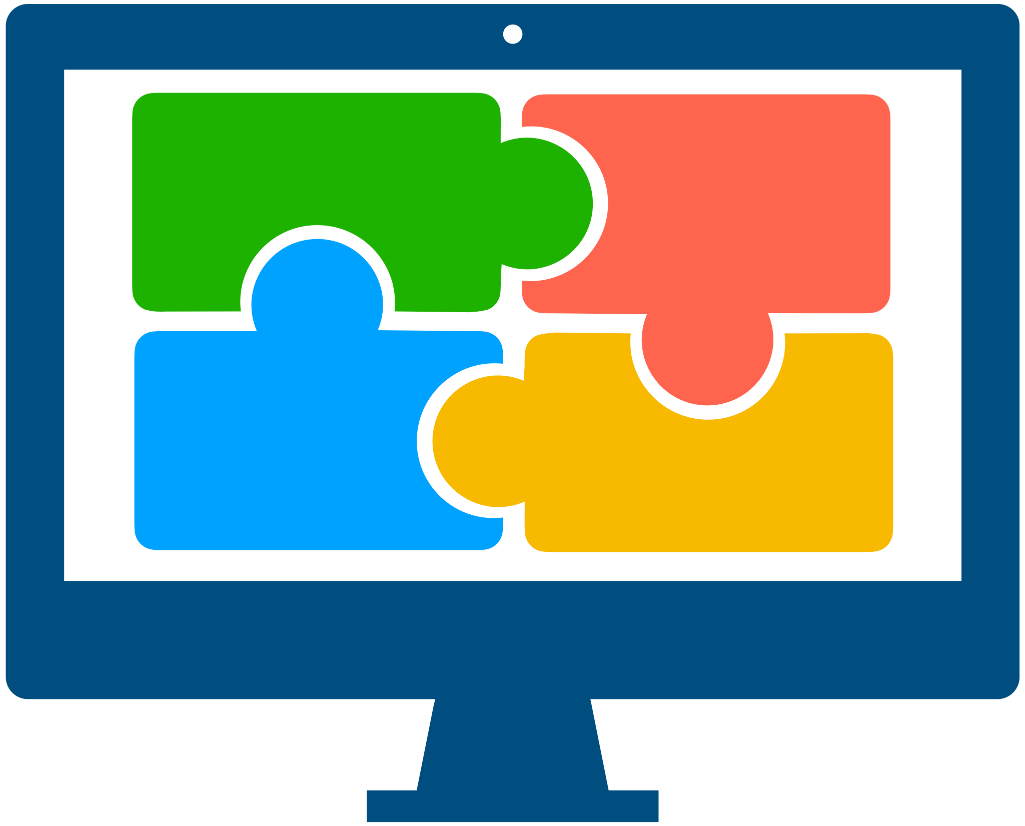

http://didawiki.di.unipi.it/doku.php/
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PSC 2020/21 (375AA, 9CFU)

Principles for Software Composition

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11 - Haskell

Lambda notation, again

Bound variables

```
int f(int x) { return x^2 + 2*x + 5 }
int f(int y) { return y^2 + 2*y + 5 }
```

$$f \triangleq \lambda x.\ x^2 + 2x + 5$$
$$f \triangleq \lambda y.\ y^2 + 2y + 5$$

```
let f x = x^2 + 2*x + 5
let f y = y^2 + 2*y + 5
```

they are all the same!
names of local variables are not important:
alpha-conversion

Free variables

$x^2 + 2x + 5$

$y^2 + 2y + 5$

they are not the same!
names of global variables matter

$\lambda x.\ x^2 + 2x + 5$

$\lambda x.\ y^2 + 2y + 5$

the same enclosing context
can make a difference

$\lambda x.\ t$

we say it binds the occurrences of x in t

$\lambda x.\ x^2 + 2z + 5$

$\lambda y.\ y^2 + 2z + 5$

$\lambda z.\ z^2 + 2z + 5$

are they all equivalent
(by alpha-conversion)?

