Models of computation (MOD)
Appello straordinario – Nov 14, 2019

[**Ex. 1**] Suppose we add to Aexp the expression a_0 **div** a_1, whose operational semantics is defined by:

$$\frac{\langle a_0, \sigma\rangle \to n \quad \langle a_1, \sigma\rangle \to m \neq 0}{\langle a_0 \ \mathbf{div}\ a_1, \sigma\rangle \to n/m}$$

where $\cdot/\cdot$ denotes integer division.

1. Prove that termination is no longer guaranteed for arithmetic expressions.
2. Prove that determinacy still holds for arithmetic expressions.
3. How should be changed the denotational semantics of arithmetic expressions to make it consistent with the operational one?

[**Ex. 2**] Consider the $\mathrm{CPO}_\perp$ $\mathcal{D} \stackrel{\text{def}}{=} (\wp(\mathbb{N}), \subseteq)$ and the function $f : \wp(\mathbb{N}) \to \wp(\mathbb{N})$ such that $f(S) \stackrel{\text{def}}{=} \{x \mid \exists a, b \in S.\ a \leq x \leq b\}$, where $\leq$ is the usual total order on $\mathbb{N}$.

1. Is f monotone?
2. Is f continuous?
3. What is the least fixpoint of f? Does f have other fixpoints?

[**Ex. 3**]
Consider the HOFL term

$$t \stackrel{\text{def}}{=} \mathbf{rec}\ f.\ \lambda x.\ \mathbf{if}\ x\ \mathbf{then}\ (x, \mathbf{fst}(f\ x))\ \mathbf{else}\ (\mathbf{snd}(f\ x), x)$$

1. Find the principal type of t.
2. Find the denotational semantics of t.

[**Ex. 4**] Let us consider the CCS processes

$$p \stackrel{\text{def}}{=} \mathbf{rec}\ x.(\alpha.x + \beta.\gamma.\mathbf{nil}) \qquad q \stackrel{\text{def}}{=} \mathbf{rec}\ y.\overline{\alpha}.(\gamma.y + \overline{\beta}.\mathbf{nil})$$

$$r \stackrel{\text{def}}{=} \mathbf{rec}\ z.(\gamma.z + \tau.\gamma.\mathbf{nil})$$

1. Draw the LTSs of the processes r and $s \stackrel{\text{def}}{=} (p|q)\backslash\alpha\backslash\beta$.
2. Show that r and s are not strong bisimilar.
3. Prove that r and s are weak bisimilar.