proof “ \Leftarrow ”: consider the LT property

$$E = \text{cl}(\text{Traces}(\mathcal{T}_2)) = \{\sigma : \text{pref}(\sigma) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)\}$$

Then, E is a safety property and $\mathcal{T}_2 \models E$.

By assumption: $\mathcal{T}_1 \models E$ and therefore $\text{Traces}(\mathcal{T}_1) \subseteq E$.

$$\text{Traces}_{fin}(\mathcal{T}_1) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)$$

iff for all safety properties E : $\mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$

Proof “ \Leftarrow ”: consider the LT property

$$E = \text{cl}(\text{Traces}(\mathcal{T}_2)) = \{\sigma : \text{pref}(\sigma) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)\}$$

Then, E is a safety property and $\mathcal{T}_2 \models E$.

By assumption: $\mathcal{T}_1 \models E$ and therefore $\text{Traces}(\mathcal{T}_1) \subseteq E$.

Hence: $\text{Traces}_{fin}(\mathcal{T}_1) = \text{pref}(\text{Traces}(\mathcal{T}_1))$

$$\text{Traces}_{fin}(\mathcal{T}_1) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)$$

iff for all safety properties E : $\mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$

Proof “ \Leftarrow ”: consider the LT property

$$E = \text{cl}(\text{Traces}(\mathcal{T}_2)) = \{\sigma : \text{pref}(\sigma) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)\}$$

Then, E is a safety property and $\mathcal{T}_2 \models E$.

By assumption: $\mathcal{T}_1 \models E$ and therefore $\text{Traces}(\mathcal{T}_1) \subseteq E$.

$$\begin{aligned} \text{Hence: } \text{Traces}_{fin}(\mathcal{T}_1) &= \text{pref}(\text{Traces}(\mathcal{T}_1)) \\ &\subseteq \text{pref}(E) \end{aligned}$$

$$\text{Traces}_{fin}(\mathcal{T}_1) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)$$

iff for all safety properties E : $\mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$

Proof “ \Leftarrow ”: consider the LT property

$$E = \text{cl}(\text{Traces}(\mathcal{T}_2)) = \{\sigma : \text{pref}(\sigma) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)\}$$

Then, E is a safety property and $\mathcal{T}_2 \models E$.

By assumption: $\mathcal{T}_1 \models E$ and therefore $\text{Traces}(\mathcal{T}_1) \subseteq E$.

$$\begin{aligned} \text{Hence: } \text{Traces}_{fin}(\mathcal{T}_1) &= \text{pref}(\text{Traces}(\mathcal{T}_1)) \\ &\subseteq \text{pref}(E) = \text{pref}(\text{cl}(\text{Traces}(\mathcal{T}_2))) \end{aligned}$$

$$\text{Traces}_{fin}(\mathcal{T}_1) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)$$

iff for all safety properties E : $\mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$

Proof “ \Leftarrow ”: consider the LT property

$$E = \text{cl}(\text{Traces}(\mathcal{T}_2)) = \{\sigma : \text{pref}(\sigma) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)\}$$

Then, E is a safety property and $\mathcal{T}_2 \models E$.

By assumption: $\mathcal{T}_1 \models E$ and therefore $\text{Traces}(\mathcal{T}_1) \subseteq E$.

$$\begin{aligned} \text{Hence: } \text{Traces}_{fin}(\mathcal{T}_1) &= \text{pref}(\text{Traces}(\mathcal{T}_1)) \\ &\subseteq \text{pref}(E) = \text{pref}(\text{cl}(\text{Traces}(\mathcal{T}_2))) \\ &= \text{Traces}_{fin}(\mathcal{T}_2) \end{aligned}$$

safety properties and finite trace inclusion:

If \mathcal{T}_1 and \mathcal{T}_2 are TS over AP then:

$$\text{Traces}_{fin}(\mathcal{T}_1) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)$$

iff for all safety properties E : $\mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$

safety properties and finite trace inclusion:

If \mathcal{T}_1 and \mathcal{T}_2 are TS over AP then:

$$\text{Traces}_{fin}(\mathcal{T}_1) \subseteq \text{Traces}_{fin}(\mathcal{T}_2)$$

iff for all safety properties E : $\mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$

safety properties and finite trace equivalence:

