PSC 2020/21 (375AA, 9CFU)

Principles for Software Composition

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27 - PEPA

PEPA
Performance Evaluation Process Algebra

Building models

Conceptualise your system as a Markov chain

Construct your Markov chain (infinitesimal generator matrix)

Solve your equations to derive quantitative information

Building models

Conceptualise your system as a Markov chain

Construct your Markov chain (infinitesimal generator matrix)

Solve your equations to derive quantitative information

Monolithic approach: not suitable for complex systems

PEPA project

the PEPA project started in Edinburgh in 1991

motivated by the performance analysis
of large computer and communication systems

exploit interplay between Process Algebras and CTMC

Process Algebras (PA):
compositional description of complex systems,
formal reasoning (for correctness)

CTMC:
numerical analysis

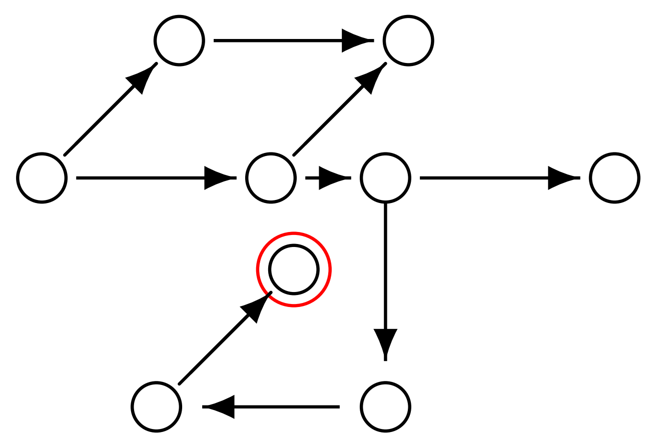
compositional construction of CTMC

PEPA meets CTMC

PA	mutual influence	CTMC
interaction designed around CTMC		ease of construction
actions have durations		design of independent components
add rates to labels		cooperation between components
probabilistic branching		explicit interaction
quantitative measures		reusable sub-models
probabilistic model checking		easy to understand models
quantitative logics		space reduction techniques
		functional verification

Formal models

qualitative vs quantitative



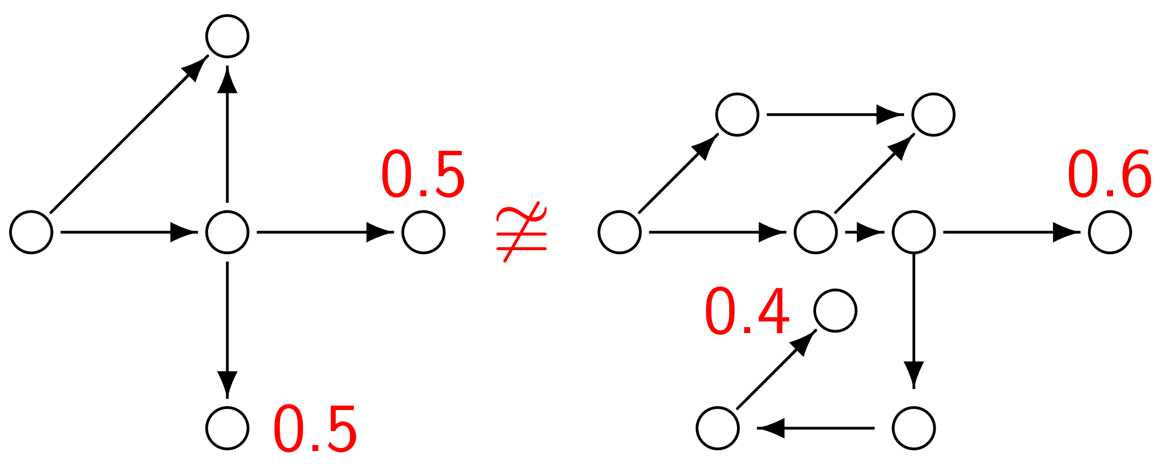
reachability:
will the system arrive to a particular state?

how long will it take the system to arrive to a particular state?

(taken from Jane Hillston's slides)

Formal models

qualitative vs quantitative



conformance:
does system behaviour
match its specification?

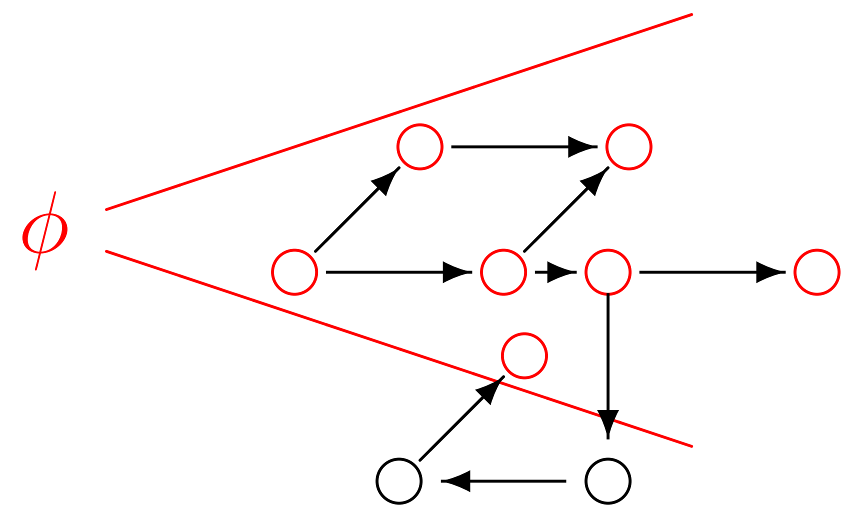
how likely is that
system behaviour will
match its specification?

does the frequency profile
of the system match
that of its specification?

(taken from Jane Hillston's slides)