Free variables: formally

$$t ::= x \mid \lambda x.\ t \mid t\ t \mid \ldots$$

$$\mathsf{fv} : LTerms \to \wp(Var)$$

$$\begin{aligned} \mathsf{fv}(x) &\triangleq \{x\} \\ \mathsf{fv}(\lambda x.\ t) &\triangleq \mathsf{fv}(t) \setminus \{x\} \\ \mathsf{fv}(t_1\ t_2) &\triangleq \mathsf{fv}(t_1) \cup \mathsf{fv}(t_2) \end{aligned}$$

$$\mathsf{fv}(\lambda x.\ x^2 + 2z + 5) = \{z\}$$

$$\mathsf{fv}(\lambda y.\ y^2 + 2z + 5) = \{z\}$$

$$\mathsf{fv}(\lambda z.\ z^2 + 2z + 5) = \emptyset$$

Alpha-conversion, again

$$\lambda x.\ t \equiv \lambda y.\ (t[{}^{y}/_{x}]) \qquad \text{if } y \notin \mathsf{fv}(\lambda x.\ t)$$

$$\lambda x.\ x^2 + 2z + 5 \equiv \lambda y.\ ((x^2 + 2z + 5)[{}^{y}/_{x}]) = \lambda y.\ y^2 + 2z + 5$$

$$\lambda x.\ x^2 + 2z + 5 \not\equiv \lambda z.\ ((x^2 + 2z + 5)[{}^{z}/_{x}]) \text{ because } z \in \mathsf{fv}(\lambda x.\ x^2 + 2z + 5)$$

Beta rule, again

$$(\lambda x.\ t)\ e \equiv t[{}^{e}/_{x}]$$

how is (capture-avoiding) substitution defined?
and why is it called “capture-avoiding”?

Capture-avoiding substitution

Substitution, 1st try

$$y[^e/_x] \triangleq \begin{cases} e & \text{if } y = x \\ y & \text{otherwise} \end{cases}$$

$$(\lambda y.\ t)[^e/_x] \triangleq \begin{cases} \lambda y.\ t & \text{if } y = x \\ \lambda y.\ (t[^e/_x]) & \text{otherwise} \end{cases}$$

$$(t_1\ t_2)[^e/_x] \triangleq t_1[^e/_x]\ (t_2[^e/_x])$$

$$t_1 \triangleq \lambda x.\ \lambda y.\ x^2 + 2y + 5 \qquad\qquad t_2 \triangleq y$$

$$\begin{aligned} t_1\ t_2 &\equiv (\lambda x.\ \lambda y.\ x^2 + 2y + 5)\ {\color{blue}y} \longleftarrow \text{free} \\ &\equiv (\lambda y.\ x^2 + 2y + 5)[^{\color{blue}y}/_x] \\ &\equiv \lambda y.\ ((x^2 + 2y + 5)[^{\color{blue}y}/_x]) \\ &\equiv \lambda y.\ {\color{blue}y}^2 + 2y + 5 \end{aligned}$$

captured variable!

Capture-avoiding

free variables occurring in e
should remain free after the application of $[{}^{e}/_{x}]$

solution: alpha-convert before substituting!

$$
\begin{aligned}
(\lambda y.\ x^2 + 2y + 5)[{}^{y}/_{x}] &\equiv (\lambda z.\ (x^2 + 2y + 5)[{}^{z}/_{y}])[{}^{y}/_{x}] \\
&\equiv (\lambda z.\ x^2 + 2z + 5)[{}^{y}/_{x}] \\
&\equiv \lambda z.\ ((x^2 + 2z + 5)[{}^{y}/_{x}]) \\
&\equiv \lambda z.\ y^2 + 2z + 5
\end{aligned}
$$

free

Substitution, 2nd try

$$y[{}^{e}/_{x}] \triangleq \begin{cases} e & \text{if } y = x \\ y & \text{otherwise} \end{cases}$$

superfluous: no free occurrences to replace

$$(\lambda y.\ t)[{}^{e}/_{x}] \triangleq \begin{cases} \lambda y.\ t & \text{if } y = x \\ \lambda z.\ (t[{}^{z}/_{y}][{}^{e}/_{x}]) & \text{otherwise, with} \\ & z \notin \mathsf{fv}(e) \cup \mathsf{fv}(\lambda y.\ t) \cup \{x\} \end{cases}$$

