Chapter 4
Biped Walking

What is the meaning of the word "walk"? Oxford Advanced Learner's Dictionary gives us a concise definition.

> **walk:**
> move along at a moderate pace by lifting up and putting down each foot in turn, so that one foot is on the ground while the other is being lifted
> (Oxford Advanced Learner's Dictionary, Oxford University Press)
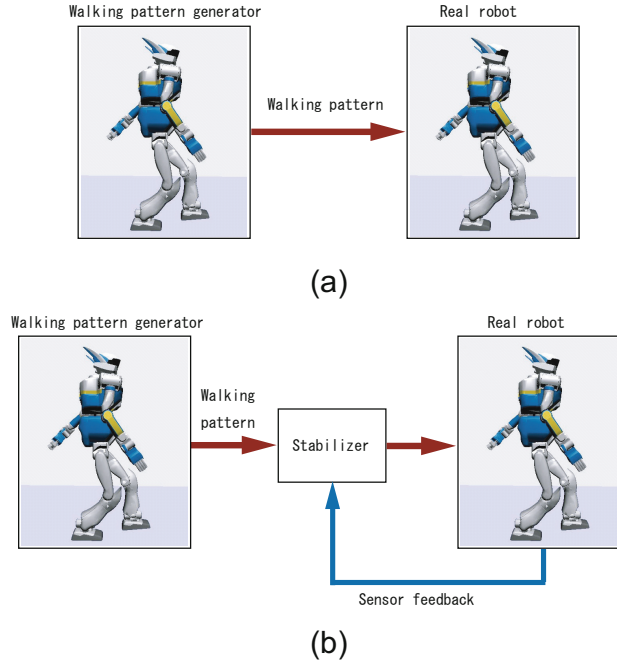
Therefore, at least one foot must be in contact with the ground at any moment during walking. There exist two kind of walking, namely, static walking and dynamic walking. In "static walking", the projection of the center of mass never leaves the support polygon during the walking. In "dynamic walking", there exist periods when the projection of the center of mass leaves the support polygon.

$$\text{Walk} \begin{cases} \text{static walk} \\ \text{dynamic walk} \end{cases}$$

Most toy robots perform static walking using large feet. This is not interesting from the view point of control engineering since it is fairly easy. Nevertheless, human feet are too small with respect to the height of center of mass to perform static walking. Indeed we are performing dynamic walking in our daily life. The walking style with which we are so accustomed can be realized by dexterous control of the whole body balance which is essentially unstable. A biped walking machine is, therefore, beyond the scope of conventional mechanical engineering. This is the reason that so many researchers and engineers are attracted to biped walking machines as well as humanoid robots.

4.1 How to Realize Biped Walking?

Figure 4.1 shows the basic framework of biped walking control that will be used throughout this chapter. A set of time series of joint angles for desired walking is called a *walking pattern*, and to create it, we use a *walking pattern*

(a)

(b)

Fig. 4.1 Basic control framework in this book: (a)In an ideal condition, a biped robot can walk by following a walking pattern. (b)In a real environment, a biped robot need a stabilizer.

generator. In an ideal situation, biped walking can be realized just by giving a walking pattern to an actual robot (Fig. 4.1(a)). For this purpose, we must prepare an accurate model of the robot, a stiff mechanism which moves exactly as commanded and a perfect horizontal floor (a huge surface plate).

On the other hand, in a real situation, a life size humanoid robot can easily fall down by floor unevenness of only a few millimeters. A humanoid proportion and mass distribution tends to quickly amplify the posture error to an unstable level. To suppress this, we need the second software, which modifies the walking pattern by using gyros, accelerometers, force sensors and other devices. This is called a *stabilizer* (Fig. 4.1(b)).