Formal models

qualitative vs quantitative



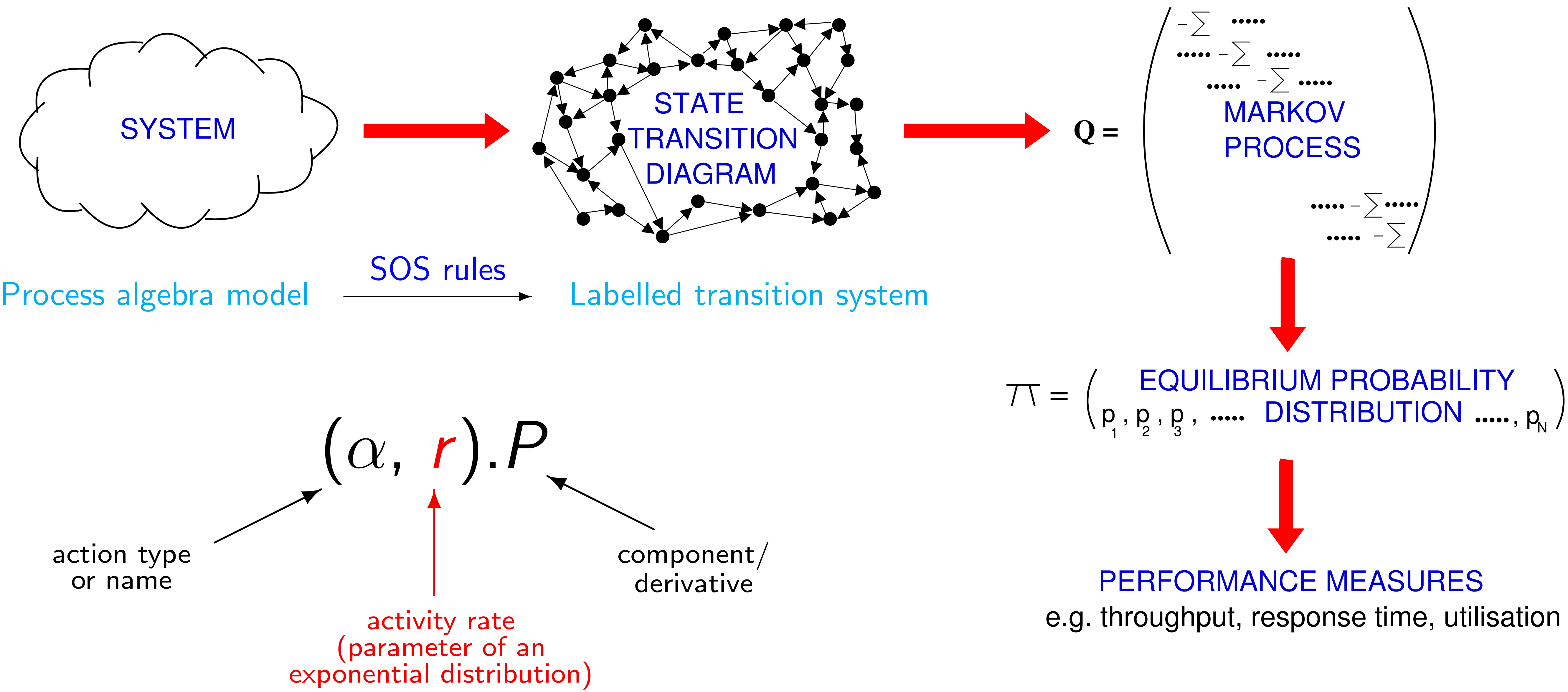
verification:
does a given property hold within the system?

Does a given property hold within the system with a given probability?

How long is it until a given probability hold?

(taken from Jane Hillston's slides)

PEPA workflow



(taken from Jane Hillston's slides)

Communication style

PEPA parallel composition is based on Hoare's CSP

CCS-style	CSP-style
actions and co-actions	no i/o distinction
binary synchronisation	multiple cooperation
conjugate sync	shared name sync
result in a silent action	result in the same name
restriction	hiding
parallel composition	cooperation combinator
one operator	parametric operator

CSP cooperation combinator

$$P \underset{L}{\bowtie} Q$$

cooperation set

interleaving

$$\frac{P_1 \xrightarrow{\alpha} Q_1 \quad \boxed{\alpha \notin L}}{P_1 \underset{L}{\bowtie} P_2 \xrightarrow{\alpha} Q_1 \underset{L}{\bowtie} P_2} \qquad \frac{P_2 \xrightarrow{\alpha} Q_2 \quad \boxed{\alpha \notin L}}{P_1 \underset{L}{\bowtie} P_2 \xrightarrow{\alpha} P_1 \underset{L}{\bowtie} Q_2}$$

cooperation

$$\frac{P_1 \xrightarrow{\alpha} Q_1 \quad P_2 \xrightarrow{\alpha} Q_2 \quad \boxed{\alpha \in L}}{P_1 \underset{L}{\bowtie} P_2 \boxed{\xrightarrow{\alpha}} Q_1 \underset{L}{\bowtie} Q_2}$$

pure interleaving

$$P \parallel Q \triangleq P \underset{\emptyset}{\bowtie} Q$$

PEPA
syntax and semantics

PEPA syntax

P, Q	$::=$	$\mathbf{nil}$	inactive process
	$\mid$	$(\alpha, r).P$	action prefix
	$\mid$	$P + Q$	choice
	$\mid$	$P \underset{L}{\bowtie} Q$	cooperation combinator
	$\mid$	P/L	hiding
	$\mid$	C	process constant

$\alpha \in \Lambda$ action

$L \subseteq \Lambda$ set of actions

$\Delta = \{C_i \triangleq P_i\}_{i \in I}$ set of process declarations

PEPA LTS

ongoing interaction
with the environment
(with other processes)
and its rate

$$P \xrightarrow{(\alpha, r)} Q$$

a process
in its current
state

the process
state after the
interaction

small-step semantics

PEPA semantics (basics)

$$\frac{}{(\alpha, r).P \xrightarrow{(\alpha,r)} P}$$

$$\frac{P_1 \xrightarrow{(\alpha,r)} Q}{P_1 + P_2 \xrightarrow{(\alpha,r)} Q} \qquad \frac{P_2 \xrightarrow{(\alpha,r)} Q}{P_1 + P_2 \xrightarrow{(\alpha,r)} Q}$$

$$\frac{C \triangleq P \in \Delta \quad P \xrightarrow{(\alpha,r)} Q}{C \xrightarrow{(\alpha,r)} Q}$$

Example

$$\text{Server} \stackrel{\triangle}{=} (get, \top).(download, \mu).(rel, \top).\text{Server}$$

extremely high rate
cannot influence the overall rate
of interacting components

$$\begin{aligned}\text{Browser} \stackrel{\triangle}{=}\ & (display, \lambda_1).(cache, m).\text{Browser} \\ + \ & (display, \lambda_2).(get, g).(download, \top).(rel, r).\text{Browser}\end{aligned}$$

a local choice
taken with probability $\frac{\lambda_i}{\lambda_1 + \lambda_2}$

Hiding and interleaving

$$\frac{P \xrightarrow{(\alpha,r)} Q \quad \boxed{\alpha \notin L}}{P/L \xrightarrow{(\alpha,r)} Q/L}$$

$$\frac{P \xrightarrow{(\alpha,r)} Q \quad \boxed{\alpha \in L}}{P/L \boxed{\xrightarrow{(\tau,r)}} Q/L}$$

$$\frac{P_1 \xrightarrow{(\alpha,r)} Q_1 \quad \boxed{\alpha \notin L}}{P_1 \underset{L}{\bowtie} P_2 \xrightarrow{(\alpha,r)} Q_1 \underset{L}{\bowtie} P_2}$$

$$\frac{P_2 \xrightarrow{(\alpha,r)} Q_2 \quad \boxed{\alpha \notin L}}{P_1 \underset{L}{\bowtie} P_2 \xrightarrow{(\alpha,r)} P_1 \underset{L}{\bowtie} Q_2}$$