$$(t_1\ t_2)[{}^{e}/_{x}] \triangleq t_1[{}^{e}/_{x}]\ (t_2[{}^{e}/_{x}])$$

Substitution, final

$$y[{}^{e}/_{x}] \triangleq \begin{cases} e & \text{if } y = x \\ y & \text{otherwise} \end{cases}$$

$$(\lambda y.\ t)[{}^{e}/_{x}] \triangleq \lambda z.\ (t[{}^{z}/_{y}][{}^{e}/_{x}]) \quad \text{with } z \notin \mathsf{fv}(e) \cup \mathsf{fv}(\lambda y.\ t) \cup \{x\}$$

$$(t_1\ t_2)[{}^{e}/_{x}] \triangleq t_1[{}^{e}/_{x}]\ (t_2[{}^{e}/_{x}])$$

Higher Order Functional Languages
Haskell

Imperative vs Functional

Imperative style

tell the machine how to compute;
a sequence of tasks to execute;
manipulation of mutable states

Purely functional style

tell the machine what to compute;
declarative style;
define what functions are,
not how to compute them;
functions have no side effects;
can’t set and change variable’s content;
manipulation of values

Declarative style

Any experience of functional programming?

Have you ever used a spreadsheet?

The value of a cell is defined in terms of those of other cells:
what is to be computed, not how it must be computed

we do not specify the order in which cells are calculated:
cells are computed according to their dependencies

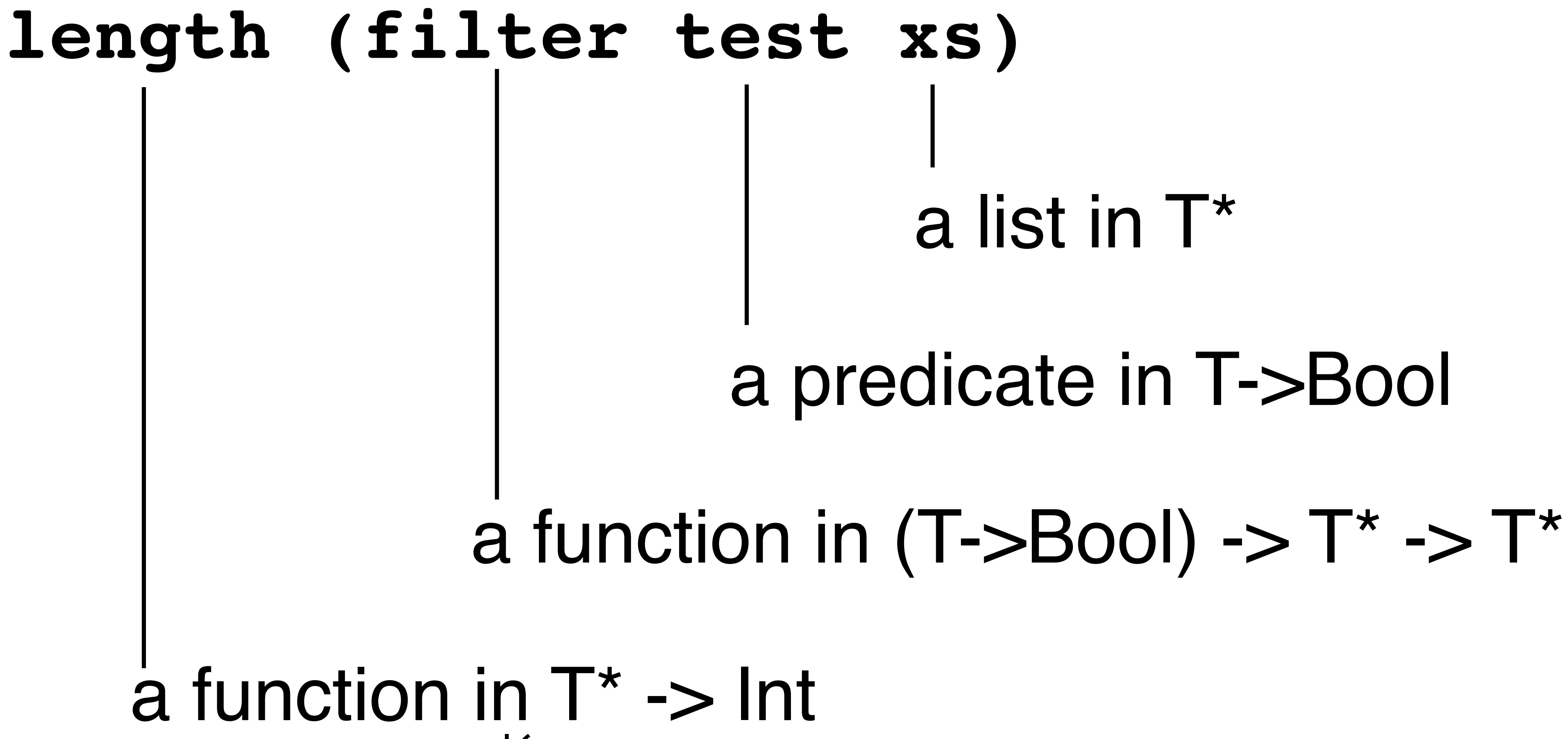
we do not decide how to allocate memory:
only those cells which are in use are allocated

