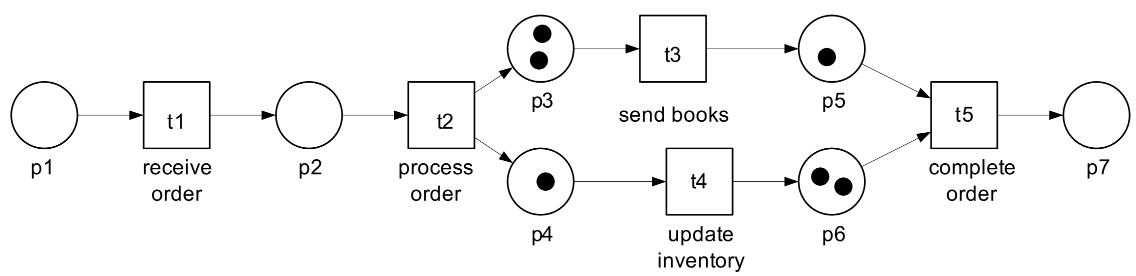
# Business Processes Modelling

## MPB (6 cfu, 295AA)

Roberto Bruni
http://www.di.unipi.it/~bruni

## 07 - Introduction to nets

# Object






Overview of the basic concepts of Petri nets

Free Choice Nets (book, optional reading)
https://www7.in.tum.de/~esparza/bookfc.html

# Why Petri nets?

Business process analysis:
**validation**: testing correctness
**verification**: proving correctness
**performance**: planning and optimization

Use of Petri nets (or alike)
visual + formal
tool supported

# Approaching Petri nets

Are you familiar with automata / transition systems?
They are fine for sequential protocols / systems
but do not capture concurrent behaviour directly

A Petri net is a mathematical model
of a parallel and concurrent system

in the same way that a finite automaton is a
mathematical model of a sequential system

# Approaching Petri nets

Petri net theory can be studied
at several level of details

We study some basics aspects, relevant to the
analysis of business processes

Petri nets have a faithful and convenient graphical
representation, that we introduce and motivate next

# Finite automata examples

# Applications

Finite automata are widely used, e.g., in
protocol analysis,
text parsing,
video game character behavior,
security analysis,
CPU control units,
natural language processing,
speech recognition,
mechanical devices
(like elevators, vending machines, traffic lights)
and many more …

# How to define an automaton

1. Identify the admissible **states** of the system
*(Optional: mark some states as error states)*

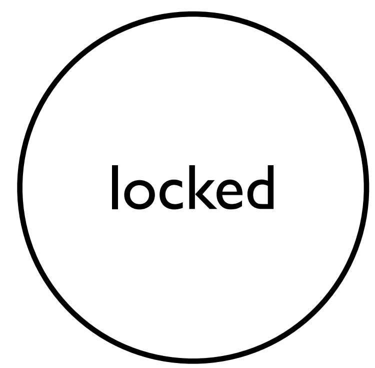
2. **Add transitions**
to move from one state to another
(no transition to recover from error states)

3. Set the **initial state**

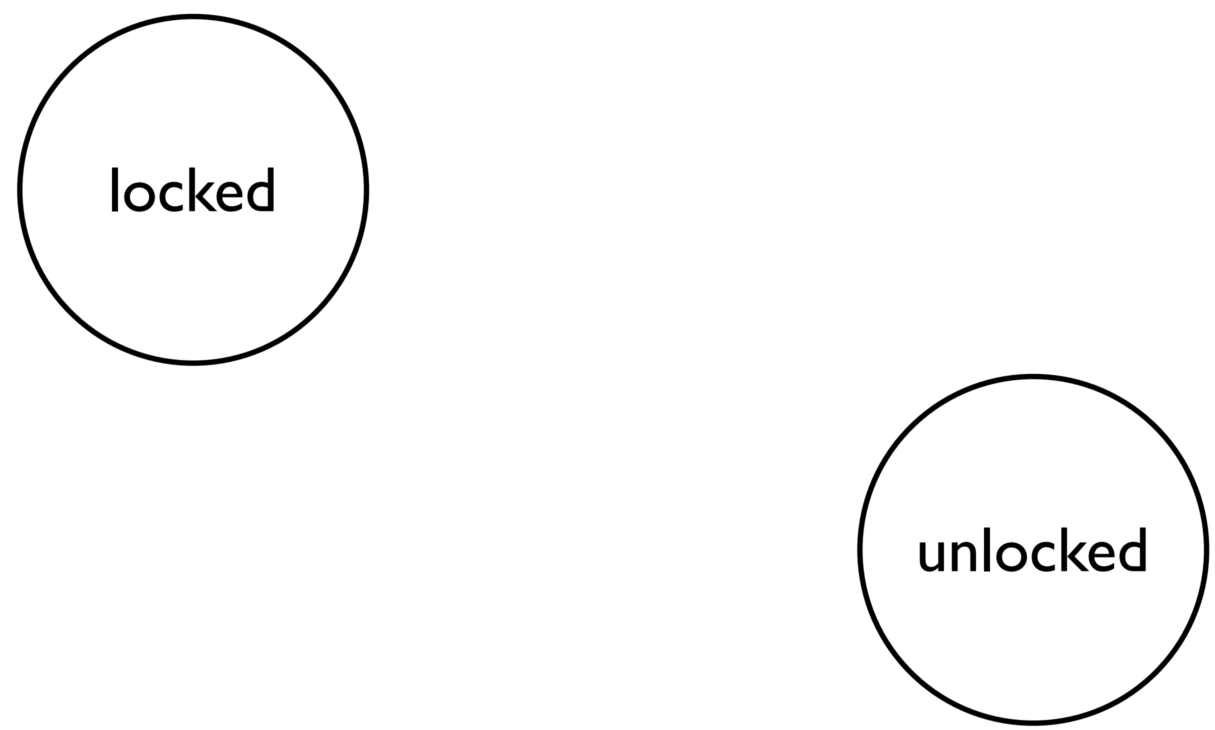
4. *(Optional: mark some states as **final states**)*

