



PSC 2021/22 (375AA, 9CFU)

Principles for Software Composition

Roberto Bruni

<http://www.di.unipi.it/~bruni/>

<http://didawiki.di.unipi.it/doku.php/magistraleinformatica/psc/start>

20 - Weak semantics

CCS syntax

p, q	$::=$	nil	inactive process
		x	process variable (for recursion)
		$\mu.p$	action prefix
		$p \setminus \alpha$	restricted channel
		$p[\phi]$	channel relabelling
		$p + q$	nondeterministic choice (sum)
		$p q$	parallel composition
		rec $x. p$	recursion

(operators are listed in order of precedence)

CCS op. semantics

$$\text{Act) } \frac{}{\mu.p \xrightarrow{\mu} p} \quad \text{Res) } \frac{p \xrightarrow{\mu} q \quad \mu \notin \{\alpha, \bar{\alpha}\}}{p \setminus \alpha \xrightarrow{\mu} q \setminus \alpha} \quad \text{Rel) } \frac{p \xrightarrow{\mu} q}{p[\phi] \xrightarrow{\phi(\mu)} q[\phi]}$$

$$\text{SumL) } \frac{p_1 \xrightarrow{\mu} q}{p_1 + p_2 \xrightarrow{\mu} q} \quad \text{SumR) } \frac{p_2 \xrightarrow{\mu} q}{p_1 + p_2 \xrightarrow{\mu} q}$$

$$\text{ParL) } \frac{p_1 \xrightarrow{\mu} q_1}{p_1 | p_2 \xrightarrow{\mu} q_1 | p_2} \quad \text{Com) } \frac{p_1 \xrightarrow{\lambda} q_1 \quad p_2 \xrightarrow{\bar{\lambda}} q_2}{p_1 | p_2 \xrightarrow{\tau} q_1 | q_2} \quad \text{ParR) } \frac{p_2 \xrightarrow{\mu} q_2}{p_1 | p_2 \xrightarrow{\mu} p_1 | q_2}$$

$$\text{Rec) } \frac{p[\mathbf{rec} \ x. \ p / x] \xrightarrow{\mu} q}{\mathbf{rec} \ x. \ p \xrightarrow{\mu} q}$$

CCS

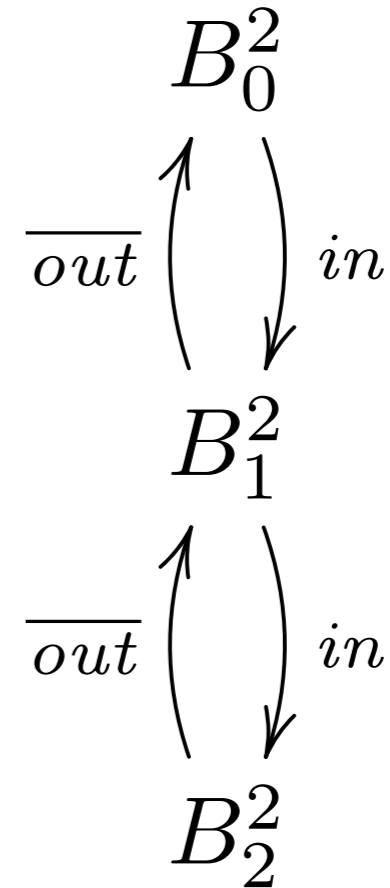
Weak transitions

Sequential buffer

$$B_0^2 \triangleq in.B_1^2$$

$$B_1^2 \triangleq in.B_2^2 + \overline{out}.B_0^2$$

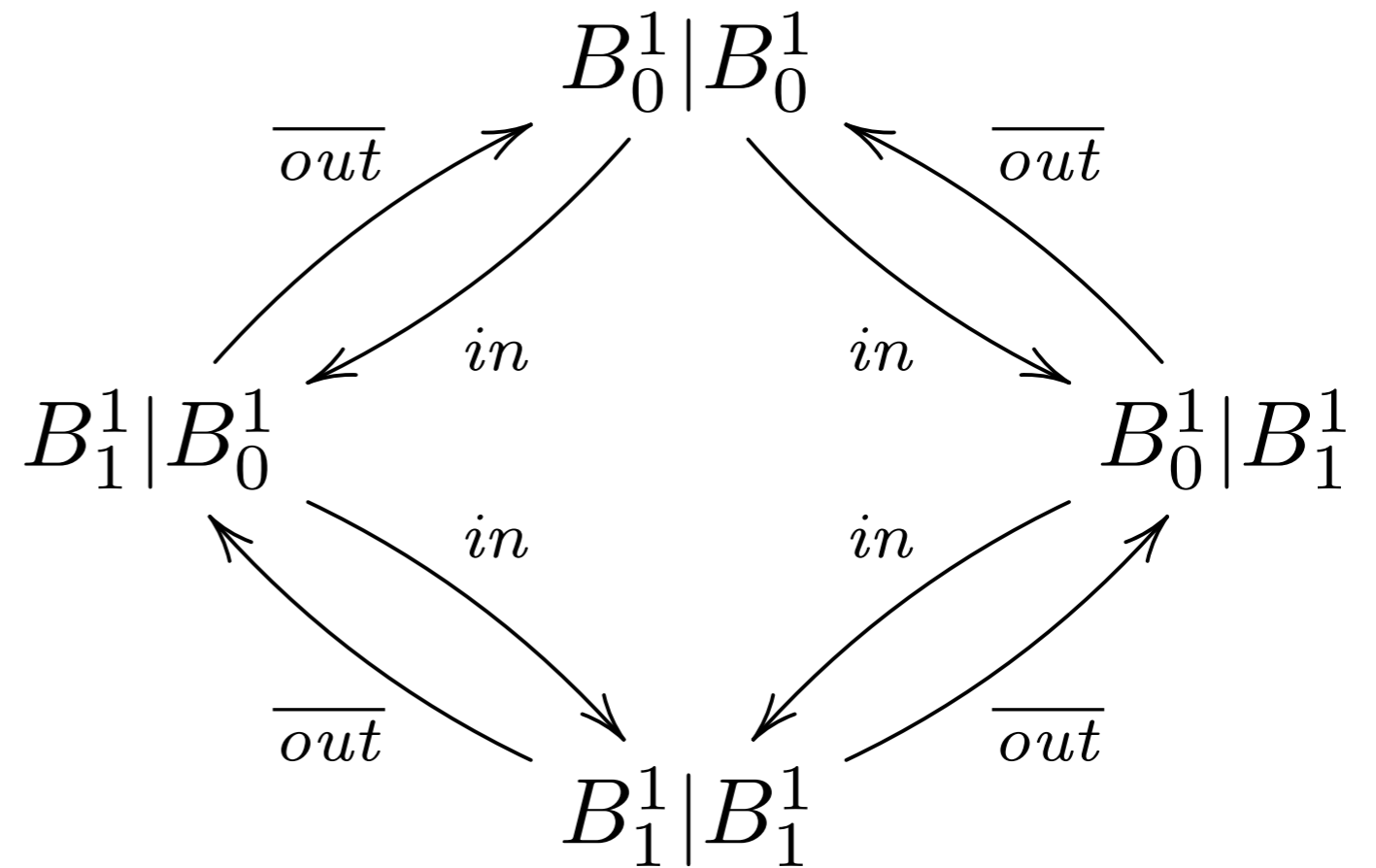
$$B_2^2 \triangleq \overline{out}.B_1^2$$