we specify the value of a cell by an expression:
its parts can be evaluated in any order

Functional style: HO

Higher-Order:
functions as values,
functions as parameters,
functions are returned,
functions are composed

how many elements of a list will pass the test?



```
length (filter test xs)
```

a list in T*

a predicate in T->Bool

a function in (T->Bool) -> T* -> T*

a function in T* -> Int

Purity: no side effects

the result of a function is determined only by its input

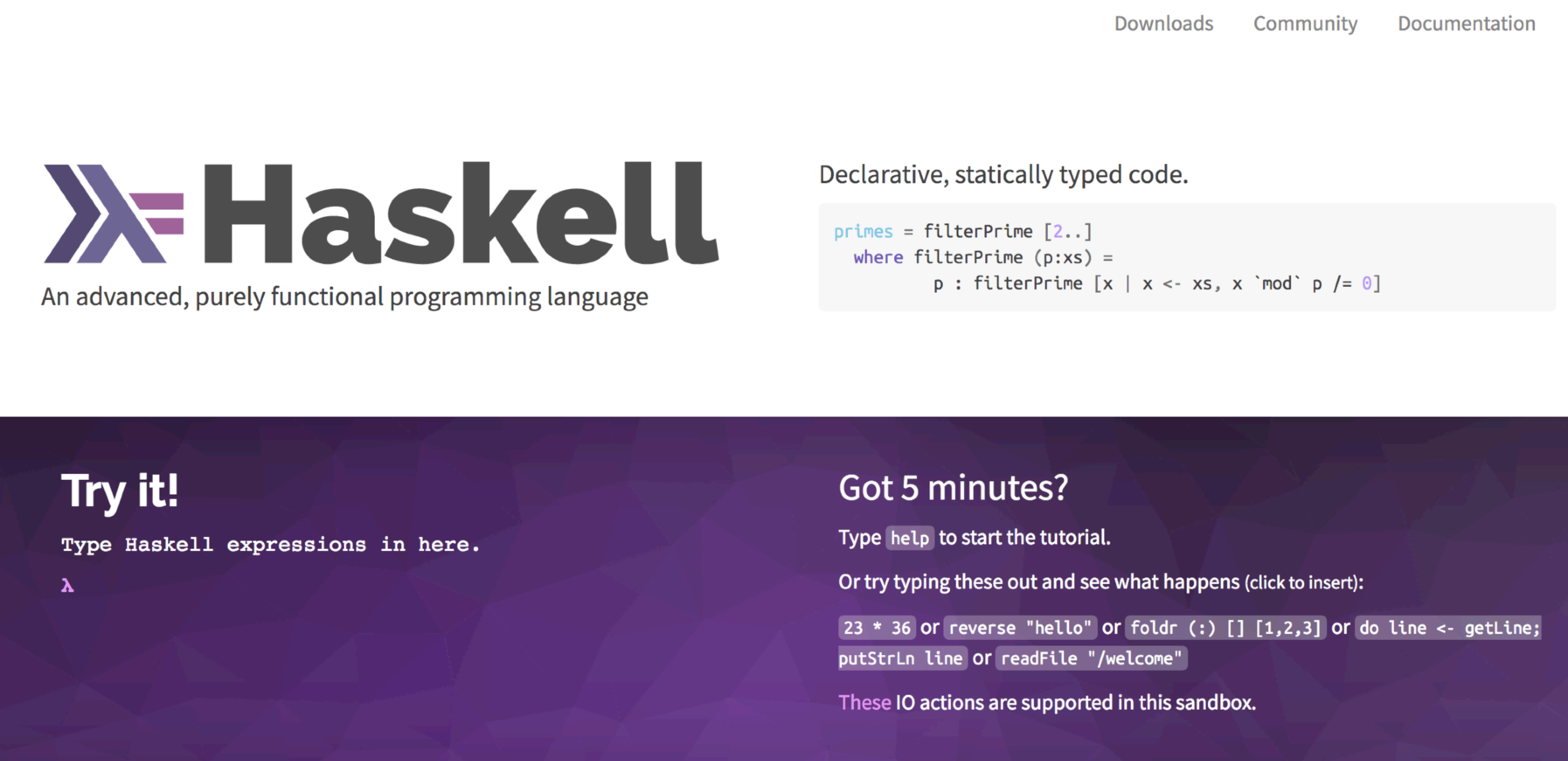
a variable is just a name bound to some (HO) value:
shorthands for expressions