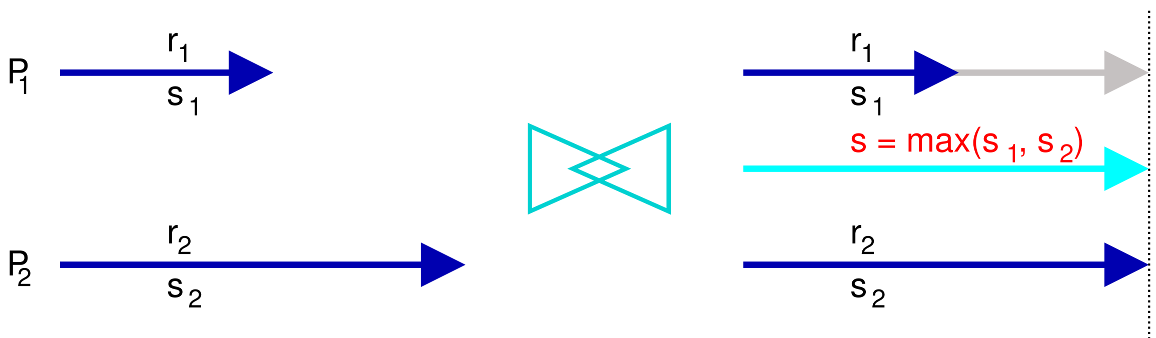
Cooperation

$$\frac{P_1 \xrightarrow{(\alpha, r_1)} Q_1 \quad P_2 \xrightarrow{(\alpha, r_2)} Q_2 \quad \boxed{\alpha \in L}}{P_1 \underset{L}{\bowtie} P_2 \xrightarrow{(\alpha, r)} Q_1 \underset{L}{\bowtie} Q_2}$$

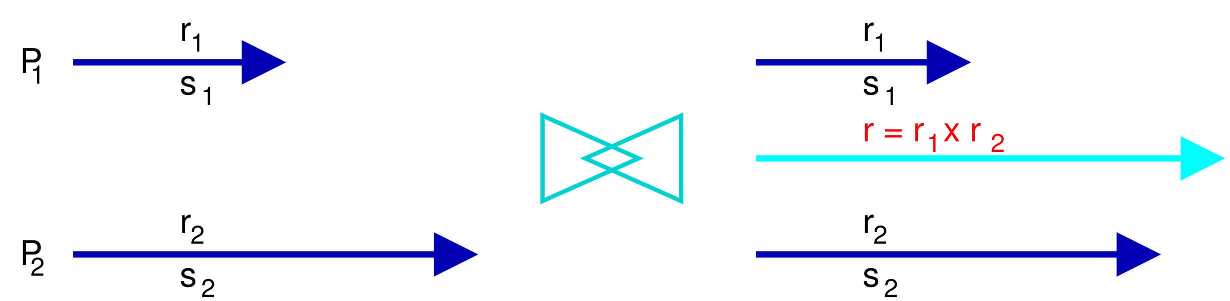
which rate should we put here?

Which rate for sync?

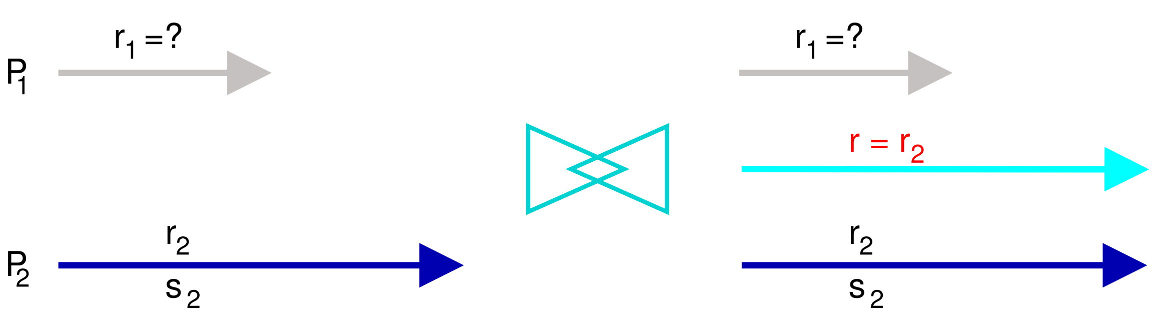
stochastic PA differ for the treatment of rates of synchronised actions



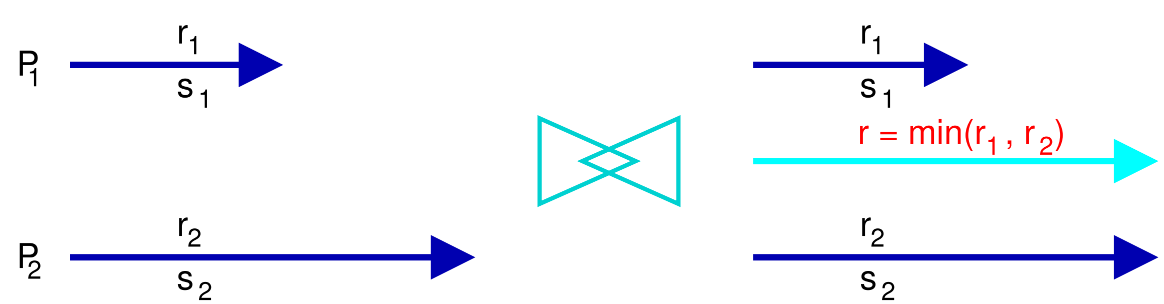
s is no longer exponentially distributed



TIPP: new rate is product of individual rates



EMPA: one participant is passive



bounded capacity: new rate is the minimum of the rates

PEPA's approach

(taken from Jane Hillston's slides)

PEPA: bounded capacity

No component can be made to carry out an action in cooperation faster than its own defined rate for the actions

thus shared actions proceed at the minimum of the rates in the participating components

the apparent rates of independent actions is instead the sum of their rates within independent concurrent components

PEPA: apparent rate

$r_\alpha(P)$ is the observed rate of action α in P

$$\begin{tikzcd}[column sep=small] & P \arrow[dl, "{(\alpha, r_1)}"'] \arrow[dr, "{(\alpha, r_n)}"] & \\ Q_1 & \cdots & Q_n \end{tikzcd}$$

$$r_\alpha(P) = r_1 + \cdots + r_n$$

PEPA: apparent rate

$r_\alpha(P)$ is the observed rate of action α in P

$$r_\alpha(\mathbf{nil}) \triangleq 0$$

$$r_\alpha((\beta, r).P) \triangleq \begin{cases} r & \text{if } \alpha = \beta \\ 0 & \text{if } \alpha \neq \beta \end{cases}$$

$$r_\alpha(P + Q) \triangleq r_\alpha(P) + r_\alpha(Q)$$

(+ is not idempotent!)

$$r_\alpha(P/L) \triangleq \begin{cases} r_\alpha(P) & \text{if } \alpha \notin L \\ 0 & \text{if } \alpha \in L \end{cases}$$

actions are interleaved

$$r_\alpha(P \underset{L}{\bowtie} Q) \triangleq \begin{cases} r_\alpha(P) + r_\alpha(Q) & \text{if } \alpha \notin L \\ \min\{r_\alpha(P), r_\alpha(Q)\} & \text{if } \alpha \in L \end{cases}$$

the slowest must be waited for

$$r_\alpha(C) \triangleq r_\alpha(P) \quad \text{if } C \triangleq P \in \Delta$$