Parallel buffer

$$B_0^1 \triangleq in.B_1^1$$

$$B_1^1 \triangleq \overline{out}.B_0^1$$

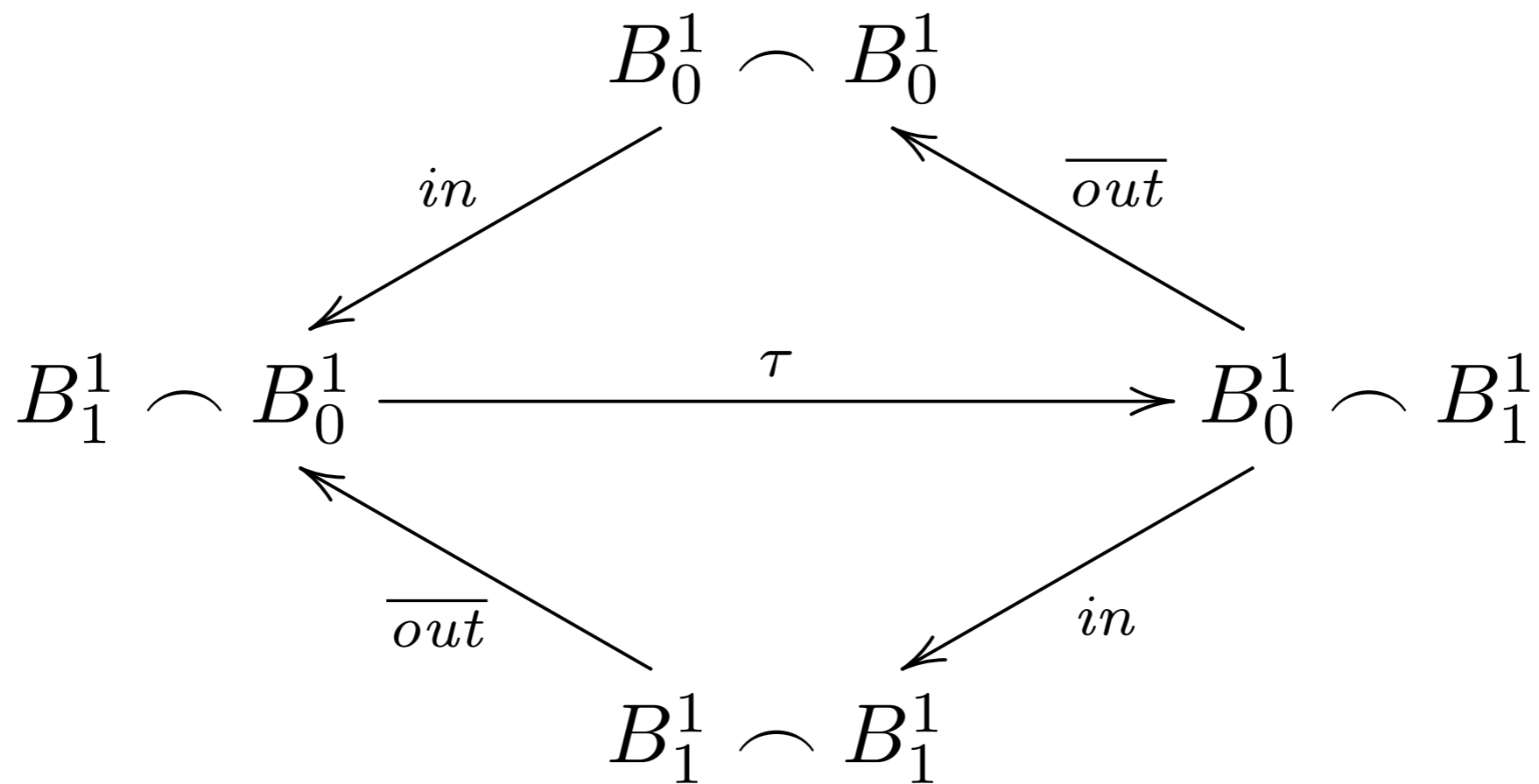


Linked buffer

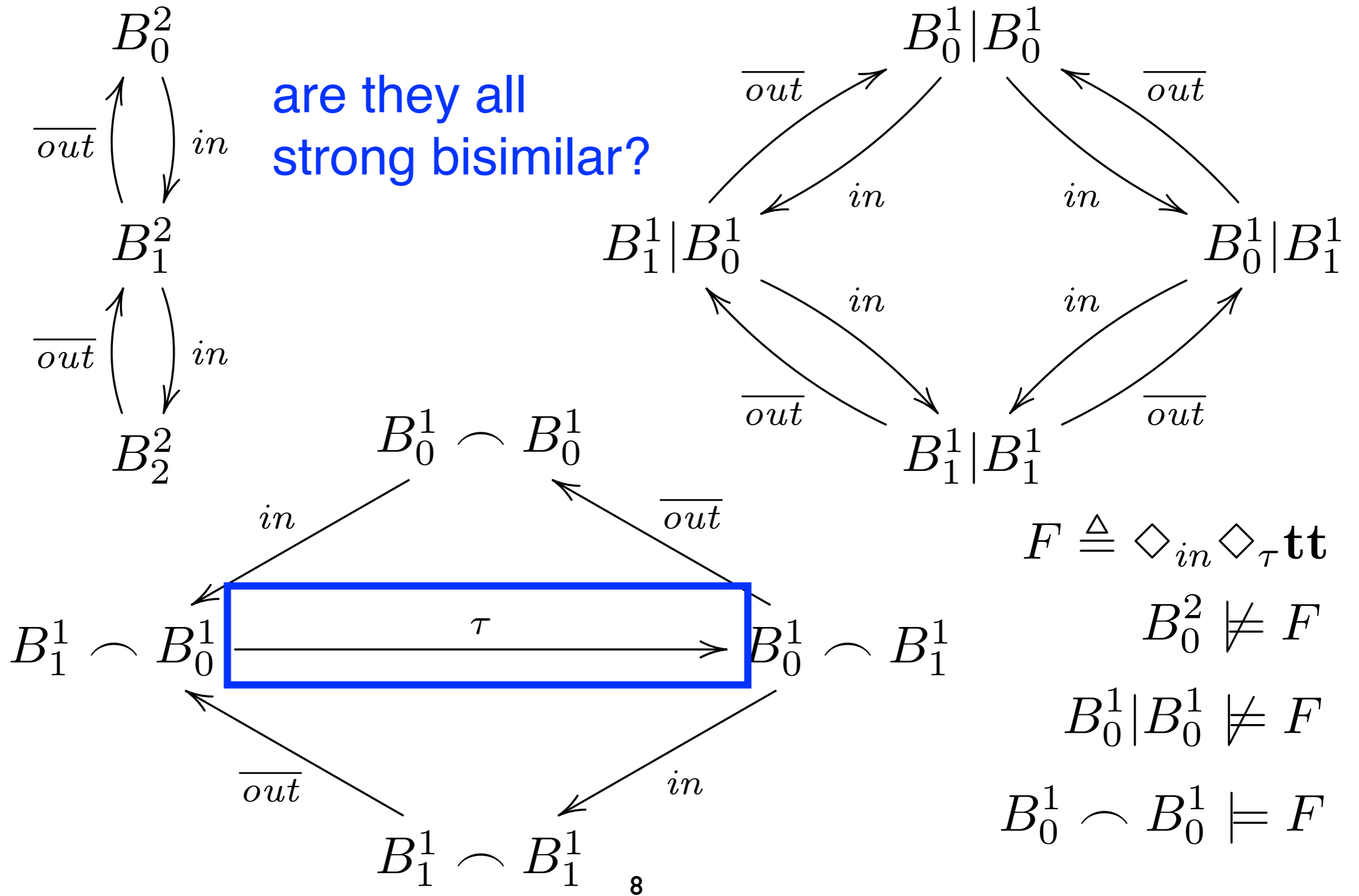
$$B_0^1 \triangleq in.B_1^1 \quad \eta(out) = c$$

$$p \frown q \triangleq (p[\eta] || q[\phi]) \setminus c$$

$$B_1^1 \triangleq \overline{out}.B_0^1 \quad \phi(in) = c$$



Comparing buffers



Silent transitions



τ -transitions are silent, non observable

they represent internal steps of the system

they can be used just for bookkeeping

can we abstract away from them?

can we find a broader equivalence?

necessary to relate an abstract specification (little use of τ)

with a concrete implementation (lots/tons of τ)

Weak bisimulation game

coarser equivalence: more power to the defender!

Alice picks a process and an ordinary transition