If \mathcal{T}_1 and \mathcal{T}_2 are TS over AP then:

$$\text{Traces}_{fin}(\mathcal{T}_1) = \text{Traces}_{fin}(\mathcal{T}_2)$$

iff \mathcal{T}_1 and \mathcal{T}_2 satisfy the same safety properties

trace inclusion

$Traces(\mathcal{T}) \subseteq Traces(\mathcal{T}')$ iff

for all LT properties E : $\mathcal{T}' \models E \implies \mathcal{T} \models E$

finite trace inclusion

$Traces_{fin}(\mathcal{T}) \subseteq Traces_{fin}(\mathcal{T}')$ iff

for all safety properties E : $\mathcal{T}' \models E \implies \mathcal{T} \models E$

trace equivalence

$Traces(\mathcal{T}) = Traces(\mathcal{T}')$ iff

\mathcal{T} and \mathcal{T}' satisfy the same LT properties

finite trace equivalence

$Traces_{fin}(\mathcal{T}) = Traces_{fin}(\mathcal{T}')$ iff

\mathcal{T} and \mathcal{T}' satisfy the same safety properties

If $Traces(\mathcal{T}) \subseteq Traces(\mathcal{T}')$
then $Traces_{fin}(\mathcal{T}) \subseteq Traces_{fin}(\mathcal{T}')$.

If $Traces(\mathcal{T}) \subseteq Traces(\mathcal{T}')$
then $Traces_{fin}(\mathcal{T}) \subseteq Traces_{fin}(\mathcal{T}')$.

correct, since

$$\begin{aligned} Traces_{fin}(\mathcal{T}) &= \text{set of all finite nonempty prefixes} \\ &\quad \text{of words in } Traces(\mathcal{T}) \\ &= \mathit{pref}(Traces(\mathcal{T})) \end{aligned}$$

If $Traces(\mathcal{T}) \subseteq Traces(\mathcal{T}')$
 then $Traces_{fin}(\mathcal{T}) \subseteq Traces_{fin}(\mathcal{T}')$.

correct, since

$$\begin{aligned} Traces_{fin}(\mathcal{T}) &= \text{set of all finite nonempty prefixes} \\ &\quad \text{of words in } Traces(\mathcal{T}) \\ &= \mathit{pref}(Traces(\mathcal{T})) \end{aligned}$$



$$Traces(\mathcal{T}) = \{ \{a\}^\omega \}$$

$$Traces_{fin}(\mathcal{T}) = \{ \{a\}^n : n \geq 1 \}$$

is **trace equivalence** the same as
finite trace equivalence ?

is **trace equivalence** the same as
finite trace equivalence ?

answer: **no**

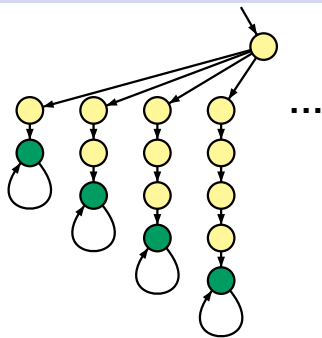
Finite trace relations versus trace relations

IS2.5-32

\mathcal{T}



\mathcal{T}'



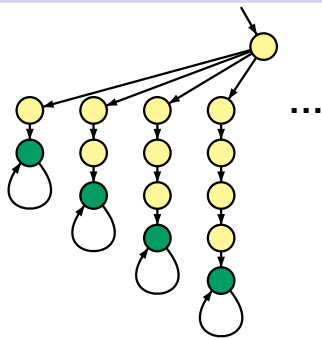
$$\text{yellow circle} \hat{=} \emptyset \quad \text{green circle} \hat{=} \{b\}$$

set of propositions

$$AP = \{b\}$$

\mathcal{T}


$$\text{Traces}(\mathcal{T}) = \{\emptyset^\omega\}$$

 \mathcal{T}'


$$\text{Yellow Circle} \hat{=} \emptyset \quad \text{Green Circle} \hat{=} \{b\}$$

