Principles for software composition 2020/21
01 - Unification, goal-oriented derivations, mathematical induction

[Ex. 1] Let $\Sigma_0 = \{0\}$ and $\Sigma_1 = \{\mathsf{s}\}$. Extend the logic program that defines the predicate $\mathsf{sum} \in \Pi_3$ (seen in classroom) to define:

1. a predicate $\mathsf{prod} \in \Pi_3$ for computing the product of two numbers;
2. a predicate $\mathsf{pow} \in \Pi_3$ for computing the power;
3. a predicate $\mathsf{div} \in \Pi_2$ for telling if the first argument divides the second.

[Ex. 2] Given the syntax in Ex. 1, solve the unification problems below

1. $G_1 \stackrel{\text{def}}{=} \{\mathsf{prod}(\mathsf{s}(x), y, \mathsf{s}(z)) \stackrel{?}{=} \mathsf{prod}(y, z, x)\}$
2. $G_2 \stackrel{\text{def}}{=} \{\mathsf{pow}(x, \mathsf{s}(y), x) \stackrel{?}{=} \mathsf{pow}(\mathsf{s}(y), z, z)\}$
3. $G_3 \stackrel{\text{def}}{=} \{\mathsf{div}(x, \mathsf{s}(y)) \stackrel{?}{=} \mathsf{div}(z, x)\ ,\ \mathsf{div}(y, \mathsf{s}(z)) \stackrel{?}{=} \mathsf{div}(u, \mathsf{s}(u))\}$

[Ex. 3] Given the logic programs in Ex. 1, write some possible goal-oriented derivations for the queries:

1. $\mathsf{sum}(x, \mathsf{s}(0), \mathsf{s}(\mathsf{s}(0)))$
2. $\mathsf{prod}(\mathsf{s}(\mathsf{s}(0)), y, \mathsf{s}(\mathsf{s}(0)))$
3. $\mathsf{div}(z, \mathsf{s}(\mathsf{s}(0)))$

[Ex. 4] Prove by mathematical induction that:

$$\forall n > 0.\ n^n \geq n!$$

[Ex. 5] Let

$$a_0 \stackrel{\text{def}}{=} 0 \qquad a_{n+1} \stackrel{\text{def}}{=} 2a_n + n$$

Prove by mathematical induction that:

$$\forall n \in \mathbb{N}.\ a_n = 2^n - n - 1$$

[Ex. 6] Let F_i denote the ith Fibonacci number.

$$F_1 \stackrel{\text{def}}{=} 1 \qquad F_2 \stackrel{\text{def}}{=} 1 \qquad F_{n+2} \stackrel{\text{def}}{=} F_n + F_{n+1}$$

Prove by mathematical induction that:

$$\forall n > 0.\ \sum_{i=1}^{n} F_i = F_{n+2} - 1$$