Bob replies possibly using many additional silent transitions

arbitrarily many, but finitely many

such sequences are called *weak* transitions

$$p \xRightarrow{\mu} q$$

what if Alice picks a silent transition?

Bob can just leave the other process idle

i.e. can choose not to move

Weak transitions

$$p \xRightarrow{\tau} q \quad \text{iff} \quad p \left(\xrightarrow{\tau} \right)^* q$$

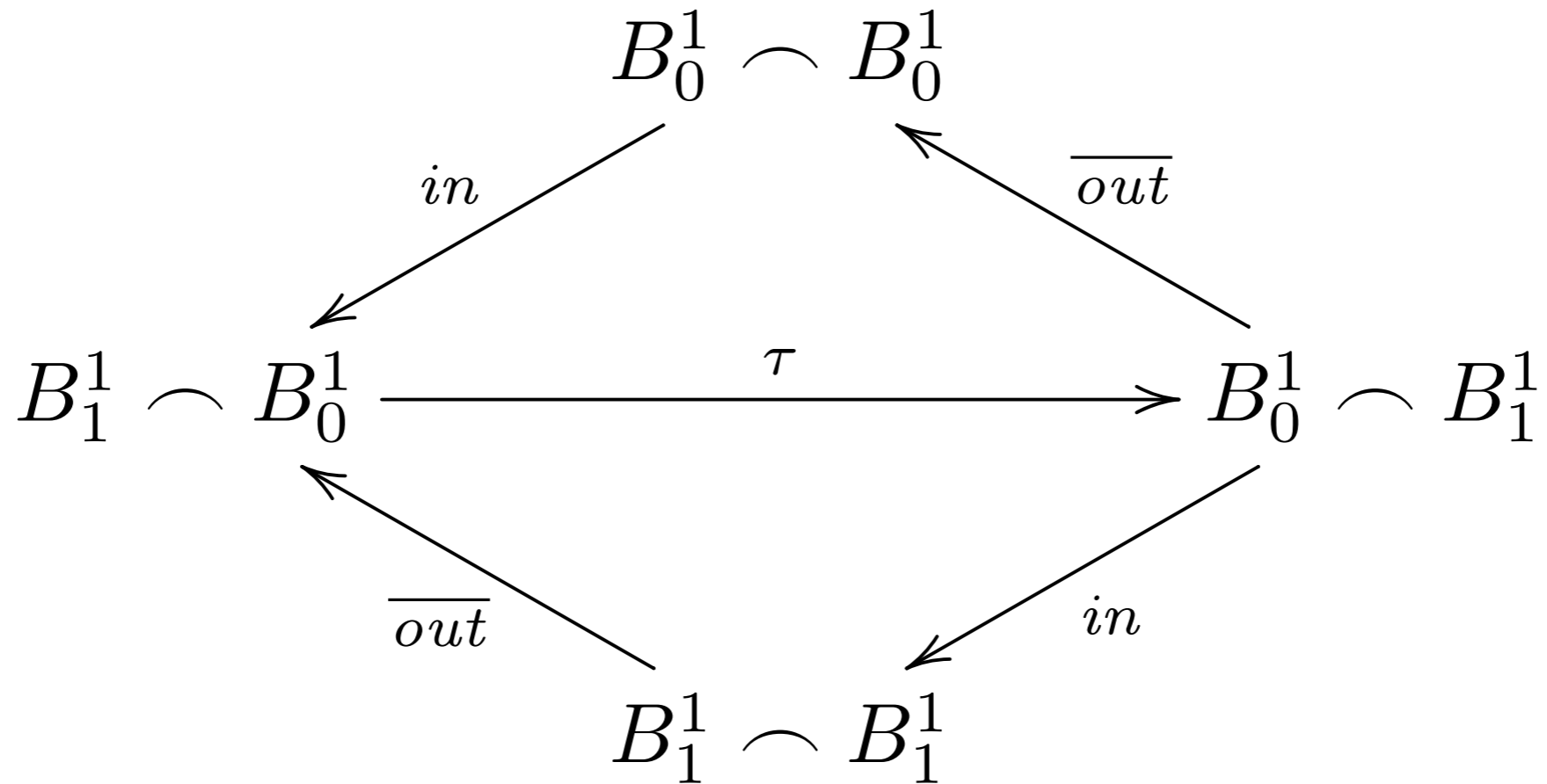
$$p = q \vee p \xrightarrow{\tau} \dots \xrightarrow{\tau} q$$

p can reach q via a (possibly empty) finite sequence of τ -transitions

$$p \xRightarrow{\lambda} q \quad \text{iff} \quad \exists p', q'. p \xRightarrow{\tau} p' \xrightarrow{\lambda} q' \xRightarrow{\tau} q$$

p can reach q via a λ -transition possibly preceded and followed by empty/finite sequences of τ -transitions