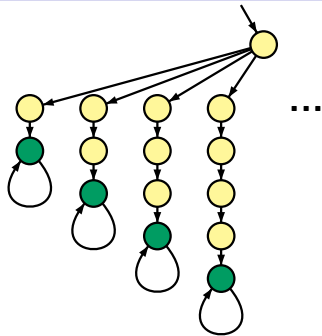
set of propositions

$$AP = \{b\}$$

\mathcal{T}


$$\text{Traces}(\mathcal{T}) = \{\emptyset^\omega\}$$

$$\text{Traces}_{\text{fin}}(\mathcal{T}) = \{\emptyset^n : n \geq 0\}$$

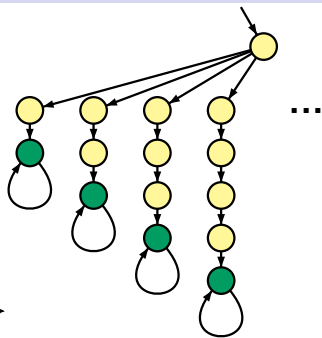
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set of propositions

$$AP = \{b\}$$

\mathcal{T}

 \mathcal{T}'


$$\text{Traces}(\mathcal{T}) = \{\emptyset^\omega\}$$

$$\text{Traces}_{\text{fin}}(\mathcal{T}) = \{\emptyset^n : n \geq 0\}$$

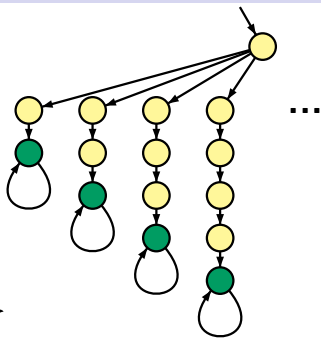
$$\text{Traces}(\mathcal{T}') = \{\emptyset^n \{b\}^\omega : n \geq 2\}$$

$$\text{yellow circle} \hat{=} \emptyset \quad \text{green circle} \hat{=} \{b\}$$

set of propositions

$$AP = \{b\}$$

\mathcal{T}

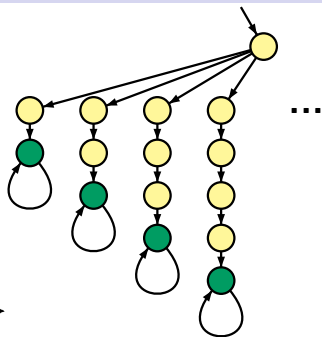
 \mathcal{T}'


$$\text{Traces}(\mathcal{T}) = \{\emptyset^\omega\}$$

$$\text{Traces}_{\text{fin}}(\mathcal{T}) = \{\emptyset^n : n \geq 0\}$$

$$\text{Traces}(\mathcal{T}') = \{\emptyset^n \{b\}^\omega : n \geq 2\}$$

$$\text{Traces}_{\text{fin}}(\mathcal{T}') = \{\emptyset^n : n \geq 0\} \cup \{\emptyset^n \{b\}^m : n \geq 2 \wedge m \geq 1\}$$

\mathcal{T}  \mathcal{T}' 

$$\text{Traces}(\mathcal{T}) = \{\emptyset^\omega\}$$

$$\text{Traces}_{\text{fin}}(\mathcal{T}) = \{\emptyset^n : n \geq 0\}$$

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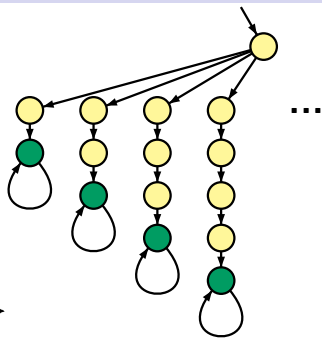
$\text{Traces}(\mathcal{T}) \not\subseteq \text{Traces}(\mathcal{T}')$, but

$\text{Traces}_{\text{fin}}(\mathcal{T}) \subseteq \text{Traces}_{\text{fin}}(\mathcal{T}')$

Finite trace relations versus trace relations

IS2.5-32

 \mathcal{T}

 \mathcal{T}'


$$\text{Traces}(\mathcal{T}) = \{\emptyset^\omega\}$$

$$\text{Traces}_{\text{fin}}(\mathcal{T}) = \{\emptyset^n : n \geq 0\}$$

$$\text{Traces}(\mathcal{T}') = \{\emptyset^n \{b\}^\omega : n \geq 2\}$$

$$\text{Traces}_{\text{fin}}(\mathcal{T}') = \{\emptyset^n : n \geq 0\} \cup \{\emptyset^n \{b\}^m : n \geq 2 \wedge m \geq 1\}$$