# Example: Turnstile

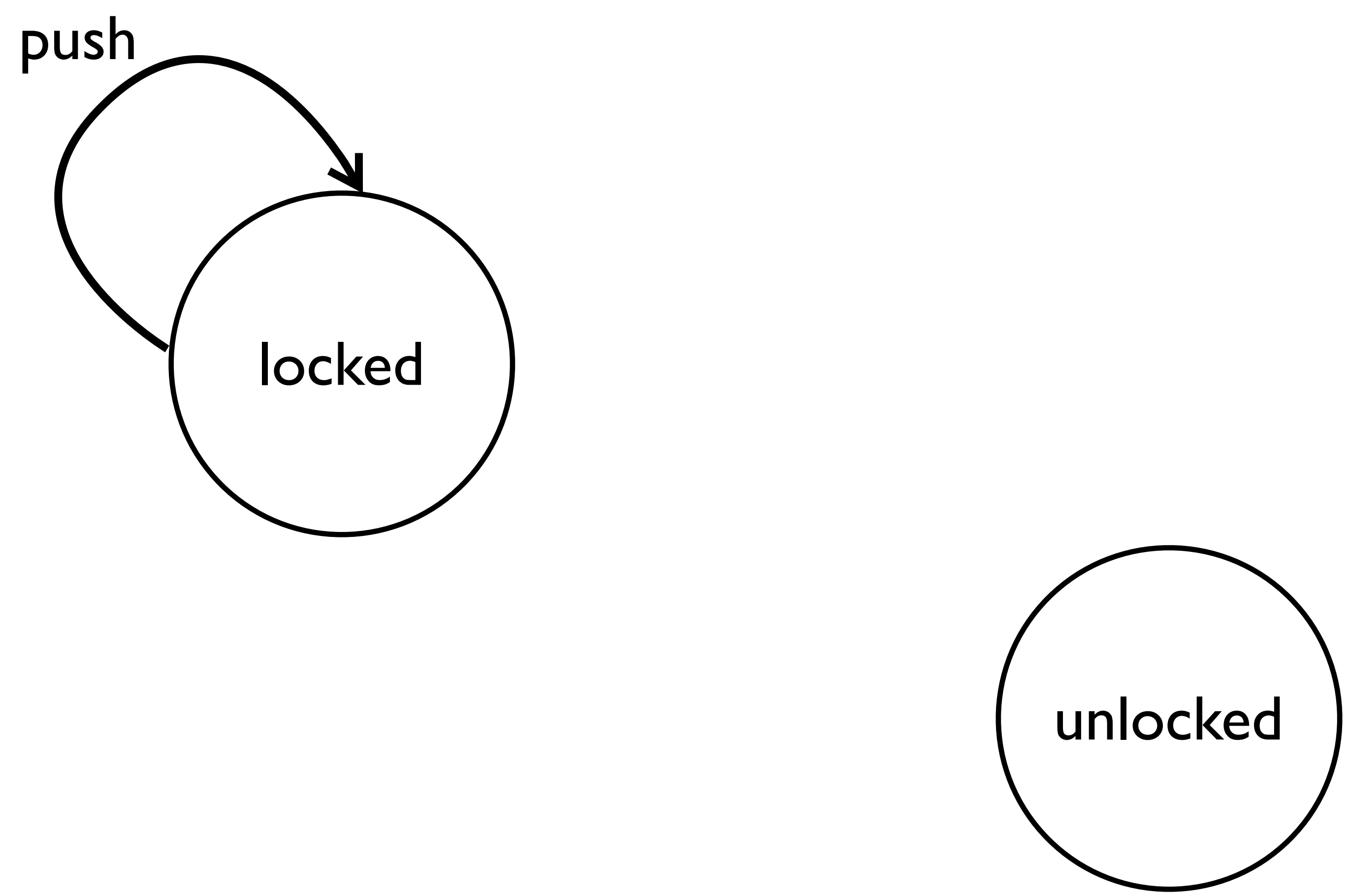
# Example: Turnstile



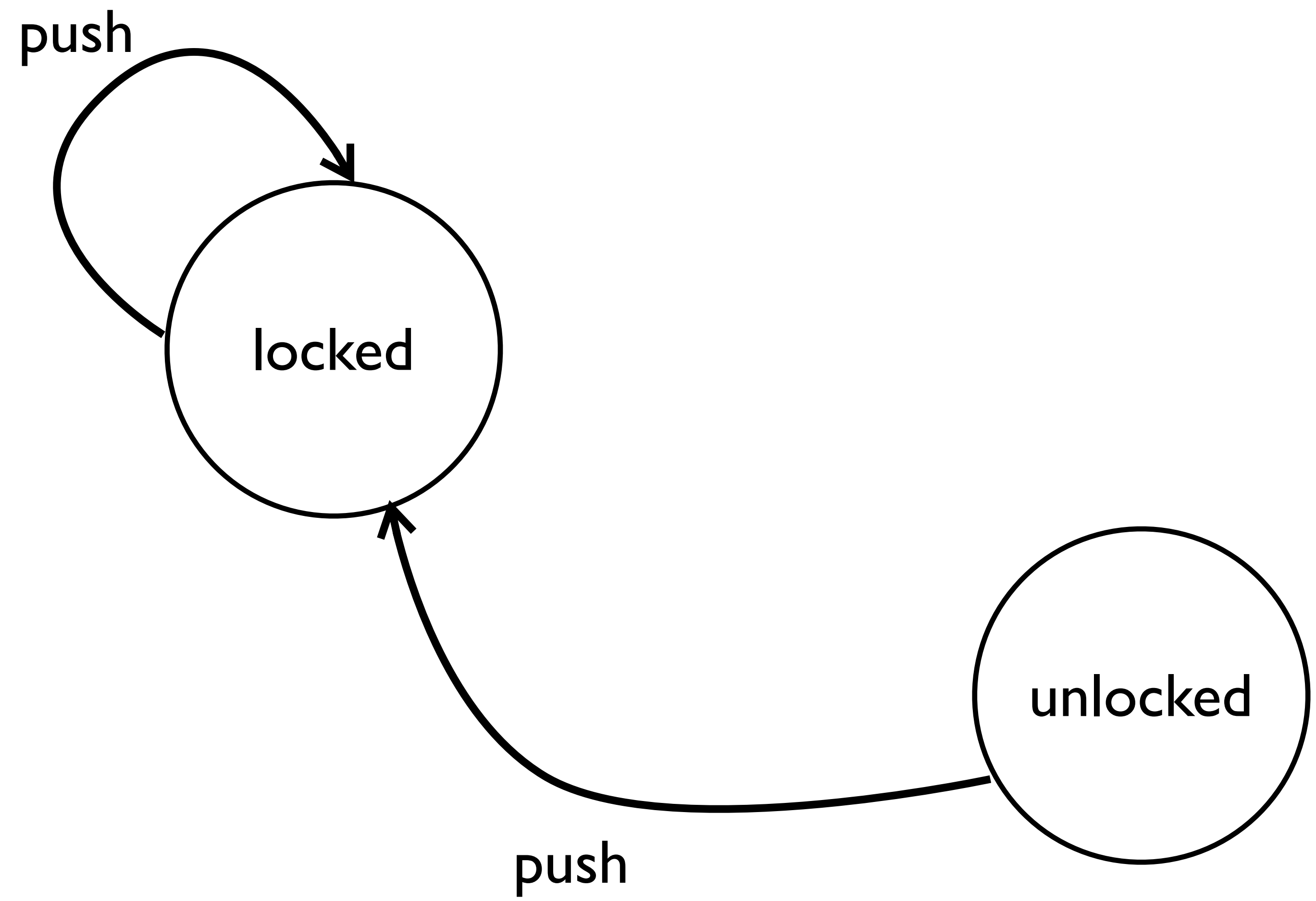
# Example: Turnstile



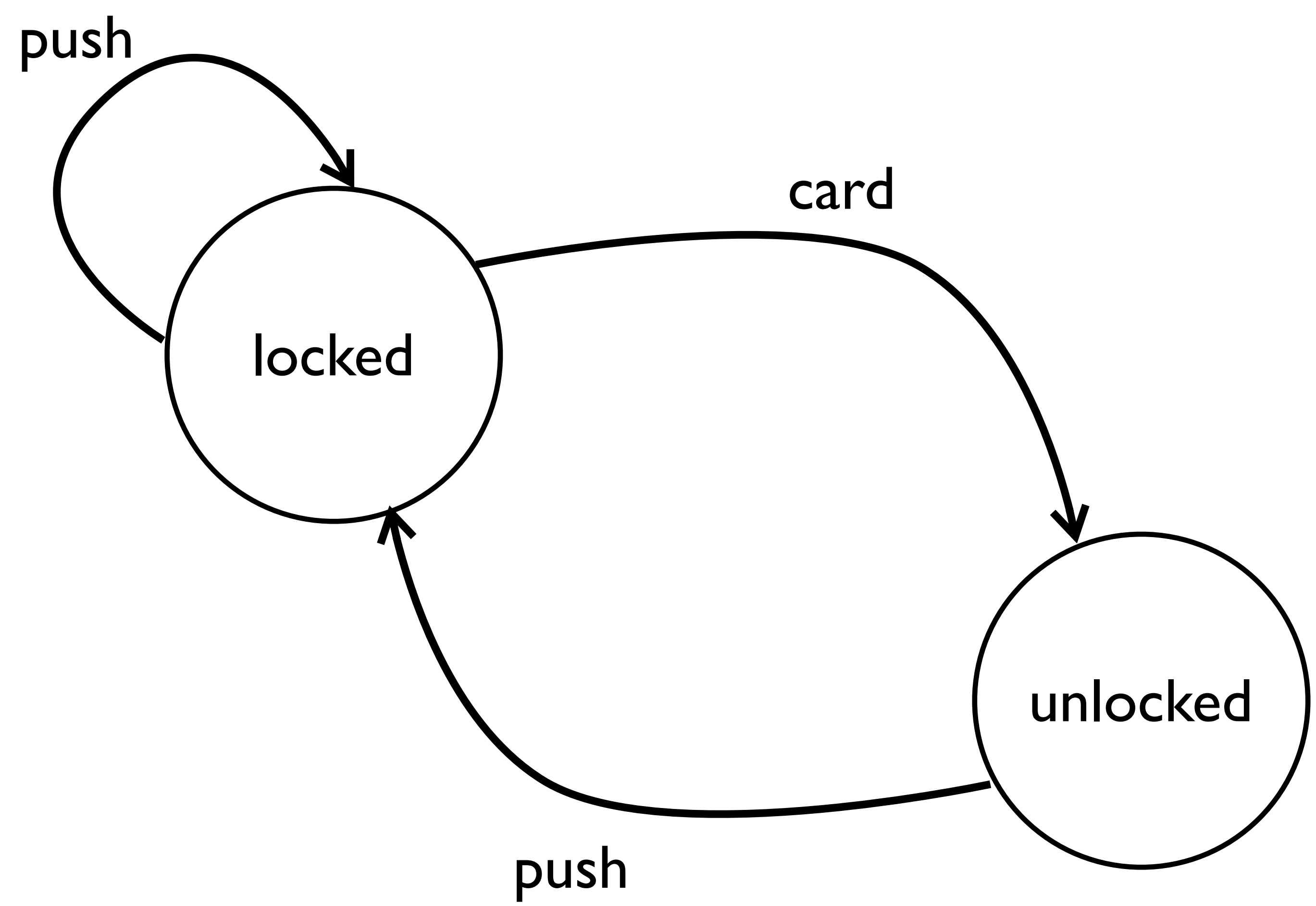