Example



$$B_0^1 \frown B_0^1 \xRightarrow{\tau} B_0^1 \frown B_0^1$$

$$B_0^1 \frown B_0^1 \xRightarrow{in} B_0^1 \frown B_1^1$$

$$B_1^1 \frown B_0^1 \xRightarrow{\overline{out}} B_0^1 \frown B_0^1$$

CCS

weak bisimulation

Weak bisimulation

\mathbf{R} is a *weak* bisimulation if

$$\forall p, q. (p, q) \in \mathbf{R} \Rightarrow \left\{ \begin{array}{l} \forall \mu, p'. p \xrightarrow{\mu} p' \Rightarrow \exists q'. q \xRightarrow{\mu} q' \wedge p' \mathbf{R} q' \\ \wedge \text{ Alice plays} \\ \forall \mu, q'. q \xrightarrow{\mu} q' \Rightarrow \exists p'. p \xRightarrow{\mu} p' \wedge p' \mathbf{R} q' \end{array} \right.$$

weak transitions

Weak bisimilarity

weak bisimilarity:

$p \approx q$ iff $\exists \mathbf{R}$ a weak bisimulation with $(p, q) \in \mathbf{R}$

TH. weak bisimilarity is an equivalence relation

TH. any strong bisimulation is a weak bisimulation

Cor. strong bisimilarity implies weak bisimilarity

Weaker bisimilarity?

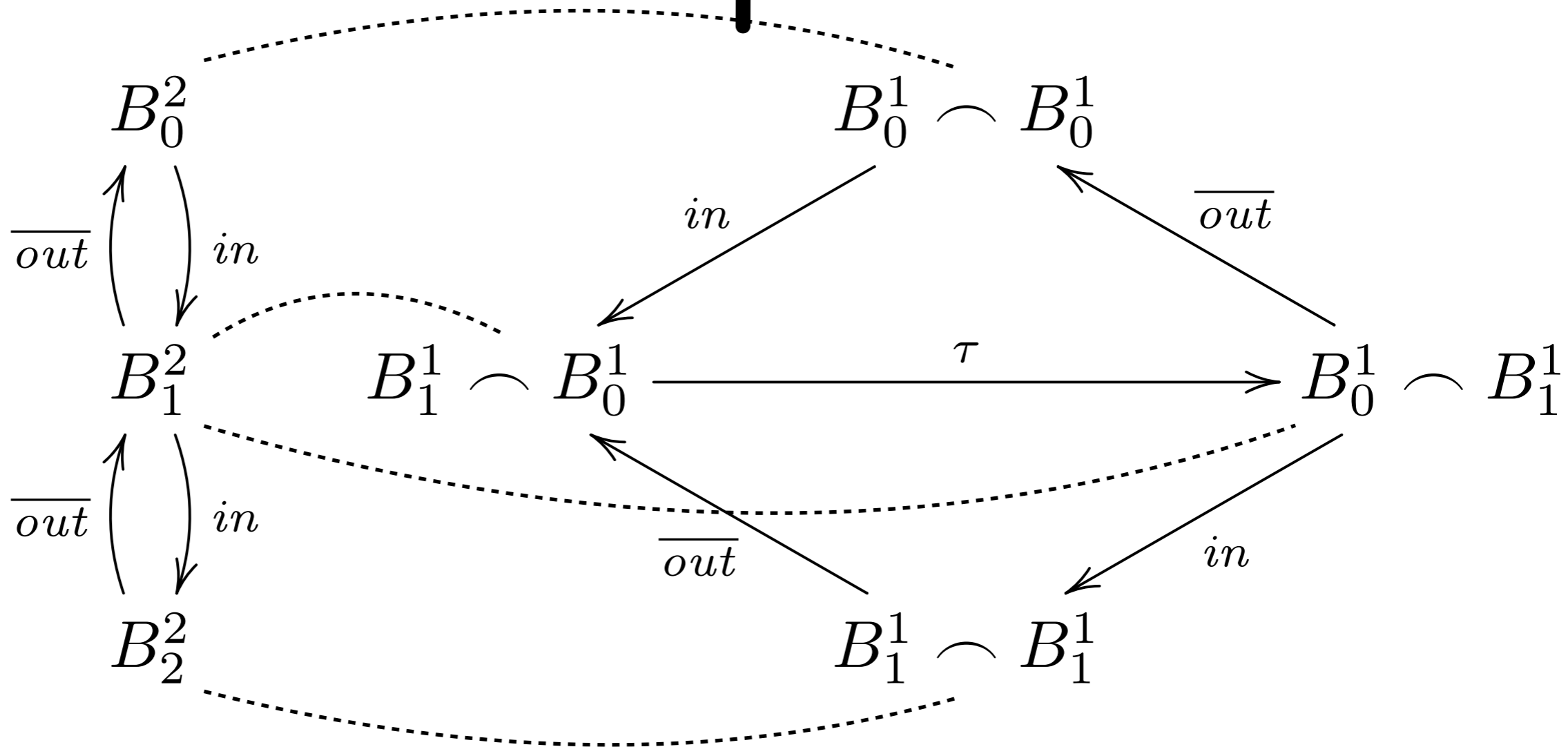
what if we give extra power to Alice as well?

