Principles for software composition 2017/18

Exam – June 13, 2018

[Ex. 1] (regular exam only)

Prove by rule induction that $\forall a \in \mathbf{Aexp}, \sigma, m.\ P(\langle a, \sigma\rangle \to m)$, where:

$$P(\langle a, \sigma\rangle \to m) \stackrel{\text{def}}{=} \forall \sigma', k.\ (\ \langle a, \sigma'\rangle \to k \neq m \Rightarrow \exists y.\ \sigma'(y) \neq \sigma(y)\ )$$

[Ex. 2] (regular exam only)

Consider the domain $(\mathbb{N}_\perp^\infty, \sqsubseteq)$, where $\mathbb{N}_\perp^\infty = \mathbb{N} \cup \{\perp, \infty\}$ and

$$\forall x, y \in \mathbb{N}_\perp^\infty.\ x \sqsubseteq y \Leftrightarrow (x = y \vee x = \perp \vee y = \infty)$$

1. Prove that $(\mathbb{N}_\perp^\infty, \sqsubseteq)$ is a CPO with bottom.
2. Complete the definitions of the functions $f, g, h : \mathbb{N}_\perp^\infty \to \mathbb{N}_\perp^\infty$ to guarantee that they are monotone and continuous, then compute their fix.
   (a) $f(\perp) \stackrel{\text{def}}{=} 0, \quad \forall n \in \mathbb{N}.\ f(n) \stackrel{\text{def}}{=} \ldots, \quad f(\infty) \stackrel{\text{def}}{=} 0$
   (b) $g(\perp) \stackrel{\text{def}}{=} 0, \quad \forall n \in \mathbb{N}.\ g(n) \stackrel{\text{def}}{=} \infty, \quad g(\infty) \stackrel{\text{def}}{=} \ldots$
   (c) $h(\perp) \stackrel{\text{def}}{=} \ldots, \quad \forall n \in \mathbb{N}.\ (h(2n) \stackrel{\text{def}}{=} \infty,\ h(2n+1) \stackrel{\text{def}}{=} 2n), \quad h(\infty) \stackrel{\text{def}}{=} \ldots$

[Ex. 3] (regular exam / 2nd mid-term)

Consider the HOFL term

$$t \stackrel{\text{def}}{=} \mathbf{rec}\ f.\ \lambda x.\ \mathbf{if}\ x\ \mathbf{then}\ f\ x\ \mathbf{else}\ f\ (f\ x)$$

1. Find the principal type of t.
2. Show that the term $(t\ 0)$ diverges operationally.
3. Compute the (lazy) denotational semantics of t.

[Ex. 4] (regular exam / 2nd mid-term)

A teacher asks three types of questions: Easy, Medium, Difficult. She never asks two difficult questions in a row and an easy question never follows a medium question. After asking a difficult question, with probability $\frac{2}{3}$ the next question will be medium. In all the other cases, the type of the next question is chosen with equal probability among the alternatives.

1. Model the system as a DTMC.
2. What is the long term chance that the teacher asks an easy question?
3. If the first question is medium, what is the probability that the third question will be difficult?

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[Ex. 5] (2nd mid-term only)
Write a GoogleGo function `Scale` that takes an integer `k` and a channel `c` for passing integers and returns a channel `s` where all integers received on `c` will be sent after having been scaled by the factor `k`. When `c` is closed, channel `s` must be closed. When writing the function, type the channels according to their usages.

[Ex. 6] (2nd mid-term only)
Let us consider the CCS processes

$$p \stackrel{\text{def}}{=} \mathbf{rec}\ x.(\alpha.x + \overline{\alpha}.\mathbf{nil}) \qquad r \stackrel{\text{def}}{=} (p|q)\backslash\alpha$$
$$q \stackrel{\text{def}}{=} \mathbf{rec}\ y.(\overline{\alpha}.y + \alpha.\mathbf{nil}) \qquad s \stackrel{\text{def}}{=} \mathbf{rec}\ z.(\tau.\tau.z + \tau.z + \tau.\mathbf{nil})$$

1. Draw the LTSs of the processes r and s.
2. Show that r and s are not strong bisimilar.
3. Prove that r and s are weak bisimilar.