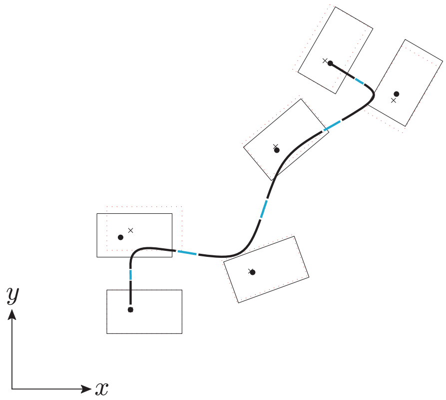

Fig. 4.31 A walking trajectory with double support periods. The CoM trajectories while double support phase are indicated by gray lines. The same parameters as were used in Fig. 4.29 are used. $z_c = 0.8, T_{sup} = 0.7, T_{dbl} = 0.1$,weights $a = 10, b = 1$.

It should be noted that while longer period of double support results smoother support exchange, it also requires undesirable quicker swing leg motion. Therefore we have a trade-off in determining T_{dbl}.

4.3.5 From Linear Inverted Pendulum to Multi-body Model

The easiest way to generate a walking pattern by using the linear inverted pendulum is to let the pelvis link follow the CoM motion of LIP. First, the real position of the CoM is calculated using a multi-body model and its position with respect to the pelvis frame is determined. After that, the position of the pelvis link is directly determined from the liner inverted pendulum assuming that the relative position of the CoM is kept constant with respect to the pelvis. In addition, we must calculate the swing foot trajectory so that it arrives the desired foot place at the specified time of touchdown.

Once we determine the trajectories for the pelvis and the both feet, the leg joint angles can be obtained by inverse kinematics as explained in Chapter 2.

This method is based on an assumption that the multi-body dynamics of the robot can be approximated by a simple inverted pendulum and its validity can be confirmed by using ZMP described in the former chapter. By calculating ZMP using multi-body model, we can evaluate the effects of swing leg reaction and errors in CoM position which were neglected in a linear inverted pendulum. Figure 4.32 shows two ZMPs, one based on the linear inverted pendulum, and one based on multi-body dynamics and the proposed

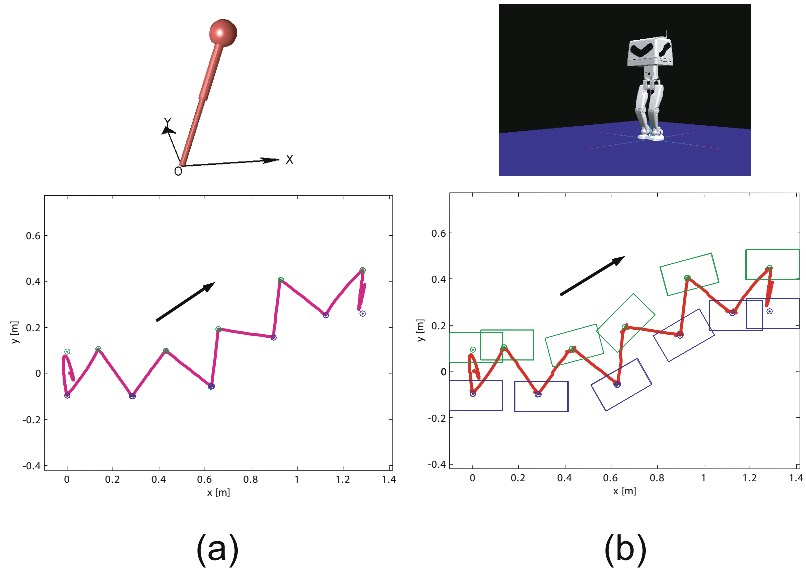

Fig. 4.32 Comparison of ZMP trajectory (a) ZMP calculated from 3D linear inverted pendulum model (b) ZMP calculated from multi-body dynamics whose pelvis link moves as 3D-LIP

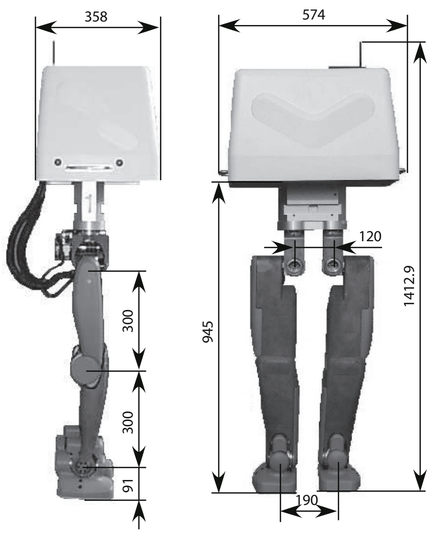

Fig. 4.33 Biped robot HRP-2L

pattern generation. Both of ZMPs are sufficiently close, hence we can conclude a multi-body dynamics can be simplified as a simple inverted pendulum in this case.