$$\forall p, q. (p, q) \in \mathbf{R} \Rightarrow \left\{ \begin{array}{l} \forall \mu, p'. p \xRightarrow{\mu} p' \Rightarrow \exists q'. q \xRightarrow{\mu} q' \wedge p' \mathbf{R} q' \\ \wedge \text{ Alice plays} \\ \forall \mu, q'. q \xRightarrow{\mu} q' \Rightarrow \exists p'. p \xRightarrow{\mu} p' \wedge p' \mathbf{R} q' \\ \text{ Bob replies} \end{array} \right.$$

weak transitions

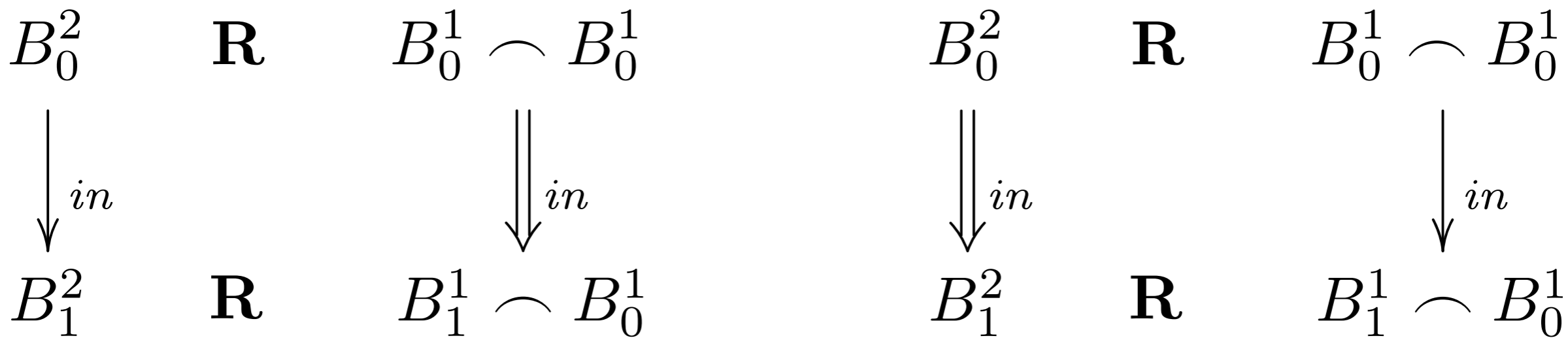
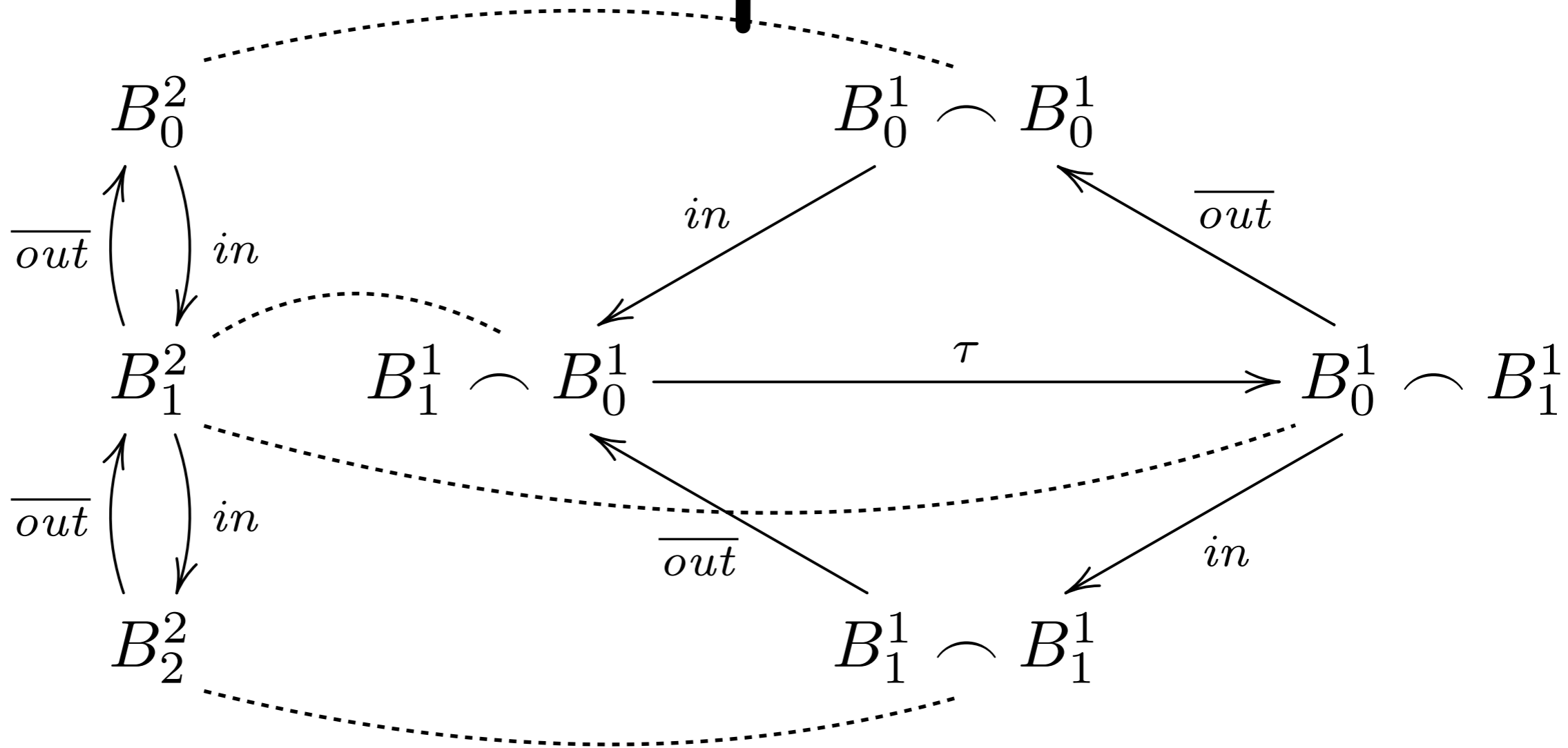
nothing changes: we still get the same weak bisimilarity