The rest of this chapter is organized as follows. In section 4.2, 4.3 and 4.4, we explain walking pattern generators and explain stabilizers in section 4.5. In section 4.6, we spotlight the history of biped walking robot research. In section 4.7, we introduce a variety of biped control methods which are not restricted in the framework of Fig. 4.1.
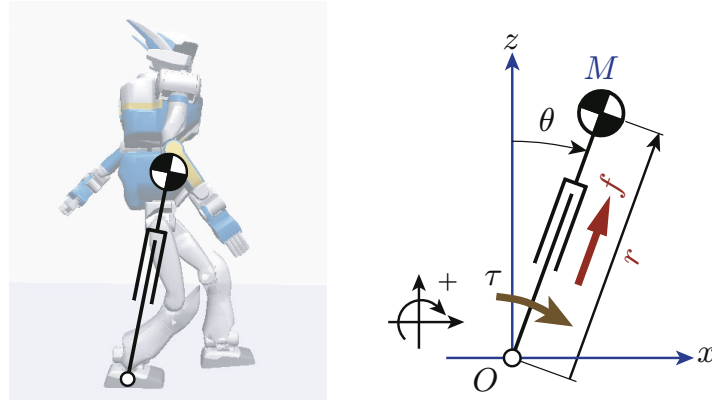
4.2 Two Dimensional Walking Pattern Generation

In this section, we consider a basic principle of biped walking in 2D and derive an algorithm for walking pattern generation.

4.2.1 Two Dimensional Inverted Pendulum

Coarse-graining is a powerful method to handle a complex system. In celestial mechanics, researchers approximate the Sun and the planets as point masses while they have their own internal structure, and they can still calculate the orbits of the solar system with sufficient accuracy. In thermodynamics, vast states of molecules, number of the order of 10^{23} are coarse-grained as a few parameters like temperature or entropy, and it makes it possible for us to predict the thermodynamic phenomenon.

Likewise, to extract an *essence* of biped locomotion, we make three assumptions as coarse-graining on a humanoid robot with more than 30 DOF made of over thousands of mechanical and electrical parts. First, we assume that all the mass of the robot is concentrated at its center of mass (CoM). Second, we assume that the robot has massless legs, whose tips contact the ground at single rotating joints. At last, we only consider the forward/backward and the up/down motions of the robot, neglecting lateral motion. In other words, we assume the robot motion is constrained to the *sagittal plane* defined by the axis of walking direction and vertical axis. With these assumptions, we model a robot as a 2D inverted pendulum as shown in Fig. 4.2.

Fig. 4.2 2D inverted pendulum: The simplest model for a human or a walking robot. It consists of the center of mass (CoM) and massless telescopic leg. We measure the inclination of the pendulum θ from vertical, positive for counter clockwise rotation.

The inputs of the pendulum are the torque τ at the pivot and the kick force f at the prismatic joint along the leg. The dynamics of the pendulum are described as a couple of differential equations[1] as follows

$$r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} - gr\sin\theta = \tau/M$$
$$\ddot{r} - r\dot{\theta}^2 + g\cos\theta = f/M.$$

We can simulate the behavior of the inverted pendulum by integrating them numerically with given input torque.

One of the important limitations is we cannot use big torque τ since the feet of biped robot is very small. If a walking robot has a point contact like a stilt, we must use

$$\tau = 0. \tag{4.1}$$

In this case, the pendulum will almost always fall down, unless the CoM is located precisely above the pivot. Even with such a simple pendulum, we have a variety of falling patterns corresponding to different kick forces, f, as illustrated in Fig. 4.3.

The most interesting case is Fig. 4.3(d) where the CoM moves horizontally under the kick force

$$f = \frac{Mg}{\cos\theta}. \tag{4.2}$$

Figure 4.4 illustrates the reason for the horizontal motion of the CoM.

Intuitively, we can say the pendulum is keeping the CoM height by extending its leg as fast as it is falling. We call this the Linear Inverted Pendulum [115][2].