# Example: Turnstile



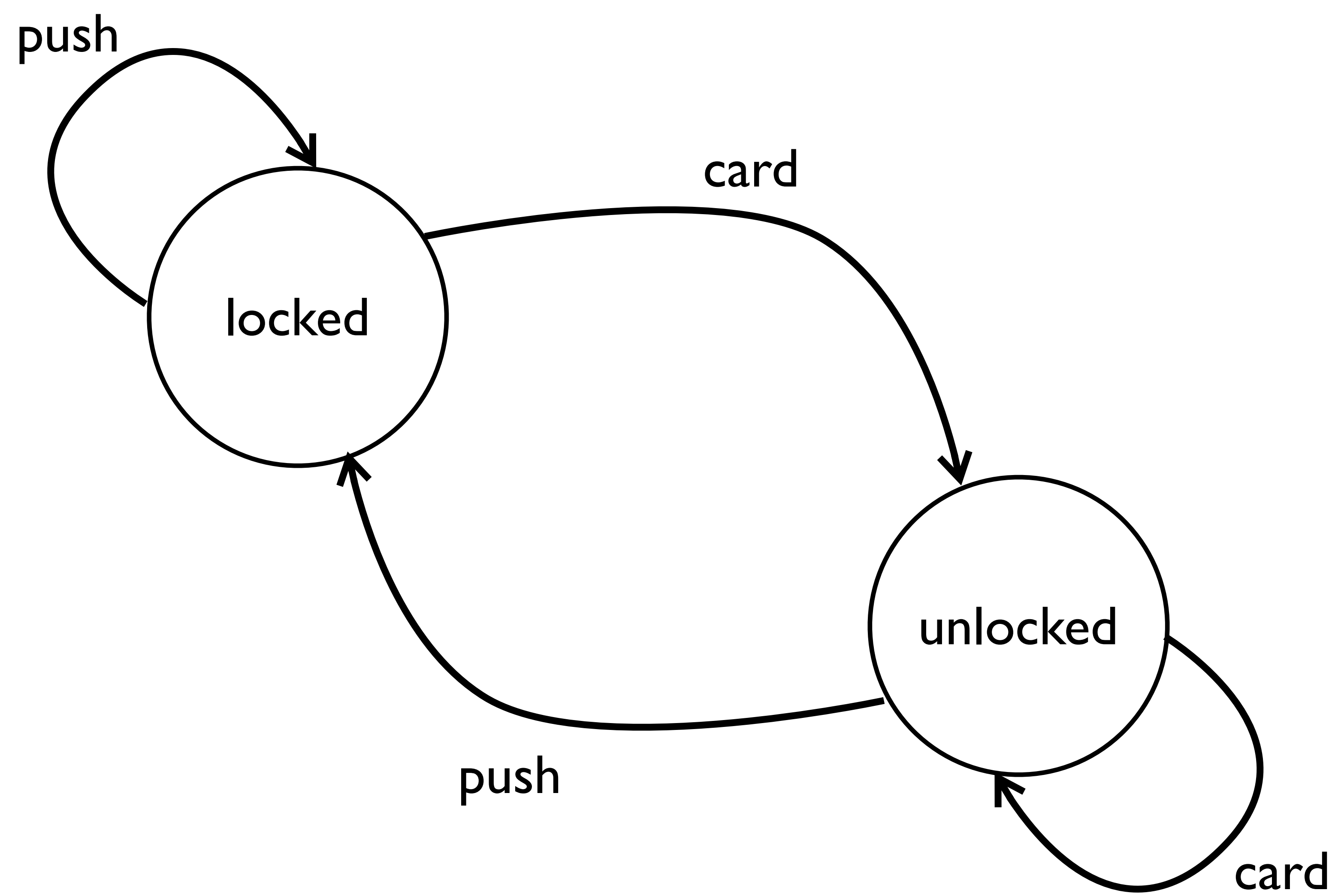
# Example: Turnstile



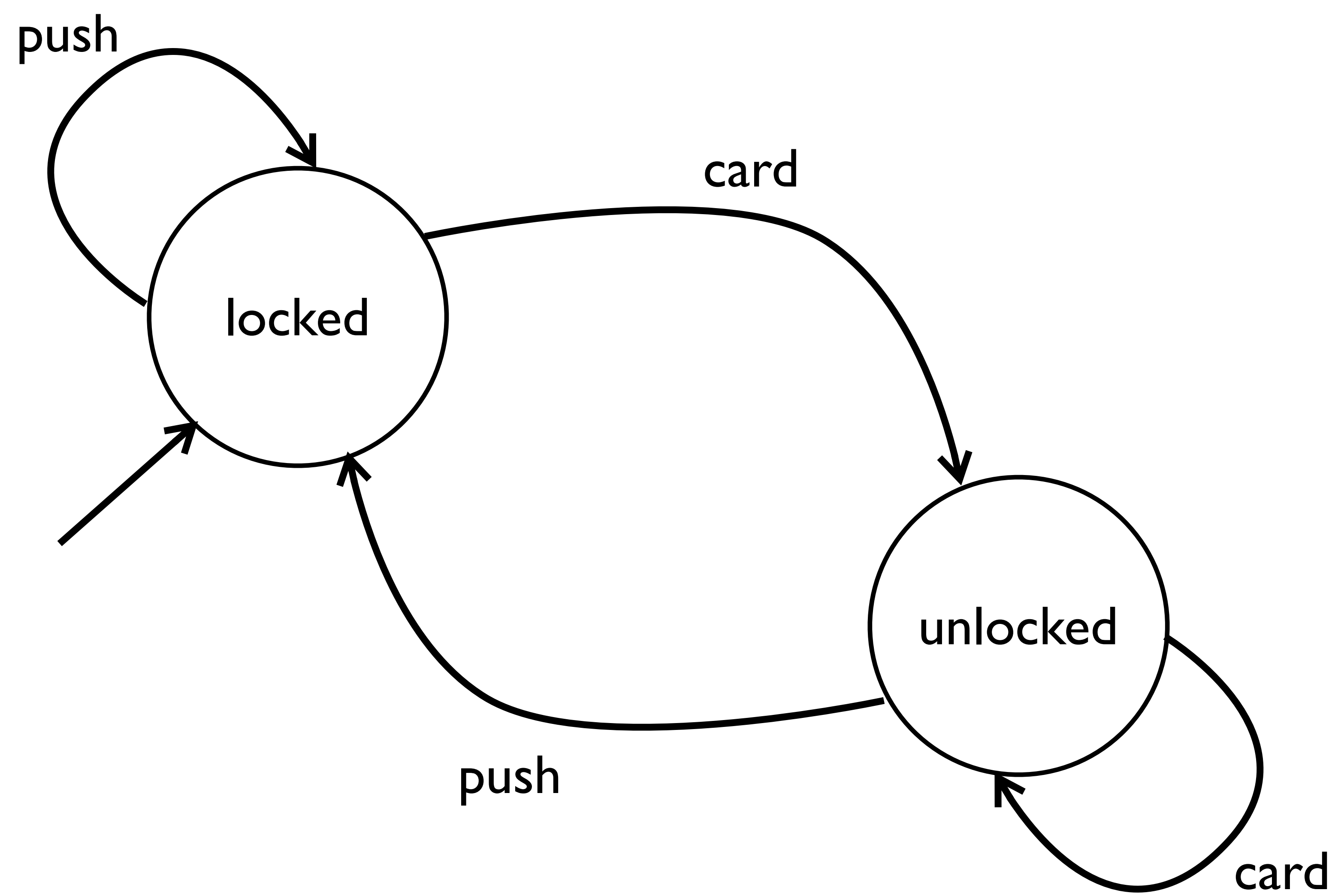
# Example: Turnstile



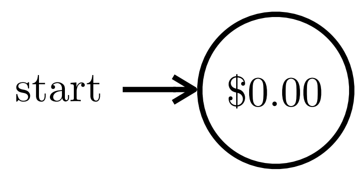