Example

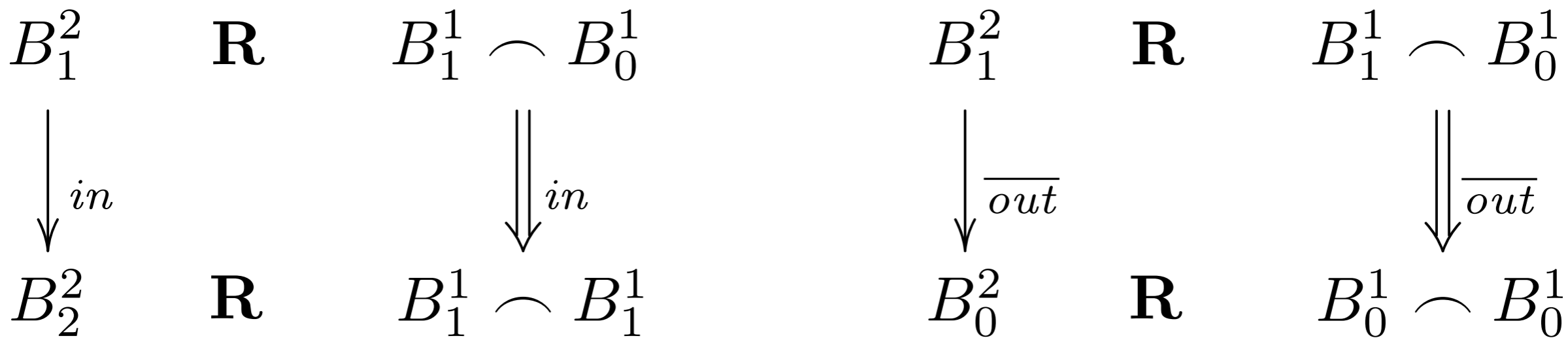
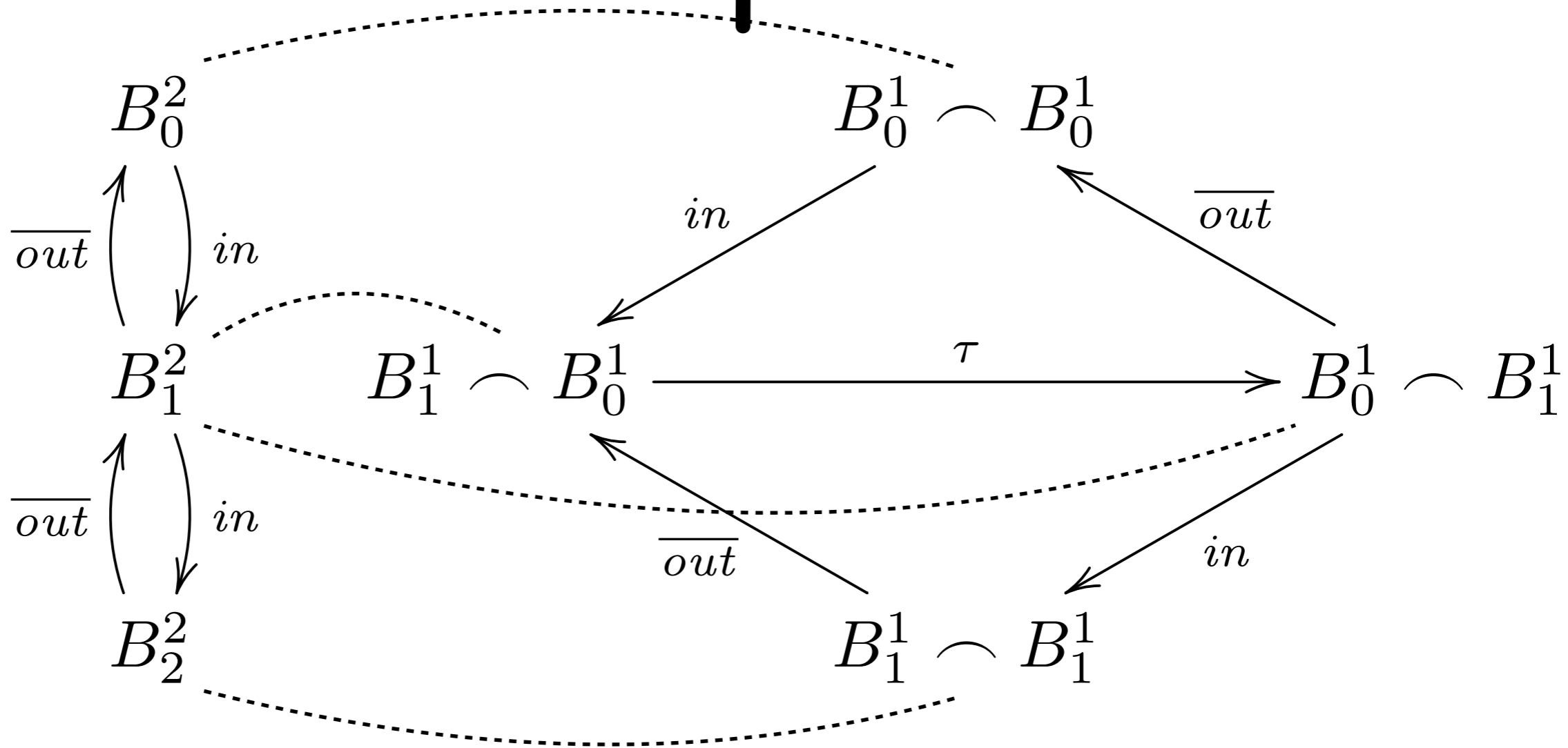


$\mathbf{R} \triangleq \left\{ \begin{array}{l} (B_0^2, B_0^1 \curvearrowright B_0^1), \\ (B_1^2, B_1^1 \curvearrowright B_0^1), \\ (B_1^2, B_0^1 \curvearrowright B_1^1), \\ (B_2^2, B_1^1 \curvearrowright B_1^1) \end{array} \right\}$ is a weak bisimulation relation