Cooperation

$$\frac{P_1 \xrightarrow{(\alpha,r_1)} Q_1 \quad P_2 \xrightarrow{(\alpha,r_2)} Q_2 \quad \boxed{\alpha \in L}}{P_1 \underset{L}{\bowtie} P_2 \xrightarrow{(\alpha,r)} Q_1 \underset{L}{\bowtie} Q_2}$$

$$r = r_\alpha(P_1 \underset{L}{\bowtie} P_2) \cdot \frac{r_1}{r_\alpha(P_1)} \cdot \frac{r_2}{r_\alpha(P_2)}$$

probability of specific action (α, r_i) among the α-transitions of P_i

apparent rate

the sum of the rates of all the α-transitions that $P_1 \underset{L}{\bowtie} P_2$ can do

Example

$$\mathsf{Server} \triangleq (get, \top).(download, \mu).(rel, \top).\mathsf{Server}$$

$$\begin{aligned}
\mathsf{S} &\triangleq (get, \top).\mathsf{S1} \\
\mathsf{S1} &\triangleq (dnd, \mu).\mathsf{S2} \\
\mathsf{S2} &\triangleq (rel, \top).\mathsf{S}
\end{aligned}$$

$$\begin{aligned}
\mathsf{Browser} &\triangleq (display, \lambda_1).(cache, m).\mathsf{Browser} \\
&+ (display, \lambda_2).(get, g).(download, \top).(rel, r).\mathsf{Browser}
\end{aligned}$$

$$\begin{aligned}
\mathsf{B} &\triangleq (dis, \lambda_1).\mathsf{B1} + (dis, \lambda_2).\mathsf{B2} \\
\mathsf{B1} &\triangleq (cac, m).\mathsf{B} \\
\mathsf{B2} &\triangleq (get, g).\mathsf{B3} \\
\mathsf{B3} &\triangleq (dnd, \top).\mathsf{B4} \\
\mathsf{B4} &\triangleq (rel, r).\mathsf{B}
\end{aligned}$$

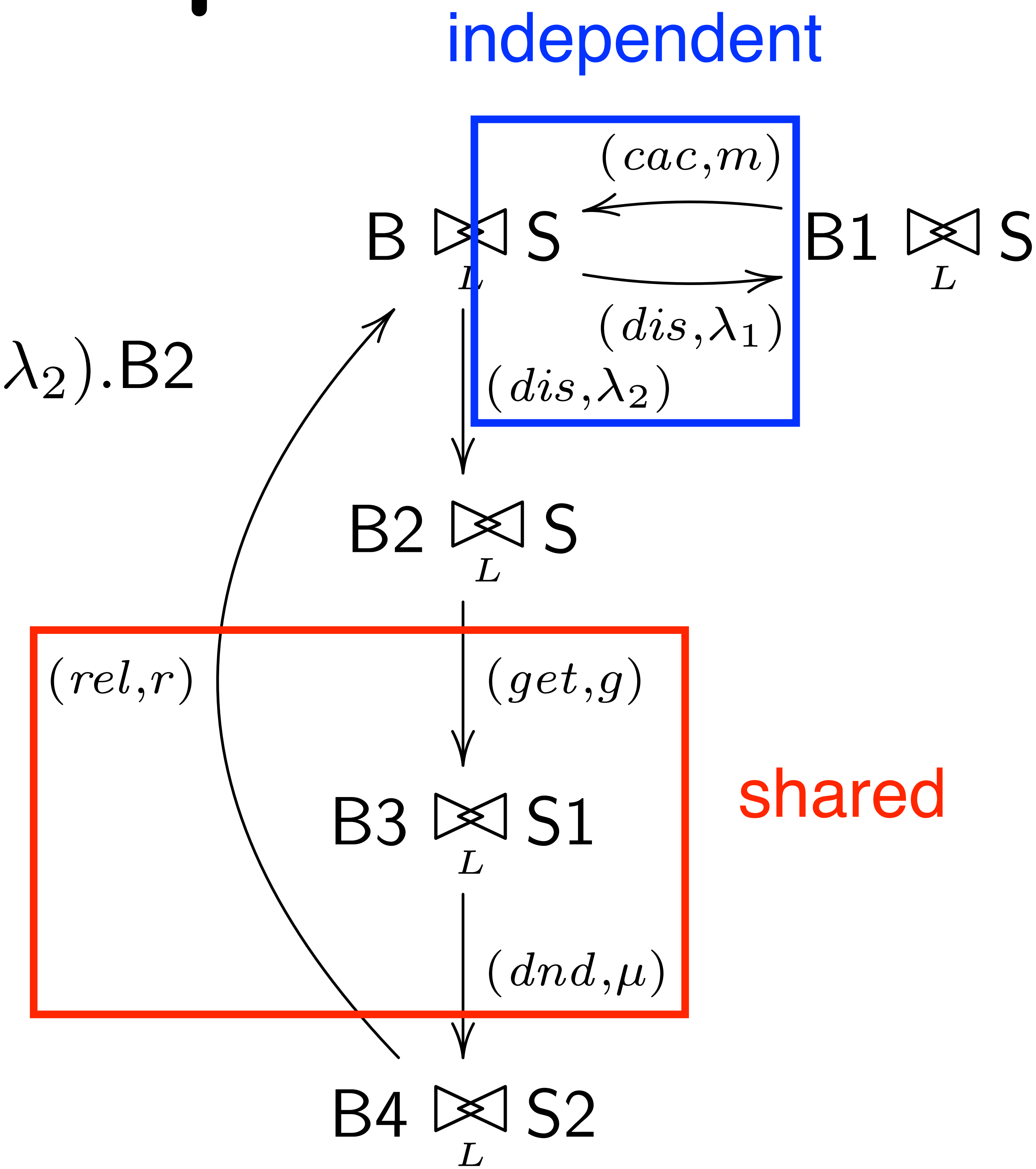
Example

$$\mathsf{S} \triangleq (get, \top).\mathsf{S1}$$
$$\mathsf{S1} \triangleq (dnd, \mu).\mathsf{S2}$$
$$\mathsf{S2} \triangleq (rel, \top).\mathsf{S}$$

$$\mathsf{B} \triangleq (dis, \lambda_1).\mathsf{B1} + (dis, \lambda_2).\mathsf{B2}$$
$$\mathsf{B1} \triangleq (cac, m).\mathsf{B}$$
$$\mathsf{B2} \triangleq (get, g).\mathsf{B3}$$
$$\mathsf{B3} \triangleq (dnd, \top).\mathsf{B4}$$
$$\mathsf{B4} \triangleq (rel, r).\mathsf{B}$$

$$L = \{get, dnd, rel\}$$

$$\mathsf{B} \bowtie_L \mathsf{S}$$



Example

$$\begin{aligned} \mathsf{S} &\triangleq (get, \top).\mathsf{S1} \\ \mathsf{S1} &\triangleq (dnd, \mu).\mathsf{S2} \\ \mathsf{S2} &\triangleq (rel, \top).\mathsf{S} \end{aligned}$$

$$L = \{get, dnd, rel\}$$

$$\begin{aligned} \mathsf{B} &\triangleq (dis, \lambda_1).\mathsf{B1} + (dis, \lambda_2).\mathsf{B2} \\ \mathsf{B1} &\triangleq (cac, m).\mathsf{B} \\ \mathsf{B2} &\triangleq (get, g).\mathsf{B3} \\ \mathsf{B3} &\triangleq (dnd, \top).\mathsf{B4} \\ \mathsf{B4} &\triangleq (rel, r).\mathsf{B} \end{aligned}$$

$$(\mathsf{B} \parallel \mathsf{B}) \underset{L}{\bowtie} \mathsf{S}$$

$$(\mathsf{B} \parallel \mathsf{B}) \underset{L}{\bowtie} \mathsf{S} \xrightarrow{(dis,\lambda_2)} (\mathsf{B2} \parallel \mathsf{B}) \underset{L}{\bowtie} \mathsf{S} \xrightarrow{(dis,\lambda_2)} (\mathsf{B2} \parallel \mathsf{B2}) \underset{L}{\bowtie} \mathsf{S}$$