# Example: Turnstile



# Example: Turnstile

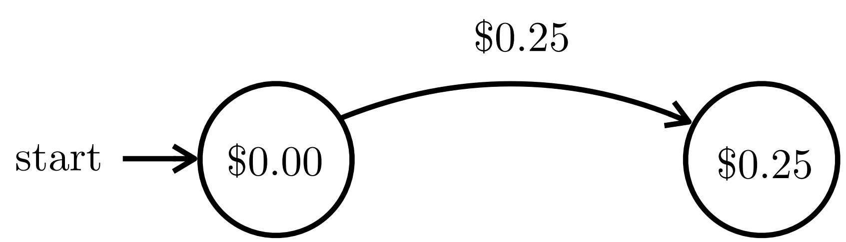


# Example: Vending Machine



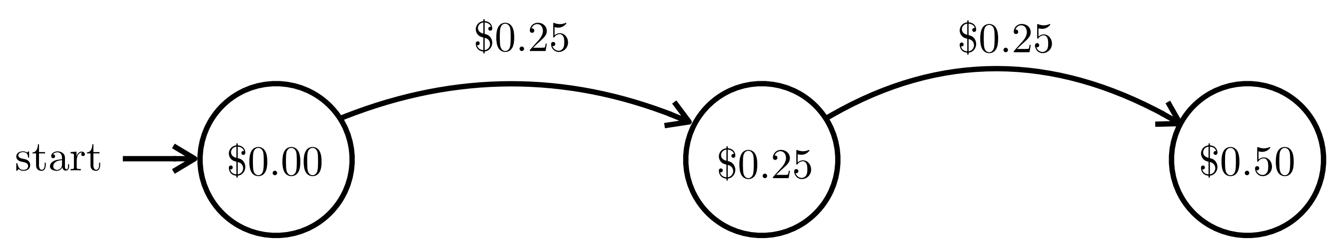

($1.25 per soda)