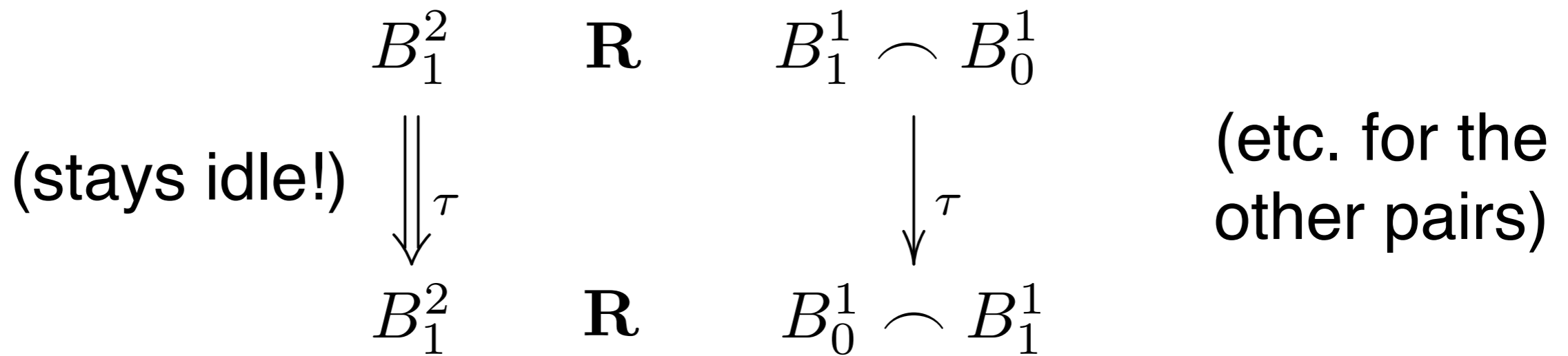
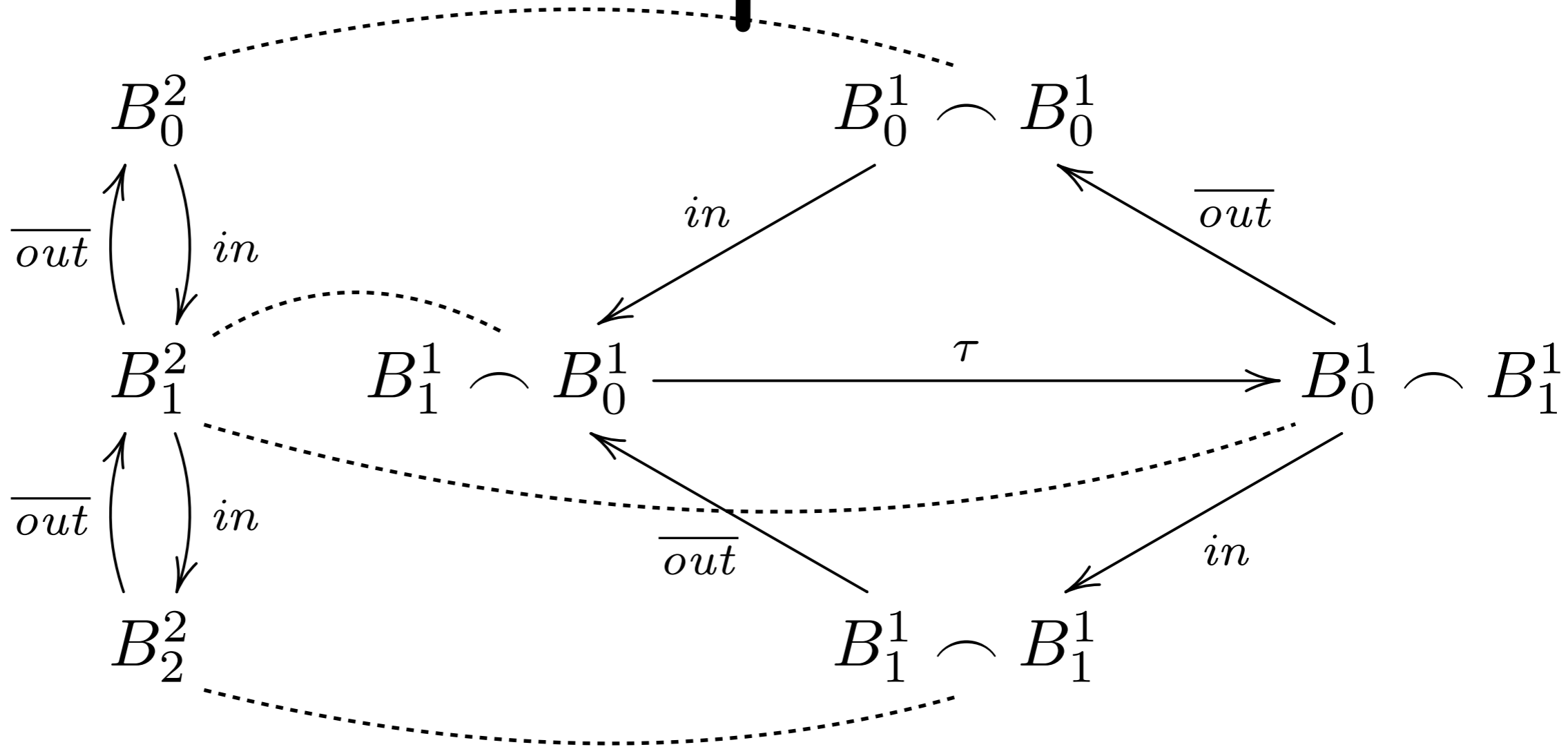
Example



Example



Example



Weak bis as a fixpoint

$$\Psi(\mathbf{R}) \triangleq \left\{ (p, q) \mid \begin{array}{l} \forall \mu, p'. p \xrightarrow{\mu} p' \Rightarrow \exists q'. q \xRightarrow{\mu} q' \wedge p' \mathbf{R} q' \\ \wedge \\ \forall \mu, q'. q \xrightarrow{\mu} q' \Rightarrow \exists p'. p \xRightarrow{\mu} p' \wedge p' \mathbf{R} q' \end{array} \right\}$$

$$\Psi : \wp(\mathcal{P} \times \mathcal{P}) \rightarrow \wp(\mathcal{P} \times \mathcal{P})$$