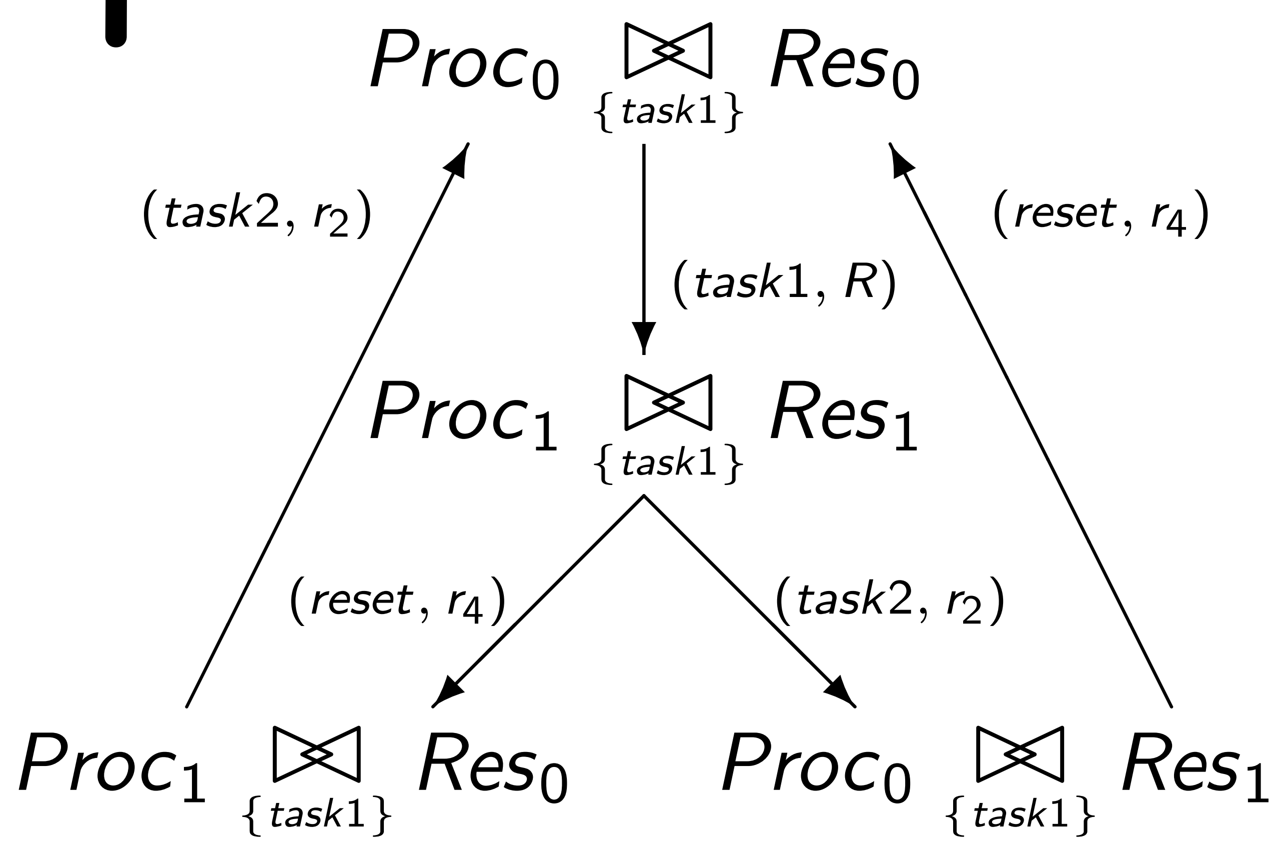
$$\begin{array}{ccc} (\mathsf{B2} \parallel \mathsf{B2}) \underset{L}{\bowtie} \mathsf{S} & \xrightarrow{(get,g)} & (\mathsf{B3} \parallel \mathsf{B2}) \underset{L}{\bowtie} \mathsf{S1} \\ \Big\downarrow{\scriptstyle (get,g)} & & \\ (\mathsf{B2} \parallel \mathsf{B3}) \underset{L}{\bowtie} \mathsf{S1} & & \end{array}$$

$$\begin{aligned} r_{get}(\mathsf{B2}) &= g \\ r_{get}(\mathsf{B2} \parallel \mathsf{B2}) &= 2g \\ r_{get}(\mathsf{S}) &= \top \\ r_{get}((\mathsf{B2} \parallel \mathsf{B2}) \underset{L}{\bowtie} \mathsf{S}) &= 2g \end{aligned}$$

Example

$$\begin{aligned}
Proc_0 &\stackrel{def}{=} (task1, r_1).Proc_1 \\
Proc_1 &\stackrel{def}{=} (task2, r_2).Proc_0 \\
Res_0 &\stackrel{def}{=} (task1, r_3).Res_1 \\
Res_1 &\stackrel{def}{=} (reset, r_4).Res_0
\end{aligned}$$

$$Proc_0 \underset{\{task1\}}{\bowtie} Res_0$$



$$R = \min(r_1, r_3)$$

$$\mathbf{Q} = \begin{pmatrix} -R & R & 0 & 0 \\ 0 & -(r_2 + r_4) & r_4 & r_2 \\ r_2 & 0 & -r_2 & 0 \\ r_4 & 0 & 0 & -r_4 \end{pmatrix}$$

$$\begin{cases} p \cdot Q = 0 \\ \sum_{i=1}^{N} p_i = 1 \end{cases}$$

(taken from Jane Hillston's slides)

Example

$$\mathbf{Q} = \begin{pmatrix} -R & R & 0 & 0 \\ 0 & -(r_2 + r_4) & r_4 & r_2 \\ r_2 & 0 & -r_2 & 0 \\ r_4 & 0 & 0 & -r_4 \end{pmatrix} \qquad \begin{cases} p \cdot Q = 0 \\ \displaystyle\sum_{i=1}^{N} p_i = 1 \end{cases}$$

$$r_1 = 2 \qquad r_2 = 2 \qquad r_3 = 6 \qquad r_4 = 8 \qquad R = \min\{r_1, r_3\} = 2$$

$$p_1 = \frac{20}{41} \qquad p_2 = \frac{4}{41} \qquad p_3 = \frac{1}{41} \qquad p_4 = \frac{16}{41}$$