variables do not vary

programs are typically shorter, maybe less efficient;
closer to semantics, ease verification of correctness;
more robust, easier to maintain

Haskell: a purely functional programming language

`http://www.haskell.org/`



Haskell

An advanced, purely functional programming language

Declarative, statically typed code.

```
primes = filterPrime [2..]
  where filterPrime (p:xs) =
          p : filterPrime [x | x <- xs, x `mod` p /= 0]
```

Try it!

`Type Haskell expressions in here.`

`λ`

Got 5 minutes?

Type `help` to start the tutorial.

Or try typing these out and see what happens (click to insert):

`23 * 36` or `reverse "hello"` or `foldr (:) [] [1,2,3]` or `do line <- getLine; putStrLn line` or `readFile "/welcome"`

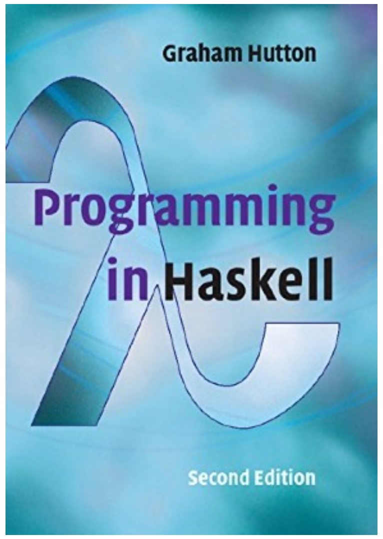
These IO actions are supported in this sandbox.

Haskell: origins

named after mathematical logician Haskell B. Curry

1987: Haskell project begun
1998: first version appear
2003: the Haskell Report was published
(first stable version)

Graham Hutton, “Programming in Haskell”, ch.1-8,14,15



Features

Referential transparency
if a function is called twice with the same argument, it returns the same result;
compiler can reason on program’s behaviour;
one can deduce a function is correct and build more complex functions by composition

Statically typed
type inference: you don’t have to label all data, their types can be figured out;
many possible errors are caught at compile time

Polymorphism
one definition of function works for many types

Overloading
different definitions of the same function-name for different types

Laziness
calculation starts only if some result is needed;
infinite data structures can be manipulated

I'M NOT LAZY
I'M JUST SAVING MY ENERGY FOR WHEN I REALLY NEED IT.
quickmeme.com

More features (less bugs)

Purity: no side effects

Typeful: types are pervasive, no dubious use of types

Concise: shorter programs, less typing (on the keyboard)

High level: closer to the algorithm description

Memory managed: programmers can focus on the algorithm

Compositionality: solve problems by composing functions that solve smaller problems

Data encapsulation and polymorphism not exclusive to OOP: modules and type classes