# Example: Vending Machine



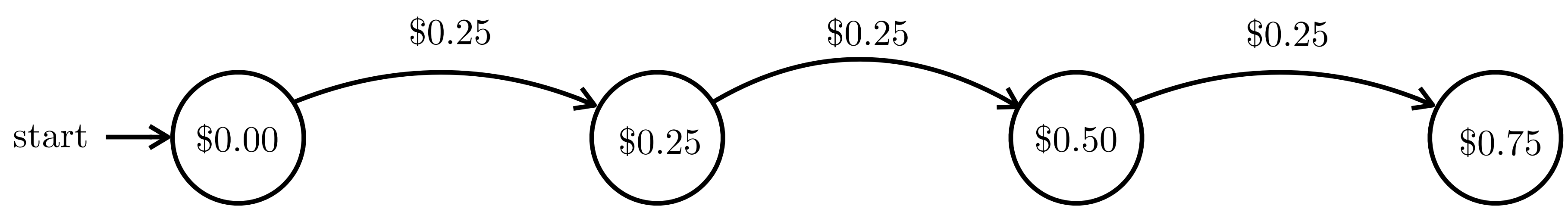

($1.25 per soda)

# Example: Vending Machine



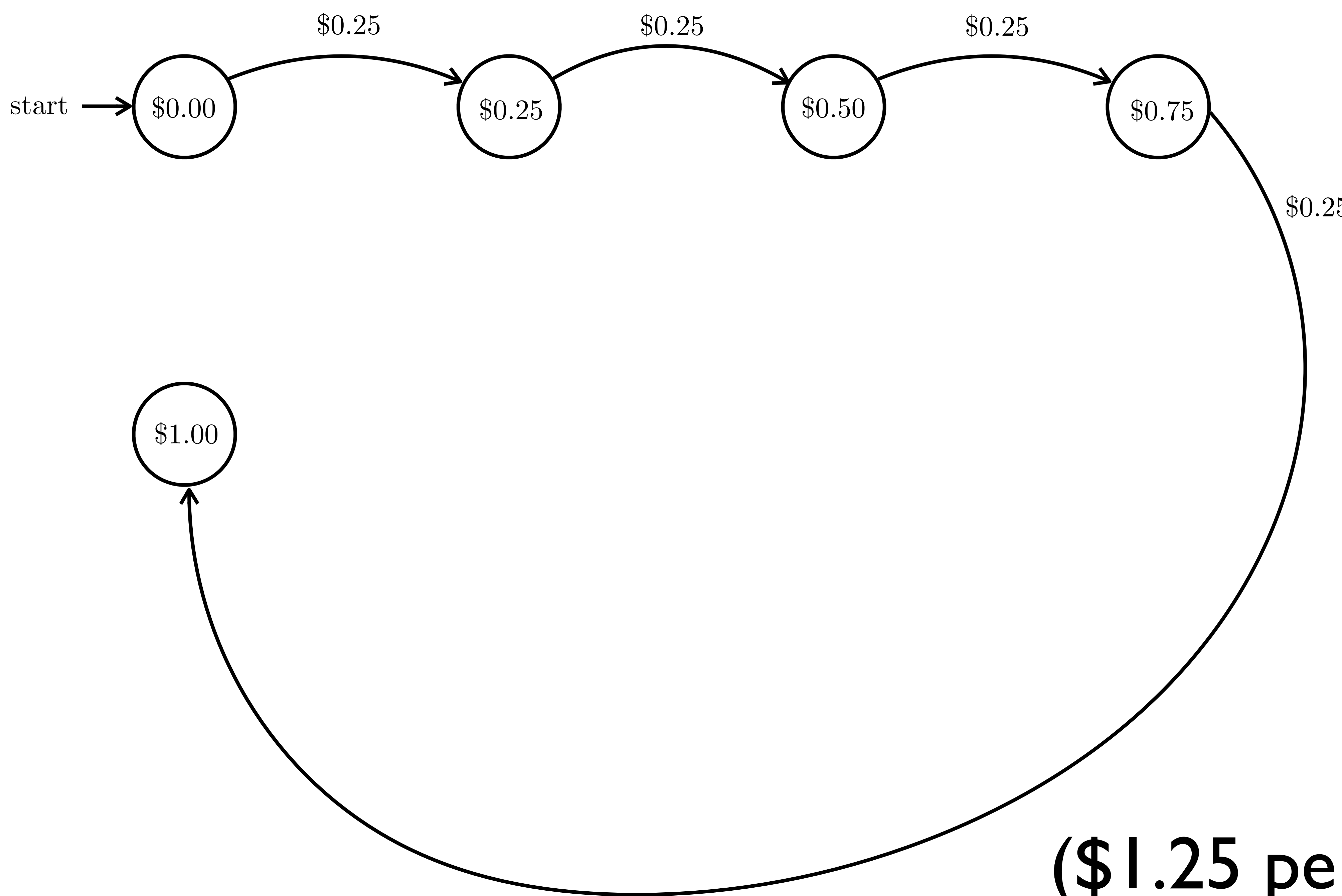

($1.25 per soda)