maps relations to relations

$$\mathbf{R} \subseteq \Psi(\mathbf{R})$$

a weak bisimulation

$$\approx = \Psi(\approx)$$

weak bisimilarity is a fixpoint

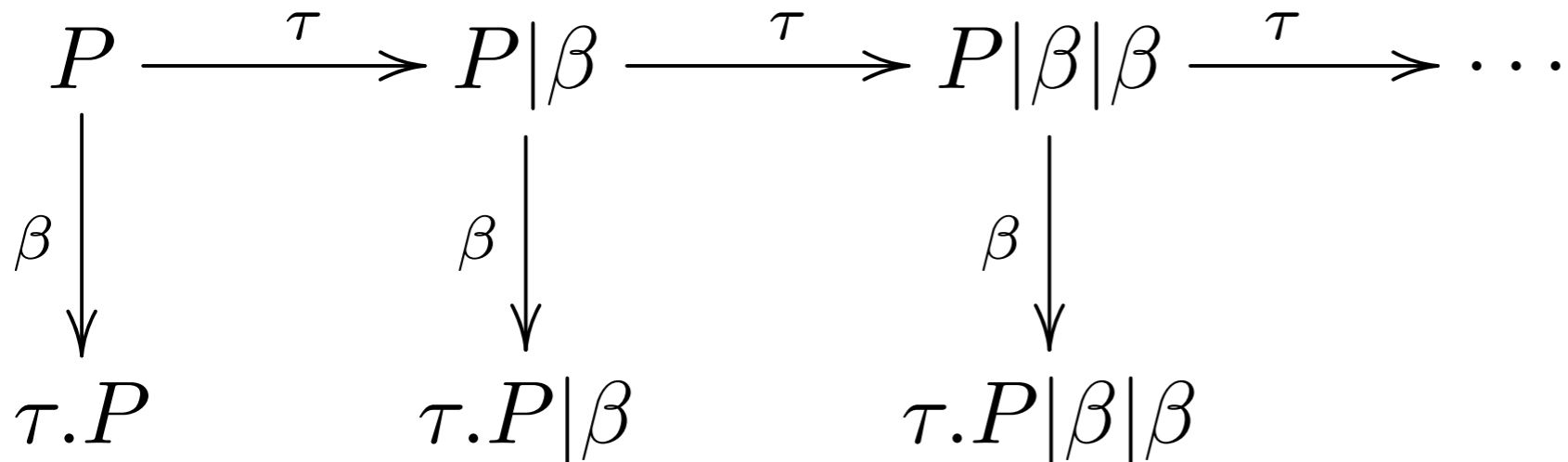
CCS

problems with weak semantics

Problems with weak bis

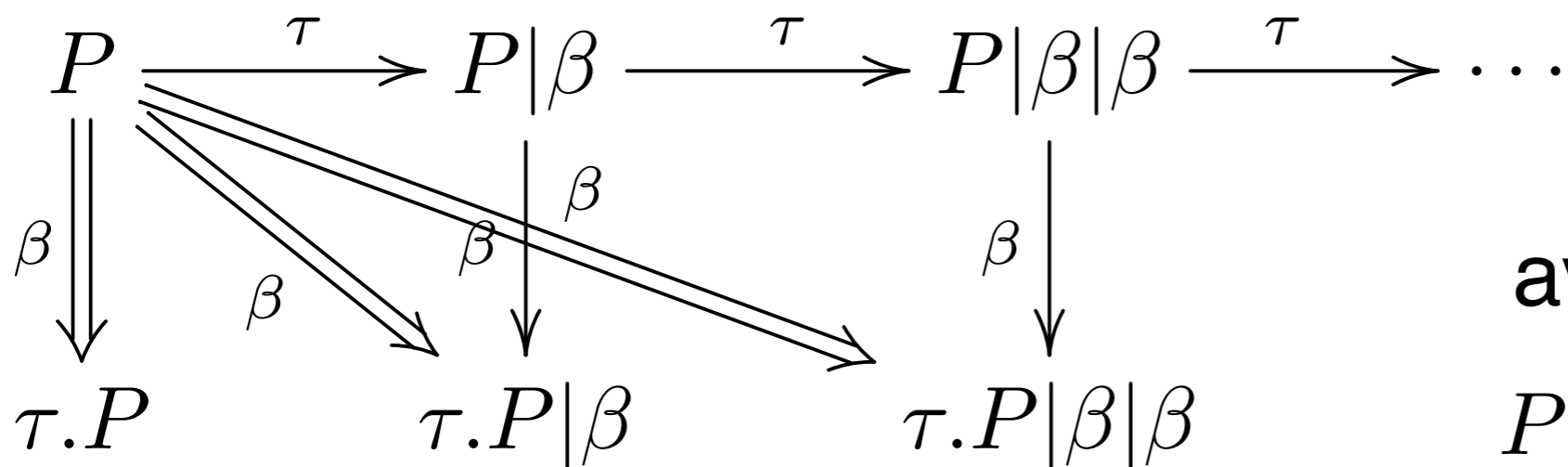
with respect to weak transitions,
 guarded processes can have infinitely branching LTS

$$P \triangleq \mathbf{rec} \ x. \ \tau.x|\beta$$



many arrows omitted

assume $p|\mathbf{nil} = p$



avoid τ -prefixes?

$$P \triangleq \mathbf{rec} \ x. \ (\alpha.x|\bar{\alpha}|\beta)\backslash\alpha$$

Problems with weak bis

weak bisimilarity is not a congruence (w.r.t. +)

take $P \triangleq \alpha$ $Q \triangleq \tau.\alpha$

if $P \xrightarrow{\alpha} \text{nil}$ then $Q \xRightarrow{\alpha} \text{nil}$

if $Q \xrightarrow{\tau} \alpha$ then $P \xRightarrow{\tau} P$

$P \approx Q$ $\mathbb{C}[P] \not\approx \mathbb{C}[Q]$

take the context

$\mathbb{C}[\cdot] \triangleq [\cdot] + \beta$

$\mathbb{C}[P] \triangleq \alpha + \beta$

$\mathbb{C}[Q] \triangleq \tau.\alpha + \beta$

Alice plays

$\mathbb{C}[Q] \xrightarrow{\tau} \alpha$

Bob can only reply

$\mathbb{C}[P] \xRightarrow{\tau} \mathbb{C}[P]$

Alice plays

$\mathbb{C}[P] \xrightarrow{\beta} \text{nil}$

Bob cannot reply

$\alpha \not\xrightarrow{\beta}$

Alice wins!

Problems with weak bis

cannot distinguish between deadlock
and silent divergence

$$\mathbf{rec } x. \tau.x \approx \mathbf{nil}$$

$$\mathbf{rec } x. \tau.x \xrightarrow{\tau} \mathbf{rec } x. \tau.x \quad \mathbf{nil} \xRightarrow{\tau} \mathbf{nil}$$

CCS

weak observational congruence

Weak obs congruence

$$p \cong q \quad \text{iff} \quad p \approx q \wedge \forall r. p + r \approx q + r$$

Equivalently

$$p \cong q \quad \text{iff} \quad \left\{ \begin{array}{l} \forall p'. p \xrightarrow{\tau} p' \quad \Rightarrow \quad \exists q', q''. q \xrightarrow{\tau} q'' \xRightarrow{\tau} q' \wedge p' \approx q' \\ \forall \lambda, p'. p \xrightarrow{\lambda} p' \quad \Rightarrow \quad \exists q'. q \xRightarrow{\lambda} q' \wedge p' \approx q' \\ \text{and vice versa} \end{array} \right.$$

not a recursive definition!
(refers to weak bisimilarity)

at the level of bisimulation game:

Bob is not allowed to use an idle move at the very first turn
(at the following turns, ordinary weak bisimulation game)

TH. \cong is the largest congruence contained in \approx

Weak obs congruence

Note: \approx is not a weak bisimulation!

$$P \triangleq \alpha$$

$$Q \triangleq \tau.\alpha$$

$$\beta.P$$

$$\approx$$

$$\beta.Q$$

$$\beta \downarrow$$

$$\beta \downarrow$$

$$P$$

$$\approx$$

$$Q$$

$$P \not\approx Q$$

$$\approx \not\subseteq \Psi(\approx)$$

Weak obs congruence

All the laws for strong bisimilarity are still valid

Additionally: Milner's τ -laws

$$p + \tau.p \cong \tau.p$$

$$\mu.(p + \tau.q) \cong \mu.(p + \tau.q) + \mu.q$$

$$\mu.\tau.p \cong \mu.p$$