Reward structure

$\mathcal{C}$ a set of PEPA components

$\rho : \mathcal{C} \to \mathbb{R}$ a reward structure

p a steady state distribution

$$R_\rho \triangleq \sum_i p_i \cdot \rho(C_i)$$

sometimes rewards are defined in terms of activities

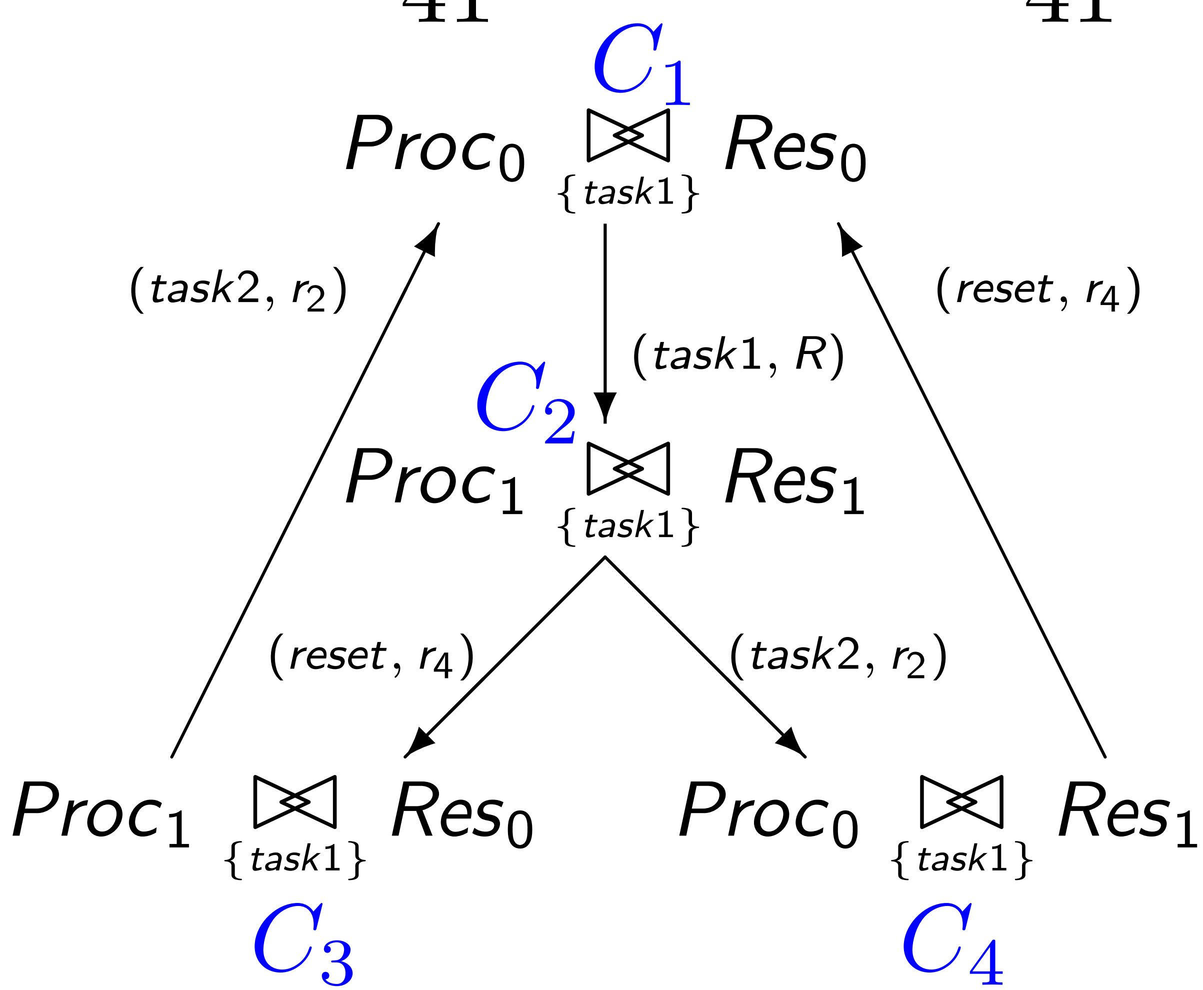
$$\rho : L \to \mathbb{R}$$

$$\rho(C) = \sum_{C \xrightarrow{(\alpha,r)} Q} \rho(\alpha)$$

Example: throughput

$$\mathbf{Q} = \begin{pmatrix} -R & R & 0 & 0 \\ 0 & -(r_2 + r_4) & r_4 & r_2 \\ r_2 & 0 & -r_2 & 0 \\ r_4 & 0 & 0 & -r_4 \end{pmatrix} \qquad \begin{cases} p \cdot Q = 0 \\ \sum_{i=1}^{N} p_i = 1 \end{cases}$$

$$p_1 = \frac{20}{41} \qquad p_2 = \frac{4}{41} \qquad p_3 = \frac{1}{41} \qquad p_4 = \frac{16}{41}$$



$$\rho(task_i) = 1 \qquad \rho(reset) = 0$$

$$\rho(C_1) = \rho(C_2) = \rho(C_3) = 1$$

$$\rho(C_4) = 0$$

$$R = \frac{20 + 4 + 1}{41} = \frac{25}{41} = 61\%$$

PEPA
further considerations

The importance of being Exp

$$
\begin{tikzcd}
 & (\alpha, r).Stop \parallel (\beta, s).Stop \arrow[dl, "(\alpha, r)"'] \arrow[dr, "(\beta, s)"] & \\
Stop \parallel (\beta, s).Stop \arrow[dr, "(\beta, s)"'] & & (\alpha, r).Stop \parallel Stop \arrow[dl, "(\alpha, r)"] \\
 & Stop \parallel Stop &
\end{tikzcd}
$$

We retain the expansion law of classical process algebra:

$$
\begin{aligned}
&(\alpha, r).Stop \parallel (\beta, s).Stop = \\
&\qquad (\alpha, r).(\beta, s).(Stop \parallel Stop) + (\beta, s).(\alpha, r).(Stop \parallel Stop)
\end{aligned}
$$