# Example: Vending Machine



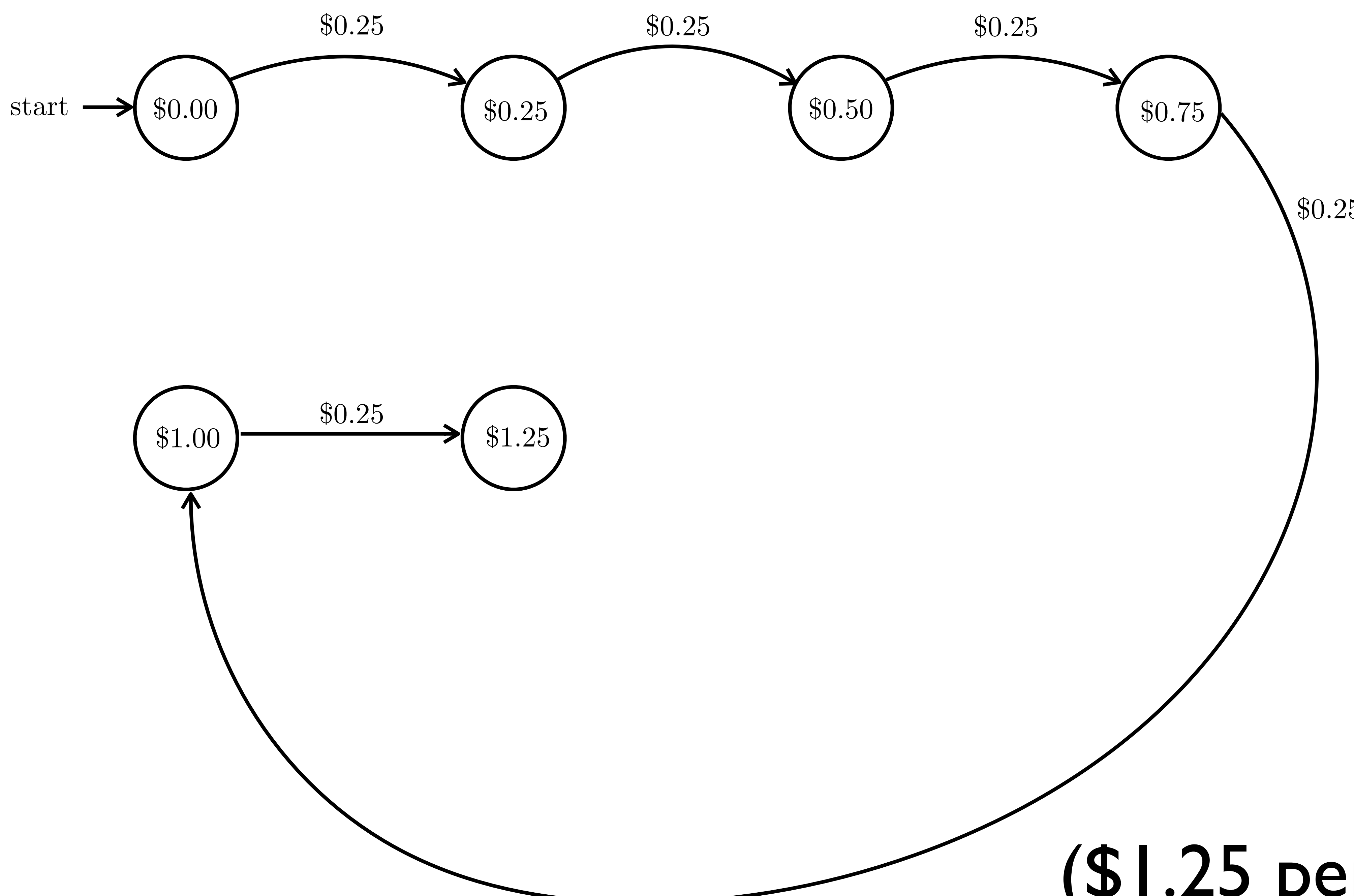

($1.25 per soda)

# Example: Vending Machine



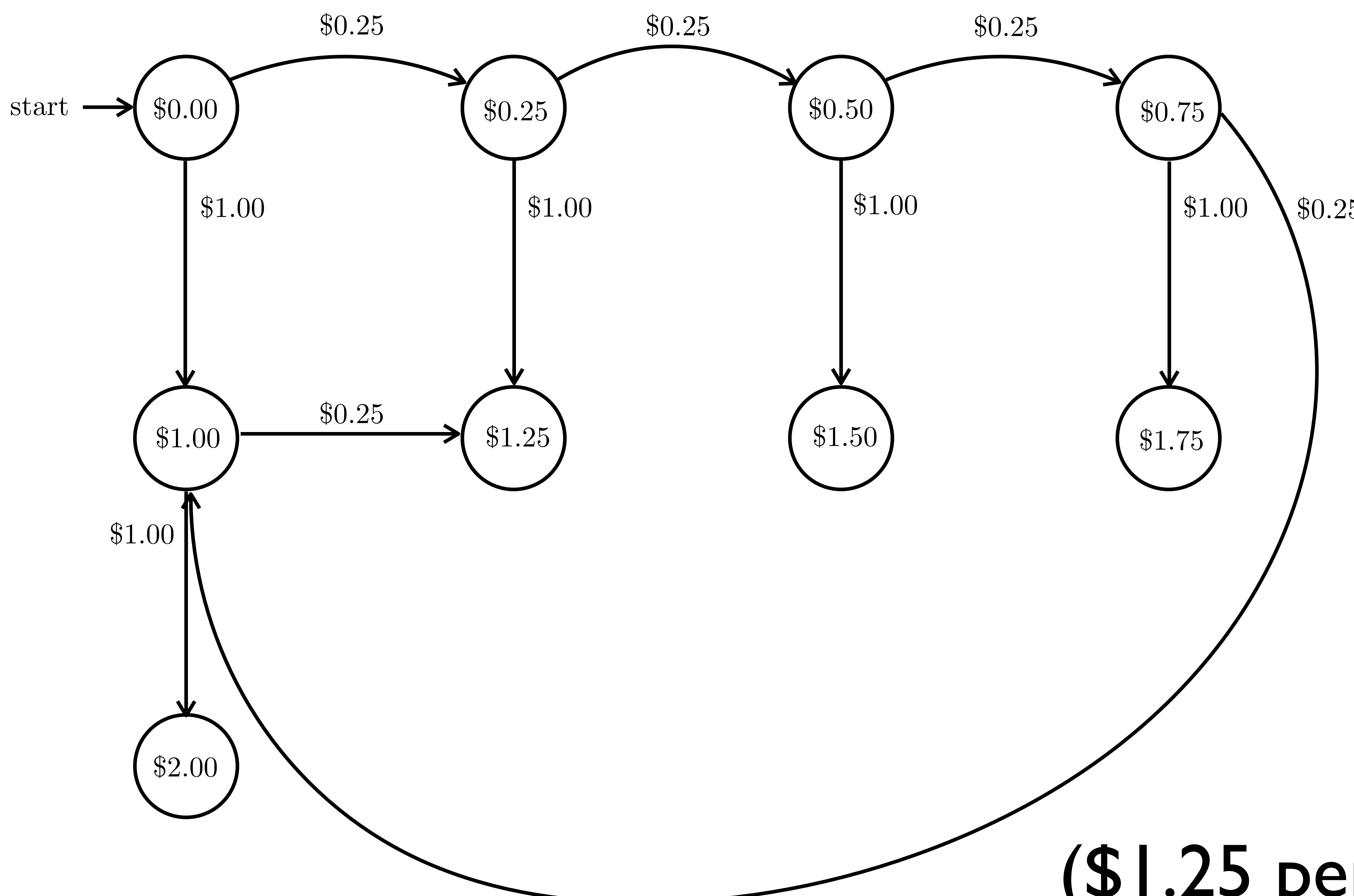

($1.25 per soda)