4.2.2 Behavior of Linear Inverted Pendulum

The Linear Inverted Pendulum provides us a mathematically easy treatment of dynamics. Let us investigate the horizontal motion.
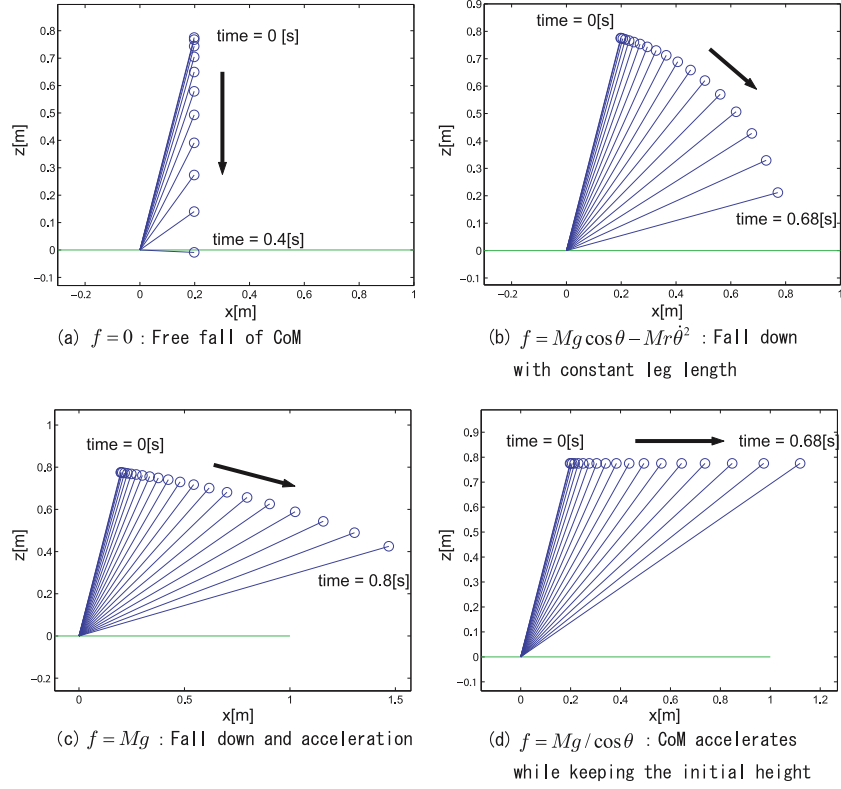
1   Horizontal Dynamics

By investigating Fig. 4.4 again, we see the horizontal component of the kick force f remains while the vertical component is canceled by gravity. The horizontal component accelerates the CoM horizontally, thus we have

$$M\ddot{x} = f\sin\theta. \tag{4.3}$$

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[1] These equations can be derived by using Lagrange's method. Joseph-Louis Lagrange (1736-1813) was a French mathematician born in Italy. He is famous as a discoverer of the *Lagrange points*, the most suitable area to place space colonies.

[2] This is the simplest version of Linear Inverted Pendulum (LIP). The concept will be extended throughout of this chapter. Also, the LIP is an example of Zero-dynamics which is discussed in the textbook by Westervelt et al. [24]

(a) $f = 0$: Free fall of CoM

(b) $f = Mg\cos\theta - Mr\dot{\theta}^2$: Fall down with constant leg length

(c) $f = Mg$: Fall down and acceleration

(d) $f = Mg/\cos\theta$: CoM accelerates while keeping the initial height

Fig. 4.3 Falling inverted pendulum under the various kick force f. The pivot torque is kept zero ($\tau = 0$) at all time.

By substituting (4.2), we get

$$M\ddot{x} = \frac{Mg}{\cos\theta}\sin\theta = Mg\tan\theta = Mg\frac{x}{z}$$

where, x, z gives the CoM of the inverted pendulum. By rewriting above equation, we obtain a differential equation for the horizontal dynamics of the CoM
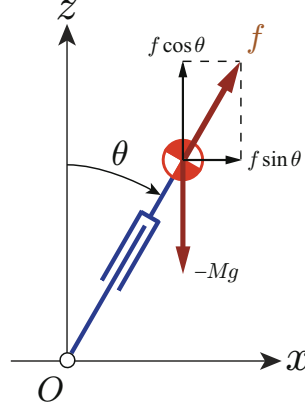
$$\ddot{x} = \frac{g}{z}x. \tag{4.4}$$