4.3.6 Implementation Example

Let us see an implementation of the proposed walking pattern generation. Figure 4.33 shows a biped robot HRP-2L which was developed in "Humanoid Robotics Project"(HRP). This robot was built to evaluate the leg part of HRP-2, the humanoid robot which was the final goal of the project. Each leg has six degrees of freedom and the robot is equipped with a Pentium II

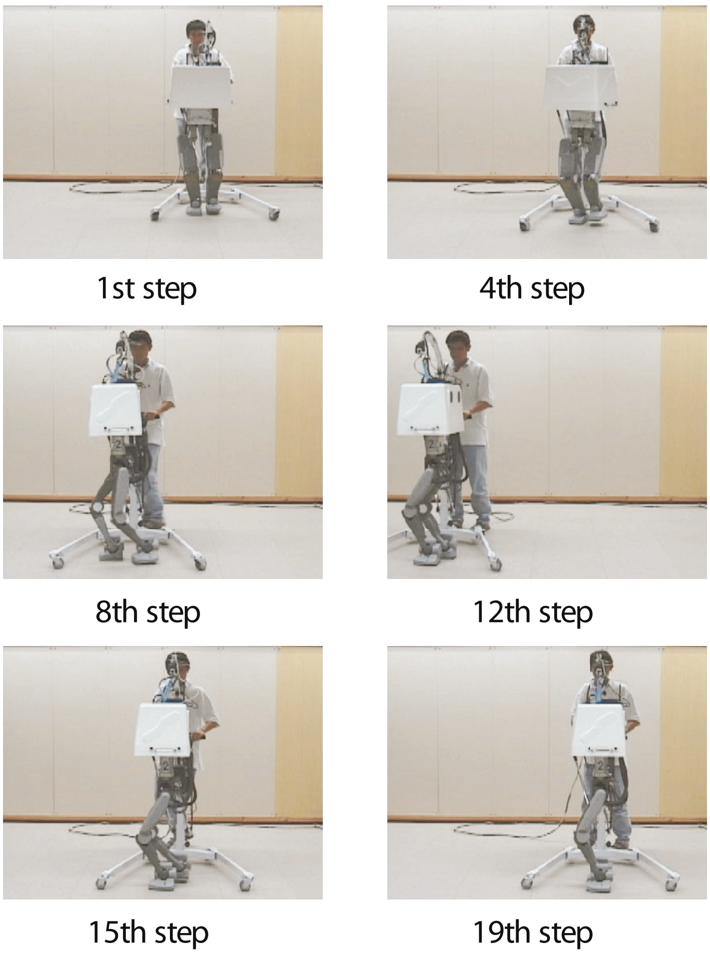

Fig. 4.34 Snapshots of real-time walking control

933MHz based on-board computer on its body part. The total weight is 58.2 [kg] including batteries of 11.4 [kg] and dummy weights of 22.6 [kg] which emulates upper body.

The algorithm of Fig. 4.25 can generate a walking pattern where at least two future steps were given. So we could build a walking control system which allows real-time step modification by specifying the walk parameter of two steps in future (s_x, s_y, s_θ) with a joystick. Figure 4.34 shows snapshots of our experiment of real-time walking control.

4.4 ZMP Based Walking Pattern Generation

4.4.1 Cart-Table Model

Let us think about a new model illustrated in Fig. 4.35. Here, a cart with mass M runs on a table whose mass is negligibly small. Although the table foot is too small to keep balance having a cart on the edge of the table, it can still keep an instantaneous balance if the cart runs with certain acceleration. We call this a *cart-table model*.

Since a cart-table model corresponds the case of a single mass at constant height in section 3.5.2, the ZMP is given as

$$p = x - \frac{z_c}{g}\ddot{x}. \tag{4.64}$$

We call this equation a *ZMP equation*.