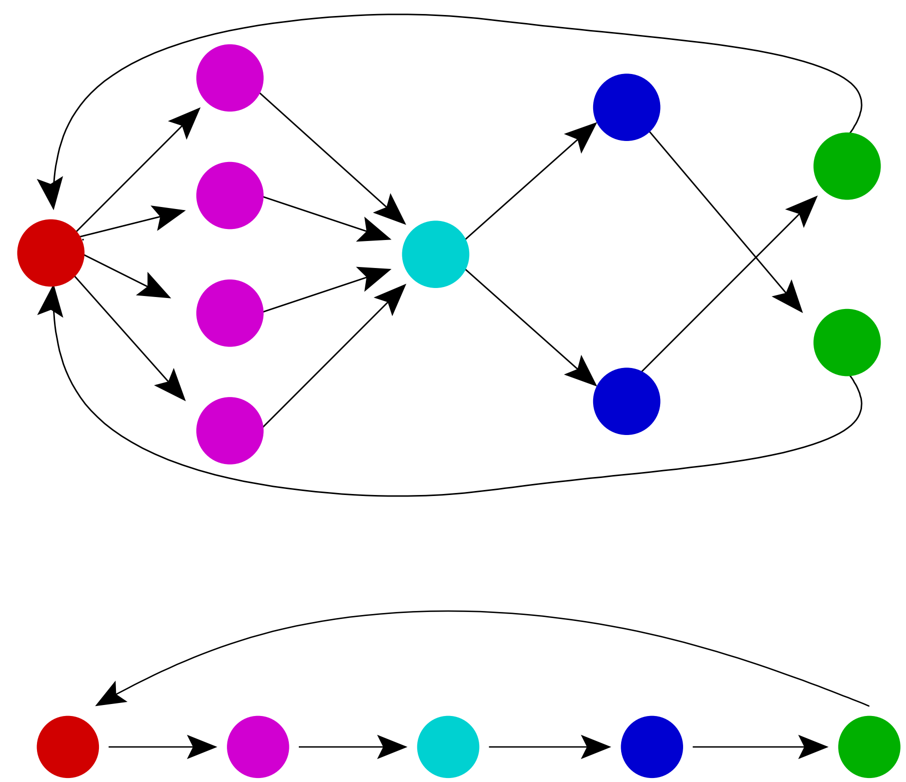
only if the negative exponential distribution is assumed.

(taken from Jane Hillston's slides)

Model aggregation

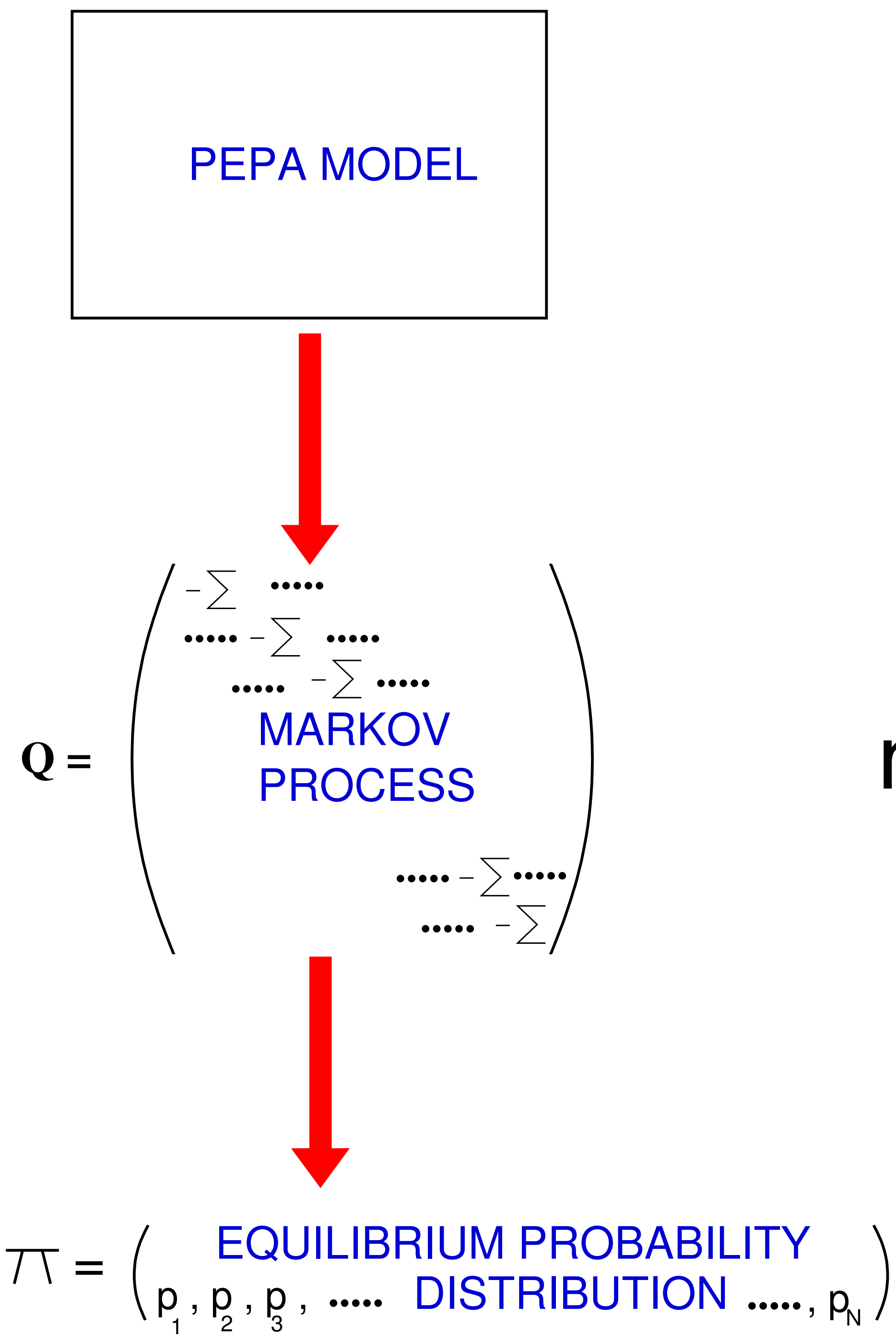
we can exploit CTMC bisimulation to reduce the state space (notion of lumpable partition)

it is the only equivalence that preserves the Markov property



(taken from Jane Hillston’s slides)