A taste of Haskell

math. notation

Haskell notation

$$f(x) = 2x + 3$$

```
f x = 2*x + 3
```

$$g(x, y) = x^2 + xy + y^2$$

```
g (x,y) = x^2 + x*y + y^2
```

$$abs(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{otherwise} \end{cases}$$

```
abs x
    | x>=0      = x
    | otherwise = -x
```

$$abs(f(g(2, 3)))$$

```
abs(f(g(3,2)))
```

set comprehension

$$\{x \mid x \in X \wedge f(x) > 5\}$$

list comprehension

```
[ x | x<-X , f x > 5 ]
```

The power of recursion

No assignments: no loops

(loops over lists exist: *list comprehension*)

Recursion is used in place of loops

```
power2 n
       | n==0 = 1
       | n>0  = 2 * power2(n-1)
```

Haskell: some principles

evaluate *expressions* (syntactic terms)

to yield *values* (abstract entities regarded as answers)

every value has an associated *type*
the association is called *typing*
you can think of types as sets of values

as expressions denote values
types are denoted by type expressions

values are first-class (passed around, returned as results)
types are not first-class

Haskell: GHCi

Interactive shell or interpreter, executing read-eval-print loop

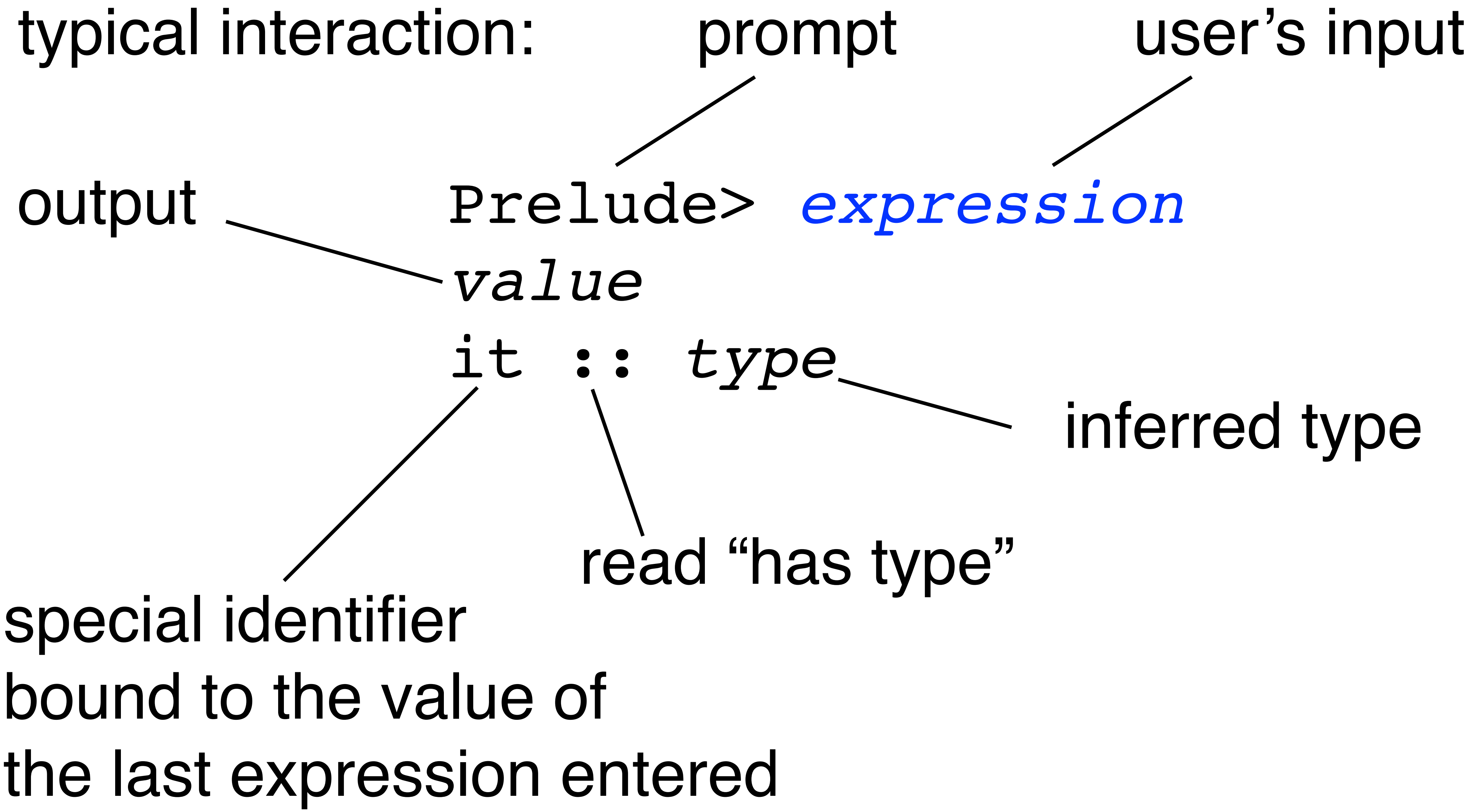
programmers enter expressions/declarations one at a time