# Example: Vending Machine



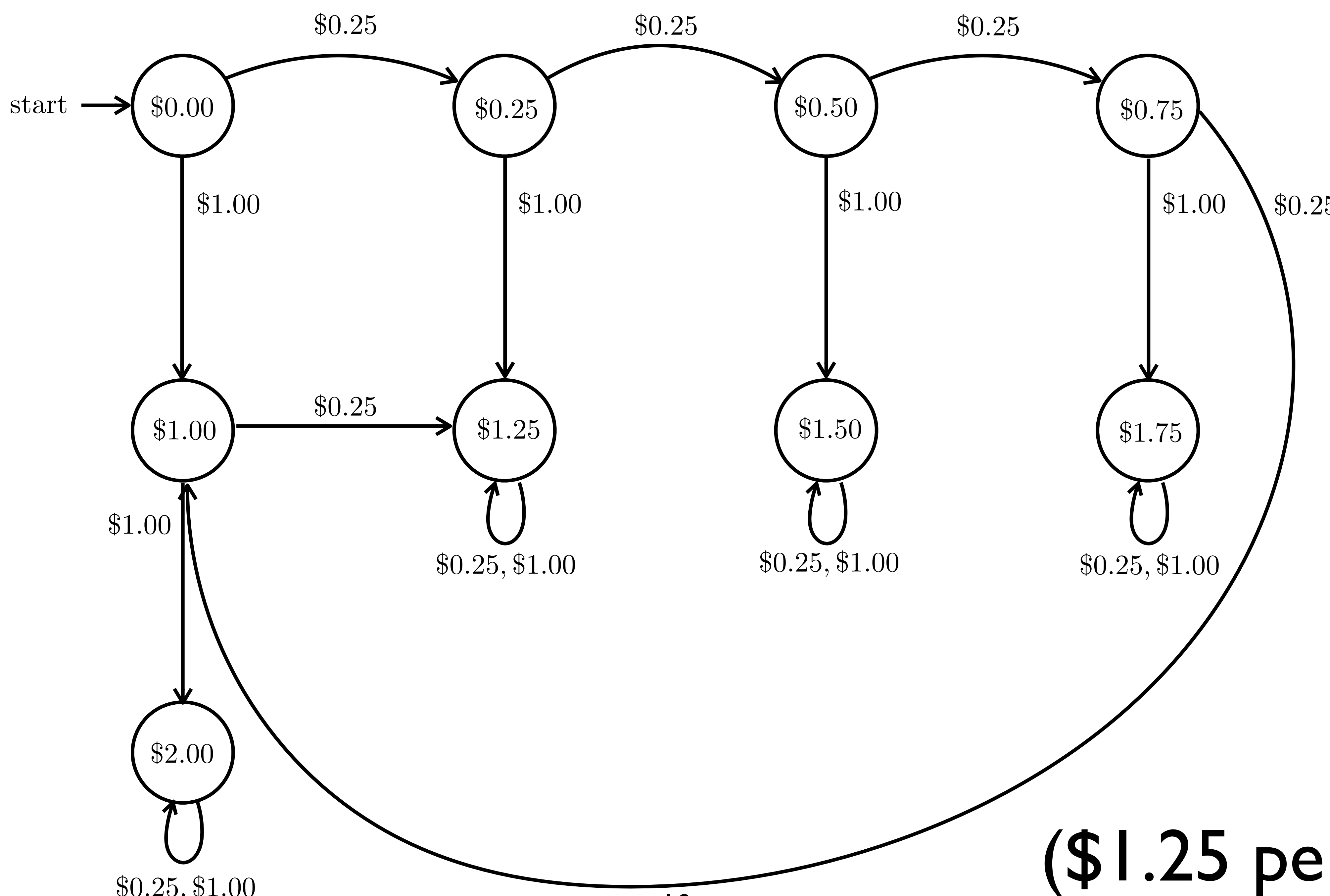

($1.25 per soda)

# Example: Vending Machine



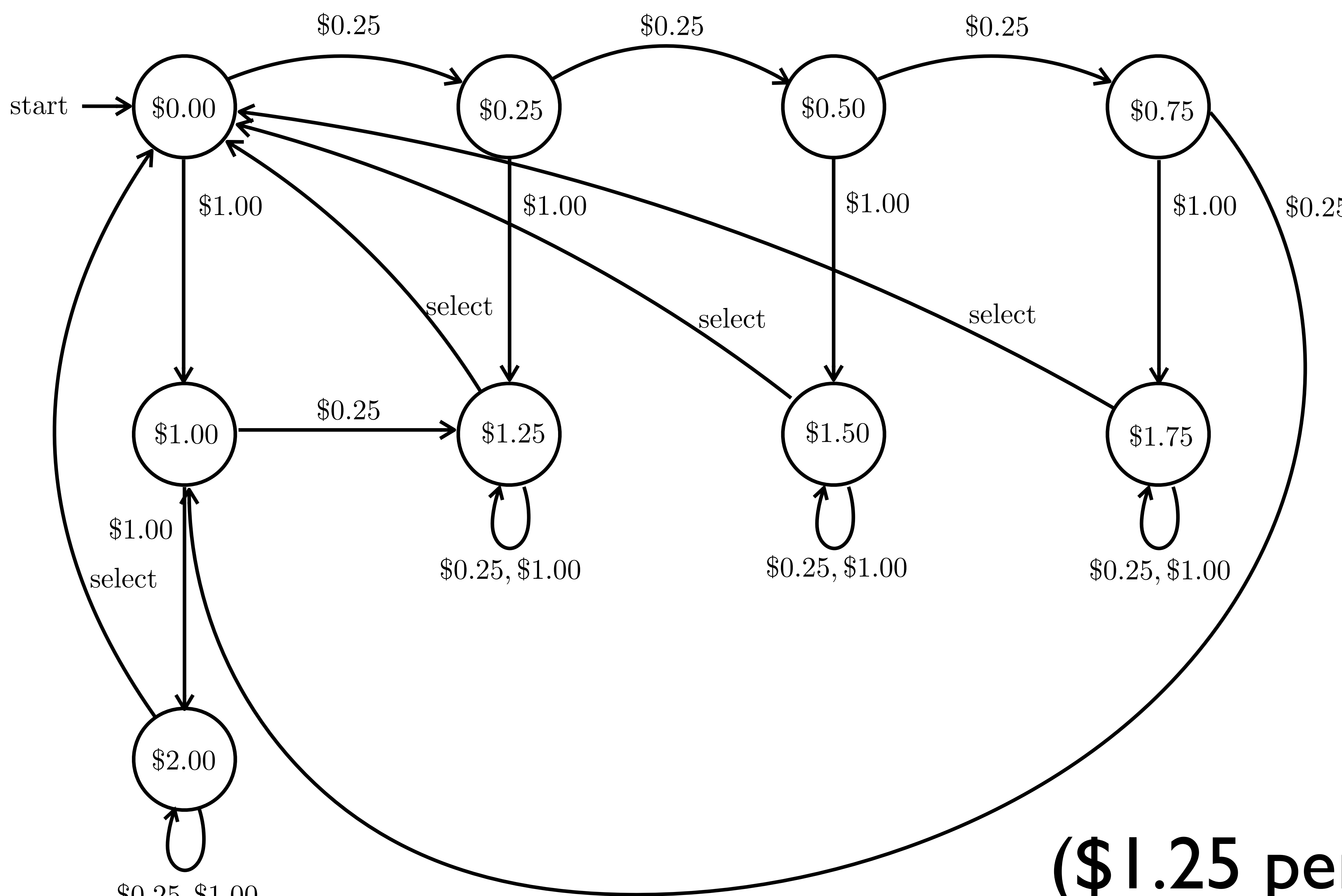

($1.25 per soda)