$\text{Traces}(\mathcal{T}) \not\subseteq \text{Traces}(\mathcal{T}')$, but
 $\text{Traces}_{\text{fin}}(\mathcal{T}) \subseteq \text{Traces}_{\text{fin}}(\mathcal{T}')$

LT property

$E \hat{=} \text{“eventually } b\text{”}$

$\mathcal{T} \not\models E, \mathcal{T}' \models E$

Suppose that \mathcal{T} and \mathcal{T}' are TS over AP such that

- (1) \mathcal{T} has no terminal states,
- (2) \mathcal{T}' is finite.

Suppose that \mathcal{T} and \mathcal{T}' are TS over AP such that

- (1) \mathcal{T} has **no terminal states**,
i.e., all paths of \mathcal{T} are infinite
- (2) \mathcal{T}' is **finite**.

Suppose that \mathcal{T} and \mathcal{T}' are TS over AP such that

- (1) \mathcal{T} has **no terminal states**,
i.e., all paths of \mathcal{T} are infinite
- (2) \mathcal{T}' is **finite**.

Then:

$$\begin{aligned} & \text{Traces}(\mathcal{T}) \subseteq \text{Traces}(\mathcal{T}') \\ \text{iff } & \text{Traces}_{\text{fin}}(\mathcal{T}) \subseteq \text{Traces}_{\text{fin}}(\mathcal{T}') \end{aligned}$$

Suppose that \mathcal{T} and \mathcal{T}' are TS over AP such that

- (1) \mathcal{T} has **no terminal states**,
i.e., all paths of \mathcal{T} are infinite
- (2) \mathcal{T}' is **finite**.

Then:

$$\begin{aligned} \text{Traces}(\mathcal{T}) &\subseteq \text{Traces}(\mathcal{T}') \\ \text{iff } \text{Traces}_{fin}(\mathcal{T}) &\subseteq \text{Traces}_{fin}(\mathcal{T}') \end{aligned}$$

“ \implies ”: holds for all transition systems,
no matter whether (1) and (2) hold

Suppose that \mathcal{T} and \mathcal{T}' are TS over AP such that

- (1) \mathcal{T} has **no terminal states**,
i.e., all paths of \mathcal{T} are infinite
- (2) \mathcal{T}' is **finite**.

Then:

$$\begin{aligned} \text{Traces}(\mathcal{T}) &\subseteq \text{Traces}(\mathcal{T}') \\ \text{iff } \text{Traces}_{\text{fin}}(\mathcal{T}) &\subseteq \text{Traces}_{\text{fin}}(\mathcal{T}') \end{aligned}$$

“ \implies ”: holds for all transition systems

“ \impliedby ”: suppose that (1) and (2) hold and that

$$(3) \text{Traces}_{\text{fin}}(\mathcal{T}) \subseteq \text{Traces}_{\text{fin}}(\mathcal{T}')$$

Show that $\text{Traces}(\mathcal{T}) \subseteq \text{Traces}(\mathcal{T}')$

Suppose that \mathcal{T} and \mathcal{T}' are TS over AP such that

- (1) \mathcal{T} has no terminal states
- (2) \mathcal{T}' is finite
- (3) $Traces_{fin}(\mathcal{T}) \subseteq Traces_{fin}(\mathcal{T}')$

Then $Traces(\mathcal{T}) \subseteq Traces(\mathcal{T}')$

Proof:

Suppose that \mathcal{T} and \mathcal{T}' are TS over AP such that

- (1) \mathcal{T} has no terminal states
- (2) \mathcal{T}' is finite
- (3) $Traces_{fin}(\mathcal{T}) \subseteq Traces_{fin}(\mathcal{T}')$