they are type checked, compiled and executed

if an expression does not parse correctly
or does not pass the type-checking phase of the compiler,
no code is generated and no code is executed

once an identifier is defined it is available at subsequent lines

GHCi expressions



typical interaction:

prompt

user’s input

output

```
Prelude> expression
value
it :: type
```

inferred type

read “has type”

special identifier
bound to the value of
the last expression entered

GHCi declarations

typical interaction:

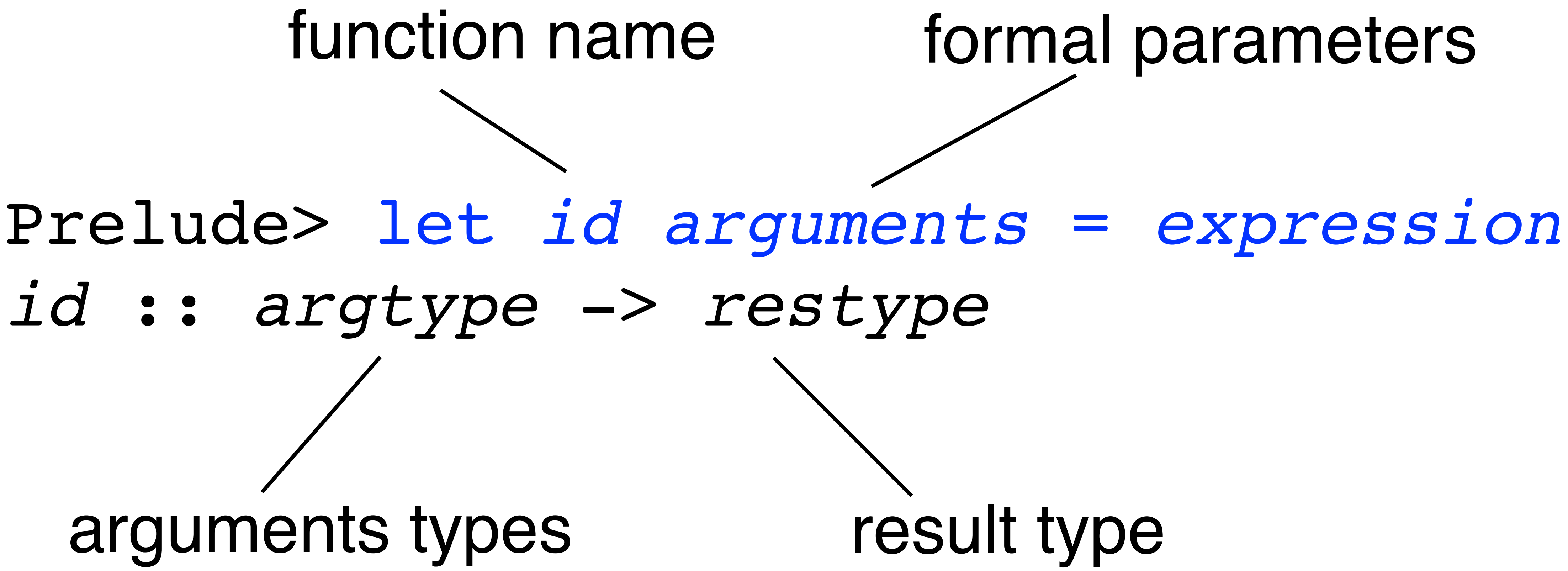
keyword

```
Prelude> let id = expression
id :: type
```

defining symbol

GHCi declarations

more generally:



