



PSC 2021/22 (375AA, 9CFU)

Principles for Software Composition

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17b - CCS: guarded processes

CCS

guarded processes

CCS op. semantics

$$\text{Act) } \frac{}{\mu.p \xrightarrow{\mu} p} \quad \text{Res) } \frac{p \xrightarrow{\mu} q \quad \mu \notin \{\alpha, \bar{\alpha}\}}{p \setminus \alpha \xrightarrow{\mu} q \setminus \alpha} \quad \text{Rel) } \frac{p \xrightarrow{\mu} q}{p[\phi] \xrightarrow{\phi(\mu)} q[\phi]}$$

$$\text{SumL) } \frac{p_1 \xrightarrow{\mu} q}{p_1 + p_2 \xrightarrow{\mu} q} \quad \text{SumR) } \frac{p_2 \xrightarrow{\mu} q}{p_1 + p_2 \xrightarrow{\mu} q}$$

$$\text{ParL) } \frac{p_1 \xrightarrow{\mu} q_1}{p_1 | p_2 \xrightarrow{\mu} q_1 | p_2} \quad \text{Com) } \frac{p_1 \xrightarrow{\lambda} q_1 \quad p_2 \xrightarrow{\bar{\lambda}} q_2}{p_1 | p_2 \xrightarrow{\tau} q_1 | q_2} \quad \text{ParR) } \frac{p_2 \xrightarrow{\mu} q_2}{p_1 | p_2 \xrightarrow{\mu} p_1 | q_2}$$

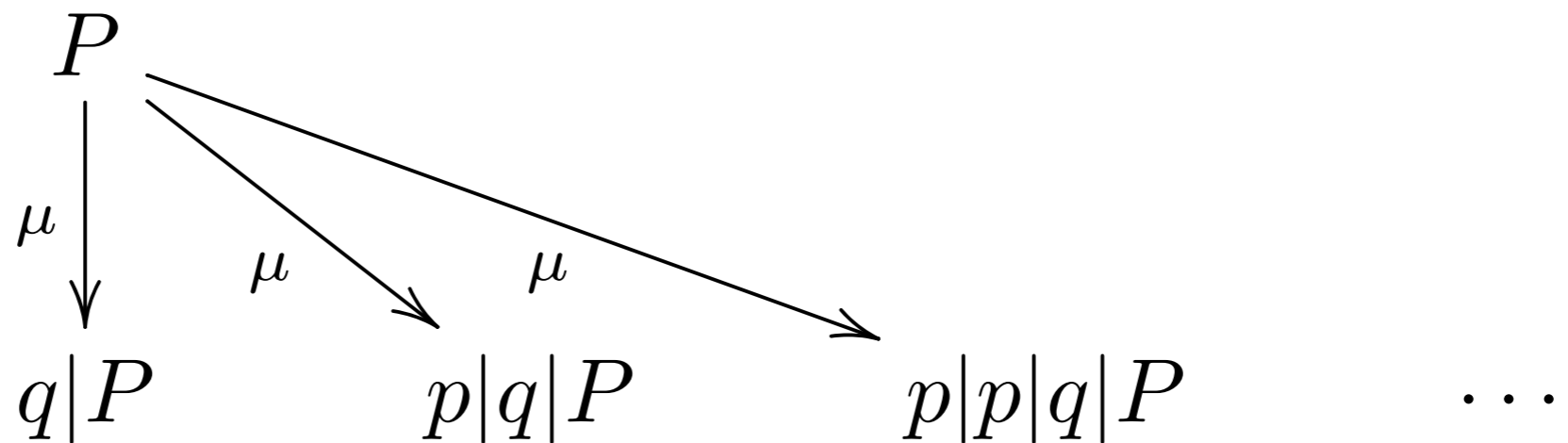
$$\text{Rec) } \frac{p[\mathbf{rec} \ x. \ p / x] \xrightarrow{\mu} q}{\mathbf{rec} \ x. \ p \xrightarrow{\mu} q}$$