Then $Traces(\mathcal{T}) \subseteq Traces(\mathcal{T}')$

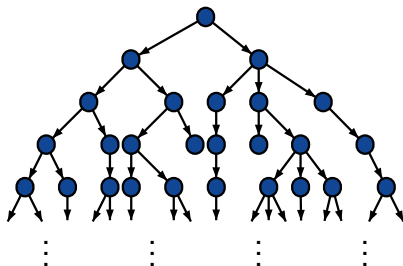
Proof: Pick some path $\pi = s_0 s_1 s_2 \dots$ in \mathcal{T} and show that there exists a path

$$\pi' = t_0 t_1 t_2 \dots \text{ in } \mathcal{T}'$$

such that $trace(\pi) = trace(\pi')$

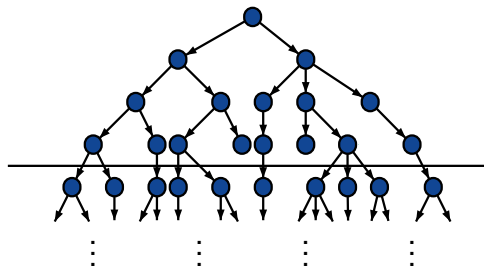
finite TS \mathcal{T}'

paths from state t_0
(unfolded into a tree)



finite TS \mathcal{T}'

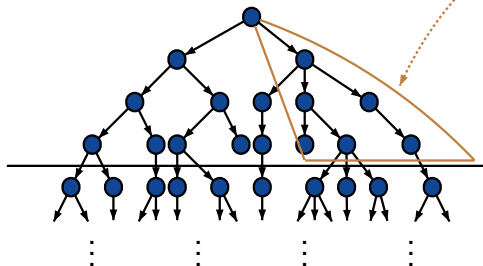
paths from state t_0
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finite until
depth $\leq n$

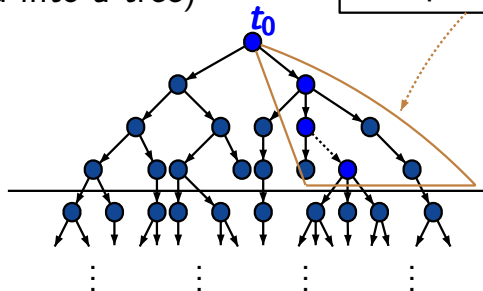
finite TS \mathcal{T}'
paths from state t_0
(unfolded into a tree)

contains all path fragments
with trace $A_0 A_1 \dots A_n$



finite until
depth $\leq n$

finite TS \mathcal{T}'
paths from state t_0
(unfolded into a tree)



contains all path fragments
with trace $A_0 A_1 \dots A_n$
in particular: $t_0 t_1 \dots t_n$

finite until
depth $\leq n$

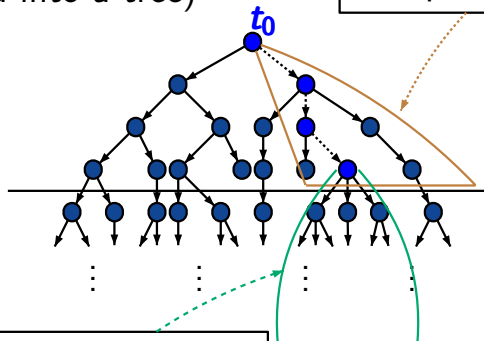
Tracesfin versus traces

IS2.5-33

finite TS \mathcal{T}'

paths from state t_0
(unfolded into a tree)

contains all path fragments
with trace $A_0 A_1 \dots A_n$
in particular: $t_0 t_1 \dots t_n$



finite until
depth $\leq n$

contains infinitely
many path fragments

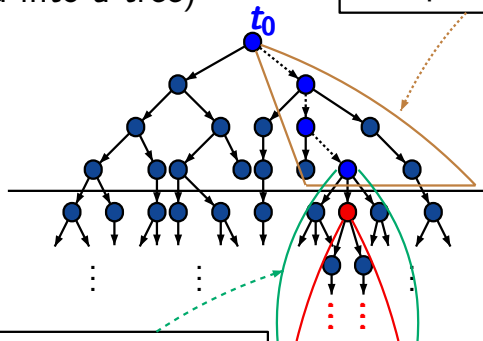
$t_n s_{n+1}^m \dots s_m^m$

Tracesfin versus traces

IS2.5-33

finite TS \mathcal{T}'
paths from state t_0
(unfolded into a tree)

contains all path fragments
with trace $A_0 A_1 \dots A_n$
in particular: $t_0 t_1 \dots t_n$



finite until
depth $\leq n$

contains infinitely
many path fragments
 $t_n s_{n+1}^m \dots s_m^m$

there exists $t_{n+1} \in \text{Post}(t_n)$
s.t. $t_{n+1} = s_{n+1}^m$ for
infinitely many m