function name

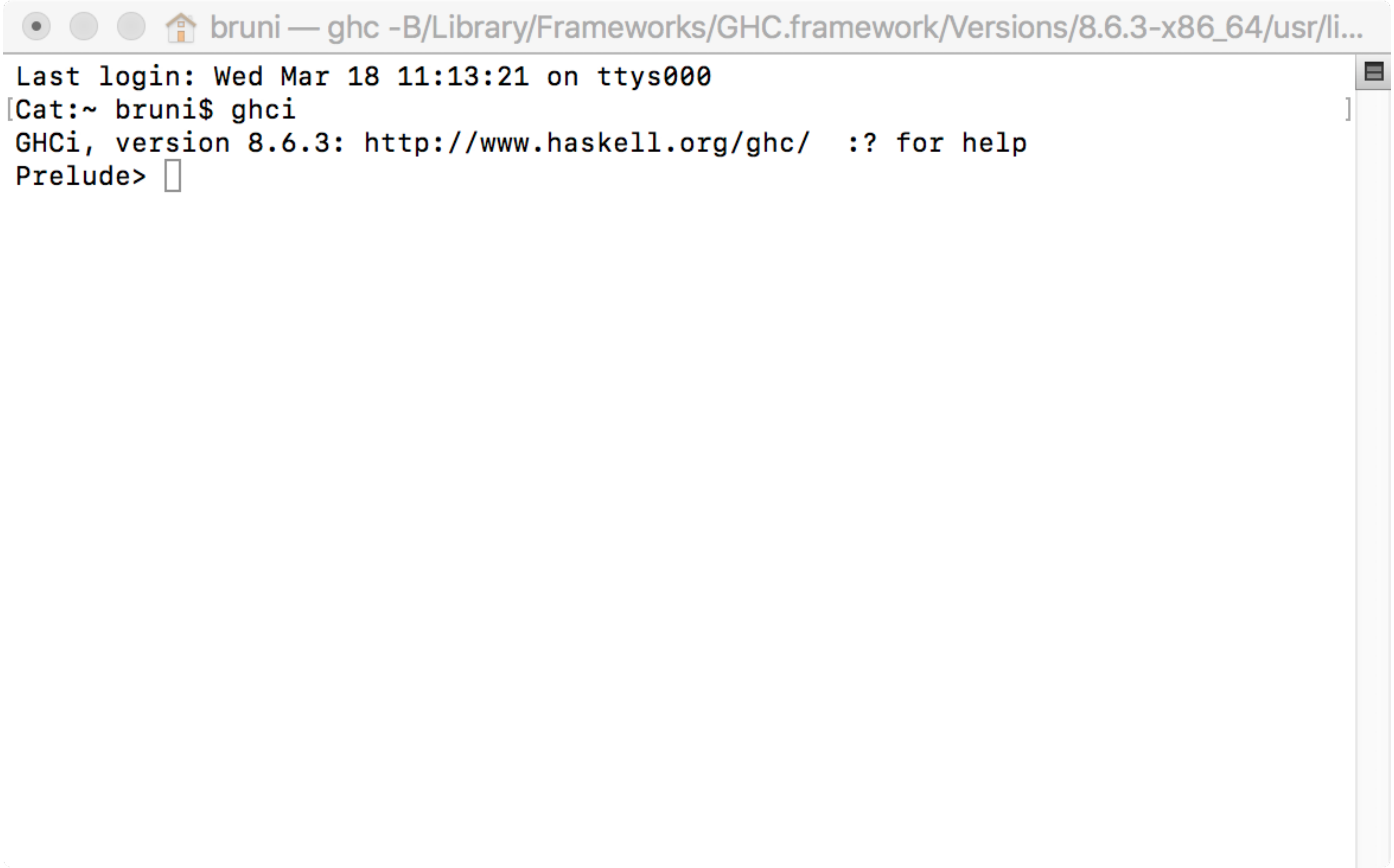
formal parameters

```
Prelude> let id arguments = expression
id :: argtype -> restype
```

arguments types

result type

GHCi session



bruni — ghc -B/Library/Frameworks/GHC.framework/Versions/8.6.3-x86_64/usr/li...

```
Last login: Wed Mar 18 11:13:21 on ttys000
Cat:~ bruni$ ghci
GHCi, version 8.6.3: http://www.haskell.org/ghc/  :? for help
Prelude> 
```