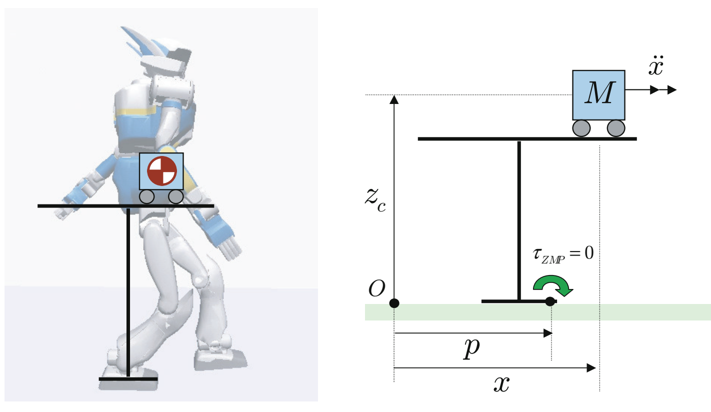

Fig. 4.35 Cart-table model: Dynamics of walking robot is approximated as a cart running on a massless table. The state of the running cart determines the center of pressure which acts from the floor, in other words, the cart changes ZMP.

On the other hand, the equation of linear inverted pendulum was given as following (Fig. 4.24).

$$\ddot{x} = \frac{g}{z_c}(x - p). \tag{4.65}$$

By regarding p as ZMP and not a foot place point as we did previously, we can treat a robot applying ankle torque and a robot in double support phase in a unified manner [134]. Moreover, we can see that (4.64) and (4.65) are the same equations with different outlooks.

A linear inverted pendulum model and a cart-table model are compared in Fig. 4.36. In a linear inverted pendulum model, the CoM motion is generated by the ZMP (Fig. 4.36(a)), and in a cart-table model, the ZMP is generated by the CoM motion (Fig. 4.36(b)). Therefore, these two models have opposite input-output causality.

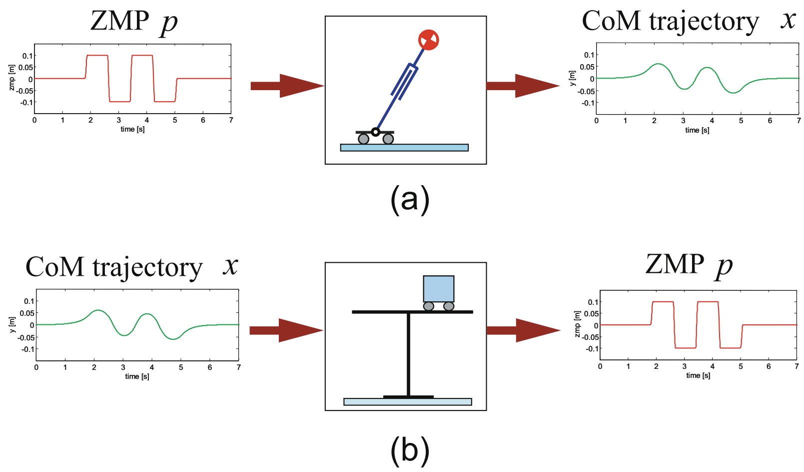

Fig. 4.36 Comparison of two models for relationship between ZMP and CoM (a) A linear inverted pendulum model inputs ZMP and outputs CoM motion. (b) A cart-table model inputs CoM motion and outputs ZMP.

As we described in the former section, a method based on a linear inverted pendulum assumes input-output relationship of Fig. 4.36(a) and the walking pattern is calculated in the following process.

(Specify target CoM motion) ⇒ (Calculate appropriate ZMP)

In this case, it is difficult to plan ZMP as expected. Indeed, we have modified the ZMP (support foot placement) in the method of the previous section.

Now, let us consider a walking pattern generation based on a cart-table model. In this case, assuming the causal relationship of Fig. 4.36(b), we calculate a walking pattern by the following manner[12].