Suppose that \mathcal{T} and \mathcal{T}' are TS over AP such that

- (1) \mathcal{T} has no terminal states
- (2) \mathcal{T}' is finite
- (3) $Traces_{fin}(\mathcal{T}) \subseteq Traces_{fin}(\mathcal{T}')$

image-finiteness
is sufficient



Then $Traces(\mathcal{T}) \subseteq Traces(\mathcal{T}')$

Suppose that \mathcal{T} and \mathcal{T}' are TS over AP such that

(1) \mathcal{T} has no terminal states

(2) \mathcal{T}' is finite

(3) $Traces_{fin}(\mathcal{T}) \subseteq Traces_{fin}(\mathcal{T}')$

Then $Traces(\mathcal{T}) \subseteq Traces(\mathcal{T}')$

image-finiteness
is sufficient

image-finiteness of $\mathcal{T}' = (S', Act, \rightarrow, S'_0, AP, L')$:

Suppose that \mathcal{T} and \mathcal{T}' are TS over AP such that

(1) \mathcal{T} has no terminal states

(2) \mathcal{T}' is finite

(3) $Traces_{fin}(\mathcal{T}) \subseteq Traces_{fin}(\mathcal{T}')$

Then $Traces(\mathcal{T}) \subseteq Traces(\mathcal{T}')$

image-finiteness
is sufficient

image-finiteness of $\mathcal{T}' = (S', Act, \rightarrow, S'_0, AP, L')$:

- for each $A \in 2^{AP}$ and state $s \in S'$:

$\{t \in Post(s) : L'(t) = A\}$ is finite

Suppose that \mathcal{T} and \mathcal{T}' are TS over AP such that

- (1) \mathcal{T} has no terminal states
- (2) \mathcal{T}' is finite
- (3) $Traces_{fin}(\mathcal{T}) \subseteq Traces_{fin}(\mathcal{T}')$

image-finiteness
is sufficient

Then $Traces(\mathcal{T}) \subseteq Traces(\mathcal{T}')$

image-finiteness of $\mathcal{T}' = (S', Act, \rightarrow, S'_0, AP, L')$:

- for each $A \in 2^{AP}$ and state $s \in S'$:
 $\{t \in Post(s) : L'(t) = A\}$ is finite
- for each $A \in 2^{AP}$: $\{s_0 \in S'_0 : L'(s_0) = A\}$ is finite

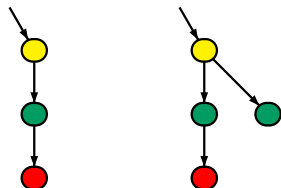
Whenever $Traces(\mathcal{T}) = Traces(\mathcal{T}')$ then
 $Traces_{fin}(\mathcal{T}) = Traces_{fin}(\mathcal{T}')$

Whenever $Traces(\mathcal{T}) = Traces(\mathcal{T}')$ then
 $Traces_{fin}(\mathcal{T}) = Traces_{fin}(\mathcal{T}')$

while the reverse direction does not hold in general
(even not for finite transition systems)

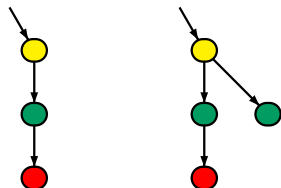
Whenever $Traces(\mathcal{T}) = Traces(\mathcal{T}')$ then
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Whenever $Traces(\mathcal{T}) = Traces(\mathcal{T}')$ then
 $Traces_{fin}(\mathcal{T}) = Traces_{fin}(\mathcal{T}')$

while the reverse direction does not hold in general
(even not for finite transition systems)



finite trace equivalent,
but *not* trace equivalent

Whenever $Traces(\mathcal{T}) = Traces(\mathcal{T}')$ then
 $Traces_{fin}(\mathcal{T}) = Traces_{fin}(\mathcal{T}')$

The reverse implication holds under additional assumptions, e.g.,

- if \mathcal{T} and \mathcal{T}' are finite and have no terminal states
- or, if \mathcal{T} and \mathcal{T}' are *AP*-deterministic