Principles for software composition 2018/19
Exam – July 23, 2019

[Ex. 1]
Let us consider expressions of the form $e ::= n \mid e?e$ with $n \in \mathbb{N}$, whose operational semantics is defined by the rules:

$$\frac{}{n \to n} \qquad \frac{e_1 \to n_1 \quad e_2 \to n_2}{e_1?e_2 \to n_1 + n_2} \qquad \frac{e_1 \to n_1 \quad e_2 \to n_2}{e_1?e_2 \to n_1 \times n_2}$$

1. Prove by structural induction that the operational semantics is terminating.
2. Show that the operational semantics is not deterministic.

[Ex. 2]
Consider the HOFL term

$$t \stackrel{\text{def}}{=} \mathbf{rec}\ f.\ \lambda x.\ \mathbf{if}\ x\ \mathbf{then}\ f\ x\ \mathbf{else}\ f\ 0$$

1. Find the principal type of t.
2. Show that the term $t\ 0$ diverges operationally.
3. Find the denotational semantics of t.

[Ex. 3]
Let us consider the CCS processes

$$p \stackrel{\text{def}}{=} \mathbf{rec}\ x.\ \tau.(x \mid \beta.\mathbf{nil}) \qquad q \stackrel{\text{def}}{=} \mathbf{rec}\ y.\ \tau.\beta.y + \tau.y$$

1. Draw (at least in part) the LTSs of the processes p and q assuming the usual laws $r|\mathbf{nil} = r$ and $(r|s)|t = r|(s|t)$.
2. Show that p and q are not strongly bisimilar.
3. Show that p and q are weakly bisimilar.

[Ex. 4]
A printing device has three states: *working*, *faulty*, *cleaning*. When it is *working* it remains in state *working* with probability 1/2 and changes state to *faulty* or *cleaning* with equal probability. Similarly, when it is *cleaning* it remains in state *cleaning* with probability 1/2 and changes state to *faulty* or *working* with equal probability. When it is *faulty* it remains *faulty* with probability 1/3 or otherwise enters the *cleaning* state.

1. Represent the system as a DTMC.
2. If the DTMC is ergodic, find the steady state distribution.
3. If there is a fault, what is the probability that the device will be working after three instants of time.