(Specify target ZMP trajectory) $\Rightarrow$ (Calculate appropriate CoM motion)

As the result, we can obtain a walking pattern which realizes the specified ZMP trajectory. Let us call such a method *ZMP based walking pattern generation.*

4.4.2 Off-Line Walking Pattern Generation

ZMP based walking pattern generation was first proposed by Vukobratović and Stepanenko in their paper published in 1972 [90], but their algorithm takes considerable computation time. Then, Takanishi et al. proposed a practical method which transforms the target ZMP pattern into a Fourier series by using FFT, solve the ZMP equation (4.64) in the frequency domain and obtains the CoM trajectory by using inverse FFT [11][13]. A pattern generator based on this method played a particularly important role in the early stage of the Humanoid Robotics Project.

In this section, we introduce a fast and efficient algorithm that was recently proposed by Nishiwaki et al. [114][14] Let us discretize the ZMP equation with a sampling time Δt. For this purpose, the acceleration $\ddot{x}$ is approximated as

$$\ddot{x}_i = \frac{x_{i-1} - 2x_i + x_{i+1}}{\Delta t^2}, \tag{4.66}$$

where $x_i \equiv x(i\Delta t)$. Using this approximation, the discretized ZMP equation is

$$\begin{aligned} p_i &= a x_{i-1} + b x_i + c x_{i+1}, \\ a_i &\equiv -z_c/(g\Delta t^2), \\ b_i &\equiv 2z_c/(g\Delta t^2) + 1, \\ c_i &\equiv -z_c/(g\Delta t^2). \end{aligned} \tag{4.67}$$

Putting the equations (4.67) in a column for the period of the specified $(1 \ldots N)$, and representing them as a single matrix equation gives

[12] There exist an infinite numbers of possible CoM motions which realize the given ZMP trajectory, however, almost all of them suffer divergence. Fig. 4.36(b) can be regarded as a mechanism which guarantees an executable solution.

[13] Later, Takanishi's method was extended to handle real-time pattern generation [40].

[14] Another fast and efficient method was proposed by Nagasaka [93].

$$\begin{bmatrix} p_1' \\ p_2 \\ \vdots \\ p_{N-1} \\ p_N' \end{bmatrix} = \begin{bmatrix} a_1 + b_1 & c_1 & 0 & & & \\ a_2 & b_2 & c_2 & \ddots & & \\ & & \ddots & & & \\ & & & \ddots & a_{N-1} & b_{N-1} & c_{N-1} \\ & & & & 0 & a_N & b_N + c_N \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N-1} \\ x_N \end{bmatrix}, \tag{4.68}$$

where p_1', p_N' are specified by using initial and terminal velocities v_1, v_N as

$$p_1' = p_1 + a_1 v_1 \Delta t, \quad p_N' = p_N - c_N v_N \Delta t.$$

Rewriting (4.68) as

$$\boldsymbol{p} = \boldsymbol{A}\boldsymbol{x}$$

gives the representation of the solution by

$$\boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{p}. \tag{4.69}$$

Although $\boldsymbol{A}$ is a huge square matrix with several thousands columns and several thousands lows, there exists an efficient algorithm to compute the inverse [136] since it is a tridiagonal matrix whose elements are all zeros except its main diagonal, the adjacent diagonals above and below it.

From this CoM trajectory, we can generate a walking pattern for a multi-body model by using the method of Section 4.3.5. Then we can calculate the ZMP trajectory for the multi-body model.

$$\boldsymbol{p}^* = RealZMP(\boldsymbol{x}). \tag{4.70}$$

The function $RealZMP()$ calculates ZMP based on a multi-body model and $\boldsymbol{p}^*$ is the obtained ZMP. The ZMP error $\boldsymbol{p}^* - \boldsymbol{p}^d$ contains information about the difference between the cart-table model and the multi-body model. Again using (4.69), we can calculate the CoM variation to compensate the ZMP error

$$\Delta\boldsymbol{x} = \boldsymbol{A}^{-1}(\boldsymbol{p}^* - \boldsymbol{p}^d).$$

The CoM trajectory is updated by

$$\boldsymbol{x} := \boldsymbol{x} - \Delta\boldsymbol{x}.$$

Going back to (4.70), we can repeat the same process until the ZMP error becomes sufficiently small.

This is a very efficient algorithm. According to Nishiwaki et al. [73] it takes only 140 [ms] in generating a walking pattern for three steps (3.2 s) of the humanoid robot H7 [113] which have 32 DOF using dual Pentium II 750MHz. They have realized joystick controlled real-time walking by generating three future steps at every step cycle and by properly connecting them.