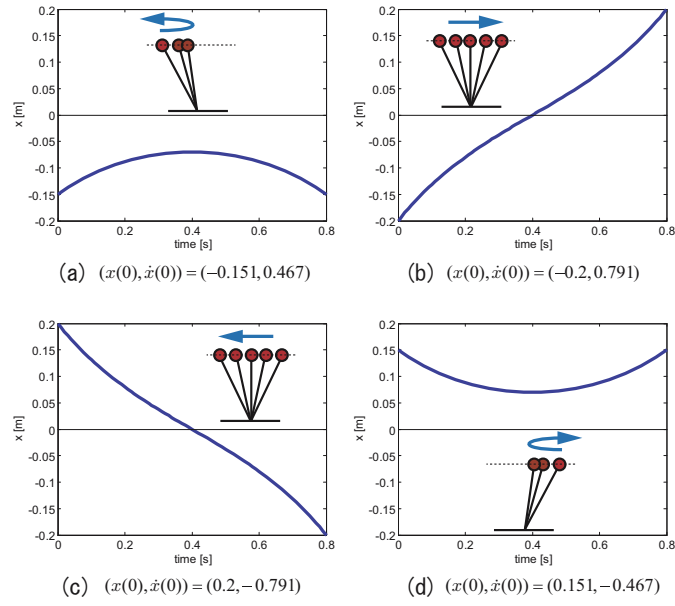Since we have constant z in a Linear Inverted Pendulum, we can easily solve this ordinary differential equation

$$x(t) = x(0)\cosh(t/T_c) + T_c\dot{x}(0)\sinh(t/T_c) \tag{4.5}$$
$$\dot{x}(t) = x(0)/T_c\sinh(t/T_c) + \dot{x}(0)\cosh(t/T_c) \tag{4.6}$$
$$T_c \equiv \sqrt{z/g}$$

Fig. 4.4 The reason for the horizontal locus of the CoM. The kick force $f = Mg/\cos\theta$ always balance with the gravity acting on the point mass.

(a) $(x(0), \dot{x}(0)) = (-0.151, 0.467)$

(b) $(x(0), \dot{x}(0)) = (-0.2, 0.791)$

(c) $(x(0), \dot{x}(0)) = (0.2, -0.791)$

(d) $(x(0), \dot{x}(0)) = (0.151, -0.467)$

Fig. 4.5 Linear Inverted Pendulum under various initial conditions. CoM height: $z = 0.8$ m.

where T_c is the time constant depending the height of the CoM and gravity acceleration.

The initial position and velocity are given by $x(0), \dot{x}(0)$, which together are called the *initial conditions*. Figure 4.5 shows the motions under various initial conditions.

2 Time for Transfer

In many cases, we want to know the time that CoM takes to travel from one point to another. When an initial condition $(x_0, \dot{x}_0)$ and a condition $(x_1, \dot{x}_1)$ at certain moment is given, they are connected by following equations by using (4.5) and (4.6)

$$x_1 = x_0 \cosh(\tau/T_c) + T_c \dot{x}_0 \sinh(\tau/T_c) \tag{4.7}$$

$$\dot{x}_1 = x_0/T_c \sinh(\tau/T_c) + \dot{x}_0 \cosh(\tau/T_c) \tag{4.8}$$

where τ is the period that the CoM takes a trip from $(x_0, \dot{x}_0)$ to $(x_1, \dot{x}_1)$.

By using the relationship of $\cosh(x) = \frac{e^x + e^{-x}}{2}$, $\sinh(x) = \frac{e^x - e^{-x}}{2}$, these equations can be rewritten as

$$x_1 = \frac{x_0 + T_c \dot{x}_0}{2} e^{\tau/T_c} + \frac{x_0 - T_c \dot{x}_0}{2} e^{-\tau/T_c}, \tag{4.9}$$

$$\dot{x}_1 = \frac{x_0 + T_c \dot{x}_0}{2T_c} e^{\tau/T_c} - \frac{x_0 - T_c \dot{x}_0}{2T_c} e^{-\tau/T_c}. \tag{4.10}$$

From (4.9)$+T_c\times$(4.10), we get

$$x_1 + T_c \dot{x}_1 = (x_0 + T_c \dot{x}_0) e^{\tau/T_c}.$$

Therefore, τ can be calculated as

$$\tau = T_c \ln \frac{x_1 + T_c \dot{x}_1}{x_0 + T_c \dot{x}_0}. \tag{4.11}$$

Similarly, we can calculate τ from (4.9)$-T_c\times$(4.10) as

$$\tau = T_c \ln \frac{x_0 - T_c \dot{x}_0}{x_1 - T_c \dot{x}_1}. \tag{4.12}$$

Basically, (4.11) and (4.12) gives exactly the same result, except in the singular case where both of the numerator and denominator go close to zero in one of the equations.