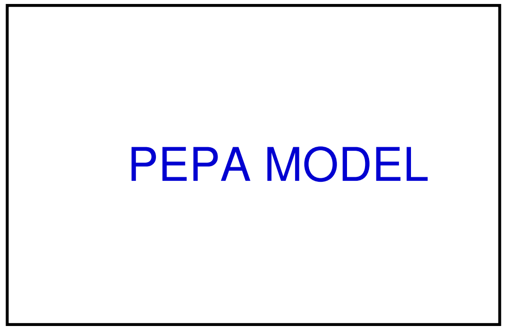
Compositionality



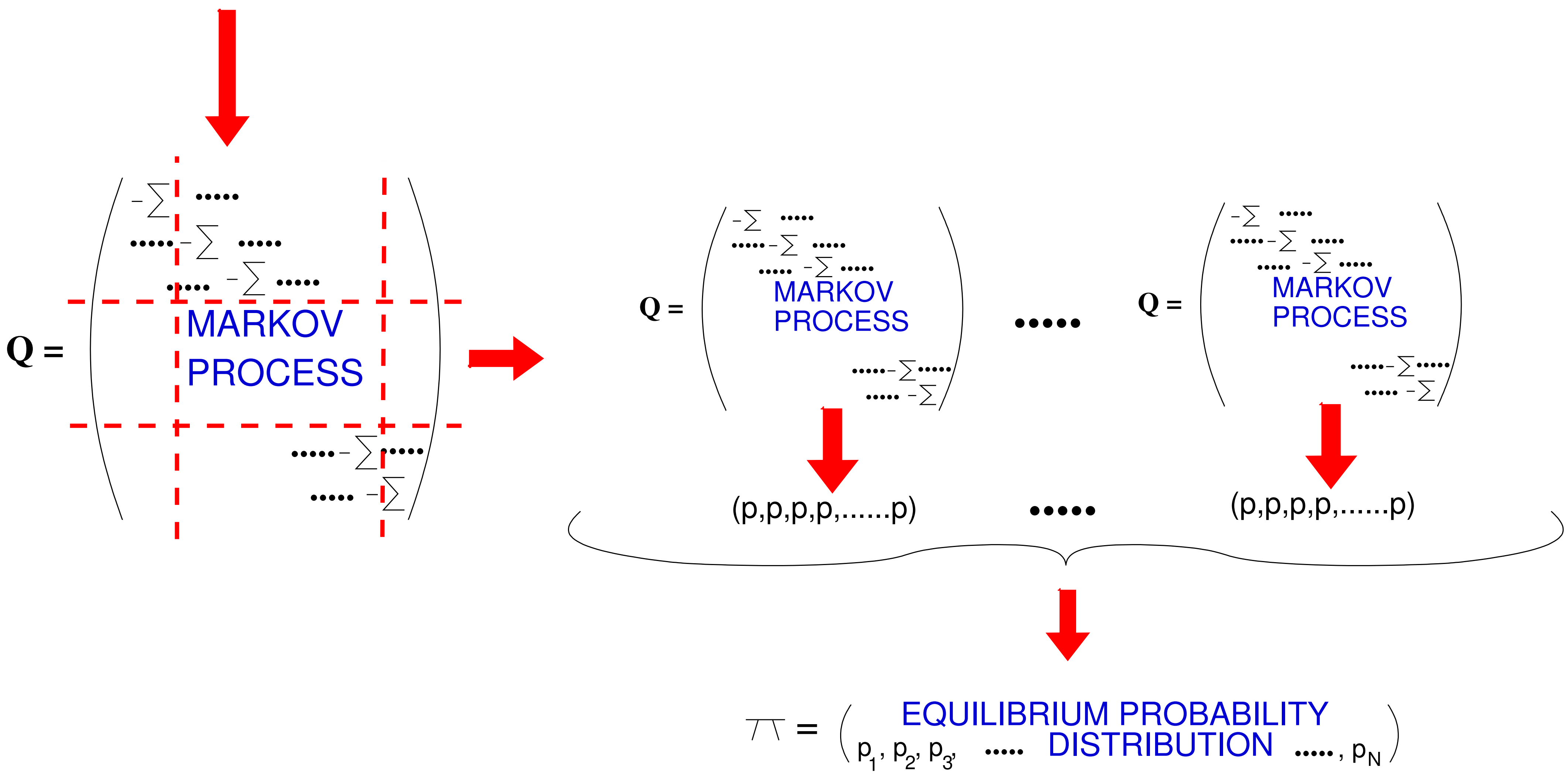
matrix size can grow very large

(taken from Jane Hillston's slides)

Compositionality

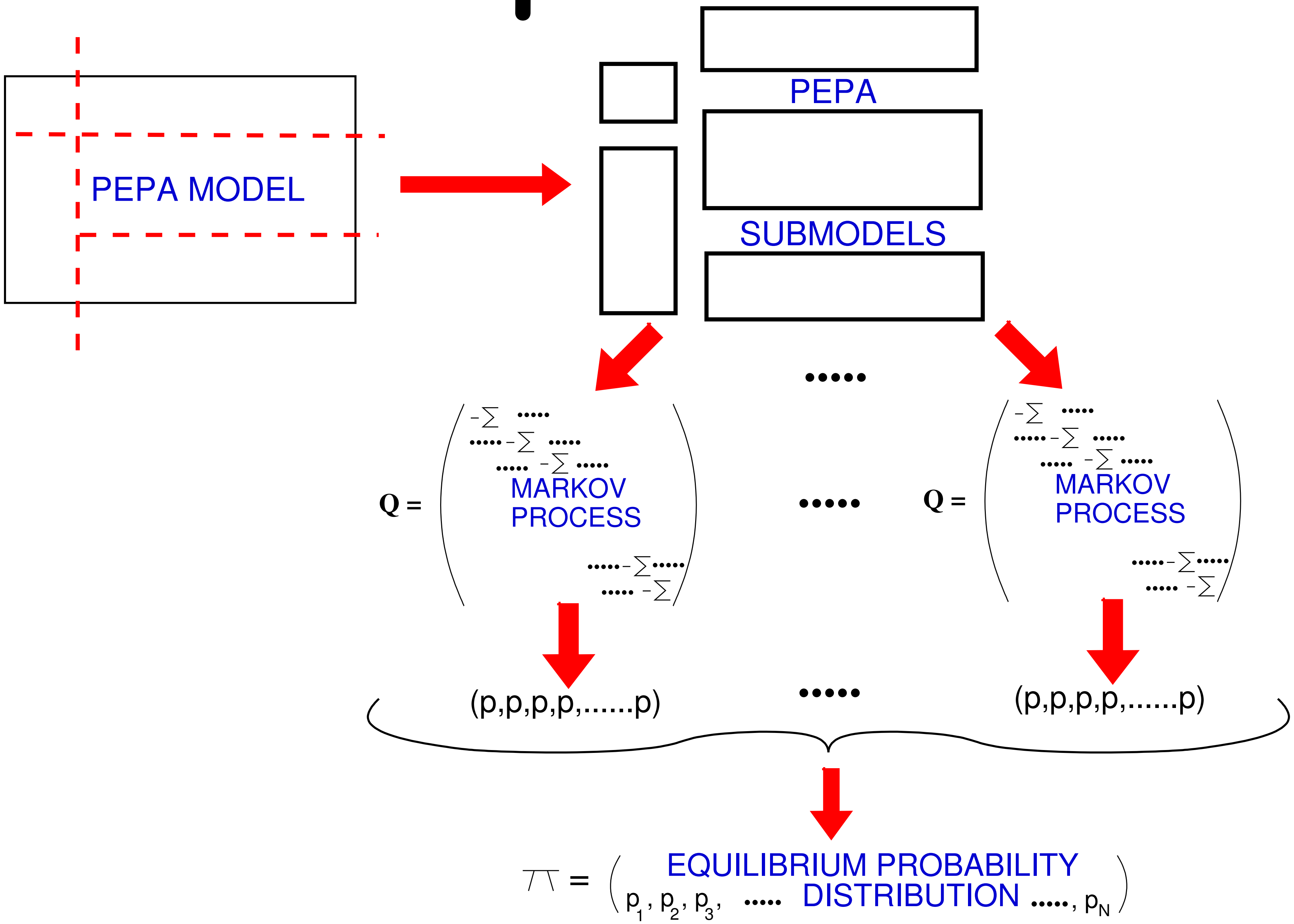


Certain structures in the matrix are known to be amenable to efficient, decomposed solution.



(taken from Jane Hillston's slides)

Compositionality



lift independent structures to the PEPA model!

(taken from Jane Hillston's slides)