CCS: guarded processes

the allowed form of recursion is very general

there are processes with infinitely many outgoing transitions

suppose $p \xrightarrow{\mu} q$ take $P \triangleq \mathbf{rec} \ x. p|x$ $P \triangleq p|P$



such processes are called *infinitely branching* and are **BAD**

CCS: guarded processes

guarded processes guarantee that process variables occur under a prefix (recursion is guarded by some action)

let X be a set of process variables

$G(p, X)$ all recursively defined names are guarded in p
if a name in X occurs free in p it is prefixed by an action

$$G(\mathbf{nil}, X) \triangleq \mathbf{true}$$

$$G(p[\phi], X) \triangleq G(p, X)$$

$$G(x, X) \triangleq x \notin X$$

$$G(p + q, X) \triangleq G(p, X) \wedge G(q, X)$$

$$G(\mu.p, X) \triangleq G(p, \emptyset)$$

$$G(p|q, X) \triangleq G(p, X) \wedge G(q, X)$$

$$G(p \setminus \alpha, X) \triangleq G(p, X) \quad G(\mathbf{rec} x. p, X) \triangleq G(p, X \cup \{x\})$$

a closed process p is *guarded* if $G(p, \emptyset)$ holds true



Exercise: guarded?

$$R \triangleq \mathbf{rec} \ x. \ \alpha.x + \beta$$

$$G(R, \emptyset)? \quad \checkmark$$

$$G(\mathbf{rec} \ x. \ \alpha.x + \beta, \emptyset) = G(\alpha.x + \beta, \{x\})$$

$$= G(\alpha.x, \{x\}) \wedge G(\beta, \{x\})$$

$$= G(x, \emptyset) \wedge G(\mathbf{nil}, \emptyset)$$

$$= x \notin \emptyset \wedge \mathbf{true}$$

$$= \mathbf{true}$$



Exercise: guarded?

$$T \triangleq \mathbf{rec} \ x. (\alpha|x) + \beta$$


$$G(T, \emptyset)? \quad \times$$

$$\begin{aligned} G(\mathbf{rec} \ x. (\alpha|x) + \beta, \emptyset) &= G((\alpha|x) + \beta, \{x\}) \\ &= G(\alpha|x, \{x\}) \wedge G(\beta, \{x\}) \\ &= G(\alpha, \{x\}) \wedge G(x, \{x\}) \wedge G(\mathbf{nil}, \emptyset) \\ &= G(\mathbf{nil}, \emptyset) \wedge x \notin \{x\} \wedge \mathbf{true} \\ &= \mathbf{true} \wedge \mathbf{false} \\ &= \mathbf{false} \end{aligned}$$



Exercise: guarded?

$U \triangleq \mathbf{rec} \ x. \ \alpha|\beta.x$

$G(U, \emptyset)?$ 

$$\begin{aligned} G(\mathbf{rec} \ x. \ \alpha|\beta.x, \emptyset) &= G(\alpha|\beta.x, \{x\}) \\ &= G(\alpha, \{x\}) \wedge G(\beta.x, \{x\}) \\ &= G(\mathbf{nil}, \emptyset) \wedge G(x, \emptyset) \\ &= \mathbf{true} \wedge x \notin \emptyset \\ &= \mathbf{true} \end{aligned}$$



Exercise: guarded?

$\text{rec } x. x$



unguarded

$\text{rec } x. \alpha. \text{rec } y. x$



guarded

$\text{rec } x. \alpha. \text{rec } y. x + y$



unguarded

$\text{rec } x. \alpha. \text{rec } y. x|y$



unguarded

$\text{rec } x. \alpha. \text{rec } y. x|\beta.y$



guarded

Properties of guarded processes

Guarded variables

TH. $G(p, X \cup \{x\}) \Rightarrow G(p, X)$

TH. $G(p, X) \wedge x \notin \text{fv}(p) \Rightarrow G(p, X \cup \{x\})$

proofs by structural induction on p (omitted)

Substitutions preserve guardedness

TH. $G(p, X) \wedge \bigwedge_{i \in [1, n]} G(p_i, X) \Rightarrow G(p[p^1 / x_1, \dots, p^n / x_n], X)$

proof by structural induction on p (omitted)

Transitions preserve guardedness

TH. $G(p, X) \wedge p \xrightarrow{\mu} q \Rightarrow G(q, \emptyset)$

proof by rule induction on $p \xrightarrow{\mu} q$ (omitted)

Guarded processes are finitely branching

TH. $G(p, \emptyset) \Rightarrow \forall \mu. \{q \mid p \xrightarrow{\mu} q\}$ is a finite set

we prove a stronger property:

TH. $X = \{x_1, \dots, x_n\} \quad \sigma = [p_1 / x_1, \dots, p_n / x_n]$

$G(p, X) \wedge \bigwedge_{i \in [1, n]} G(p_i, X) \Rightarrow \{q \mid \exists \mu. p\sigma \xrightarrow{\mu} q\}$ is a finite set

proof: by structural induction on p

we only show some cases (and omit some details)

(see next slides)

TH. $X = \{x_1, \dots, x_n\}$ $\sigma = [p_1 / x_1, \dots, p_n / x_n]$ (continue)

$G(p, X) \wedge \bigwedge_{i \in [1, n]} G(p_i, X) \Rightarrow \{q \mid \exists \mu. p\sigma \xrightarrow{\mu} q\}$ is a finite set

proof: by structural induction on p

$p = \mathbf{nil}$

$\mathbf{nil}\sigma = \mathbf{nil}$

$\{q \mid \exists \mu. \mathbf{nil} \xrightarrow{\mu} q\} = \emptyset$

$p = x$

$x \in X$ $G(x, X) = \mathbf{false}$

$x \notin X$ $x\sigma = x$ $\{q \mid \exists \mu. x \xrightarrow{\mu} q\} = \emptyset$

$p = \mu'.p'$

$\{q \mid \exists \mu. p\sigma \xrightarrow{\mu} q\} = \{p'\sigma\}$

TH. $X = \{x_1, \dots, x_n\}$ $\sigma = [p_1 / x_1, \dots, p_n / x_n]$ (continue)

$G(p, X) \wedge \bigwedge_{i \in [1, n]} G(p_i, X) \Rightarrow \{q \mid \exists \mu. p \sigma \xrightarrow{\mu} q\}$ is a finite set

proof: by structural induction on p

$p = p' \setminus \alpha$ $p \sigma = p' \sigma \setminus \alpha$ $G(p, X) = G(p', X)$

$\{q \mid \exists \mu. p \sigma \xrightarrow{\mu} q\} = \{q' \setminus \alpha \mid \exists \mu \notin \{\alpha, \bar{\alpha}\}. p' \sigma \xrightarrow{\mu} q'\}$

finite, by inductive hypothesis

$p = p'_0 + p'_1$ $p \sigma = p'_0 \sigma + p'_1 \sigma$ $G(p, X) = G(p'_0, X) \wedge G(p'_1, X)$

$\{q \mid \exists \mu. p \sigma \xrightarrow{\mu} q\} = \{q'_0 \mid \exists \mu. p'_0 \sigma \xrightarrow{\mu} q'_0\} \cup \{q'_1 \mid \exists \mu. p'_1 \sigma \xrightarrow{\mu} q'_1\}$

finite, by inductive hypotheses

TH. $X = \{x_1, \dots, x_n\}$ $\sigma = [p_1 / x_1, \dots, p_n / x_n]$ (continue)

$G(p, X) \wedge \bigwedge_{i \in [1, n]} G(p_i, X) \Rightarrow \{q \mid \exists \mu. p\sigma \xrightarrow{\mu} q\}$ is a finite set

proof: by structural induction on p

$p = \mathbf{rec} \ x.p'$ without loss of generality $x \notin X \cup \bigcup_{i \in [1, n]} \text{fv}(p_i)$

$$p\sigma = \mathbf{rec} \ x.p'\sigma$$

$$G(p, X) = G(p', X \cup \{x\})$$

$$\{q \mid \exists \mu. p\sigma \xrightarrow{\mu} q\} = \{q' \mid \exists \mu. p'\sigma[\mathbf{rec} \ x.p'\sigma / x] \xrightarrow{\mu} q'\}$$

finite, by inductive hypotheses