# Example: Vending Machine



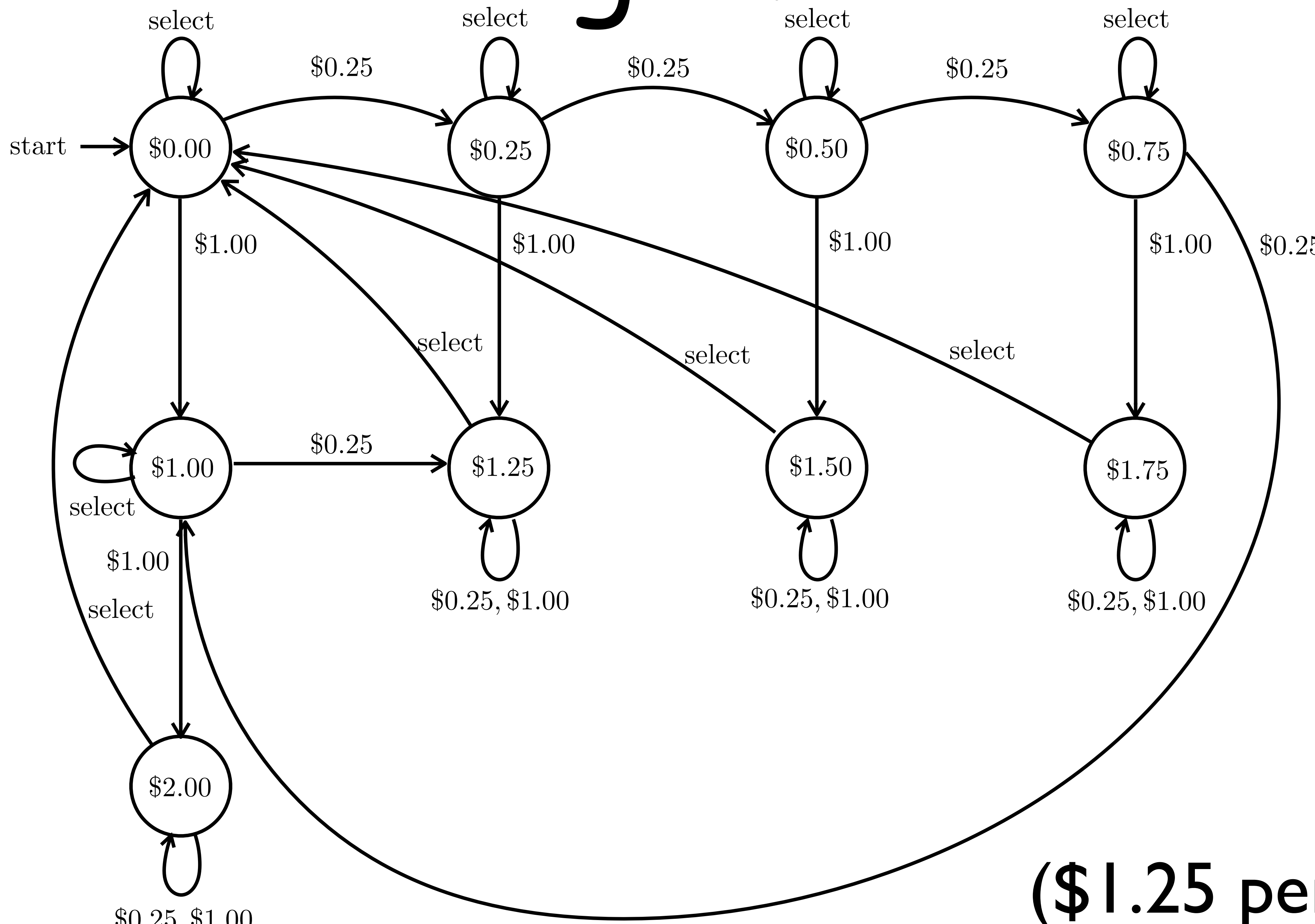

($1.25 per soda)

# Example: Vending Machine




($1.25 per soda)

# Example: Vending Machine

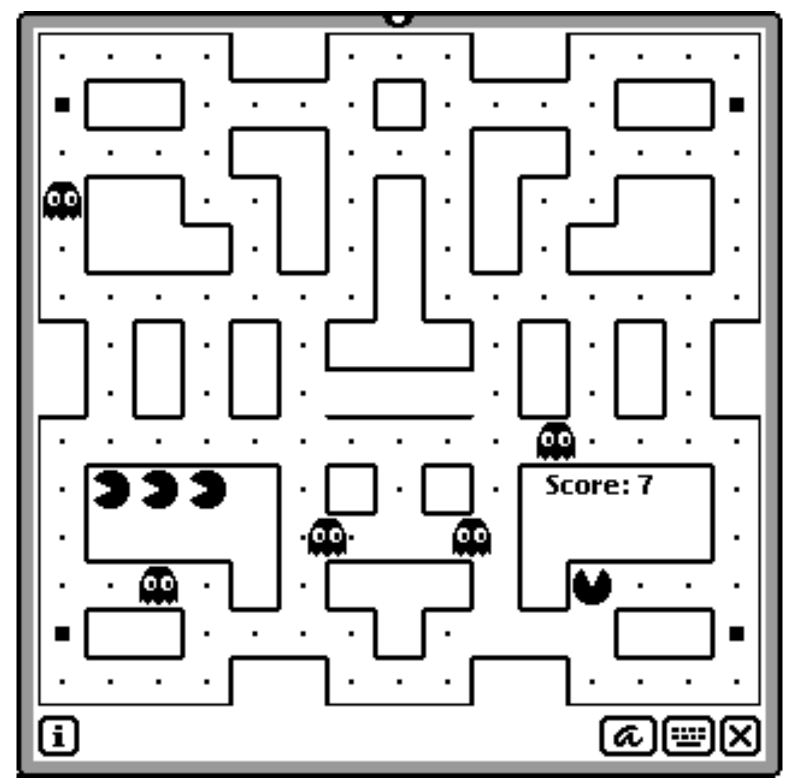



($1.25 per soda)

# Computer controlled characters for games

**States** = characters behaviours

**Transitions** = events that cause a change in behaviour




Example:
Pac-man moves in a maze
wants to eat pills
is chased by ghosts
by eating power pills, pac-man can defeat ghosts

# Example: Pac-Man Ghosts

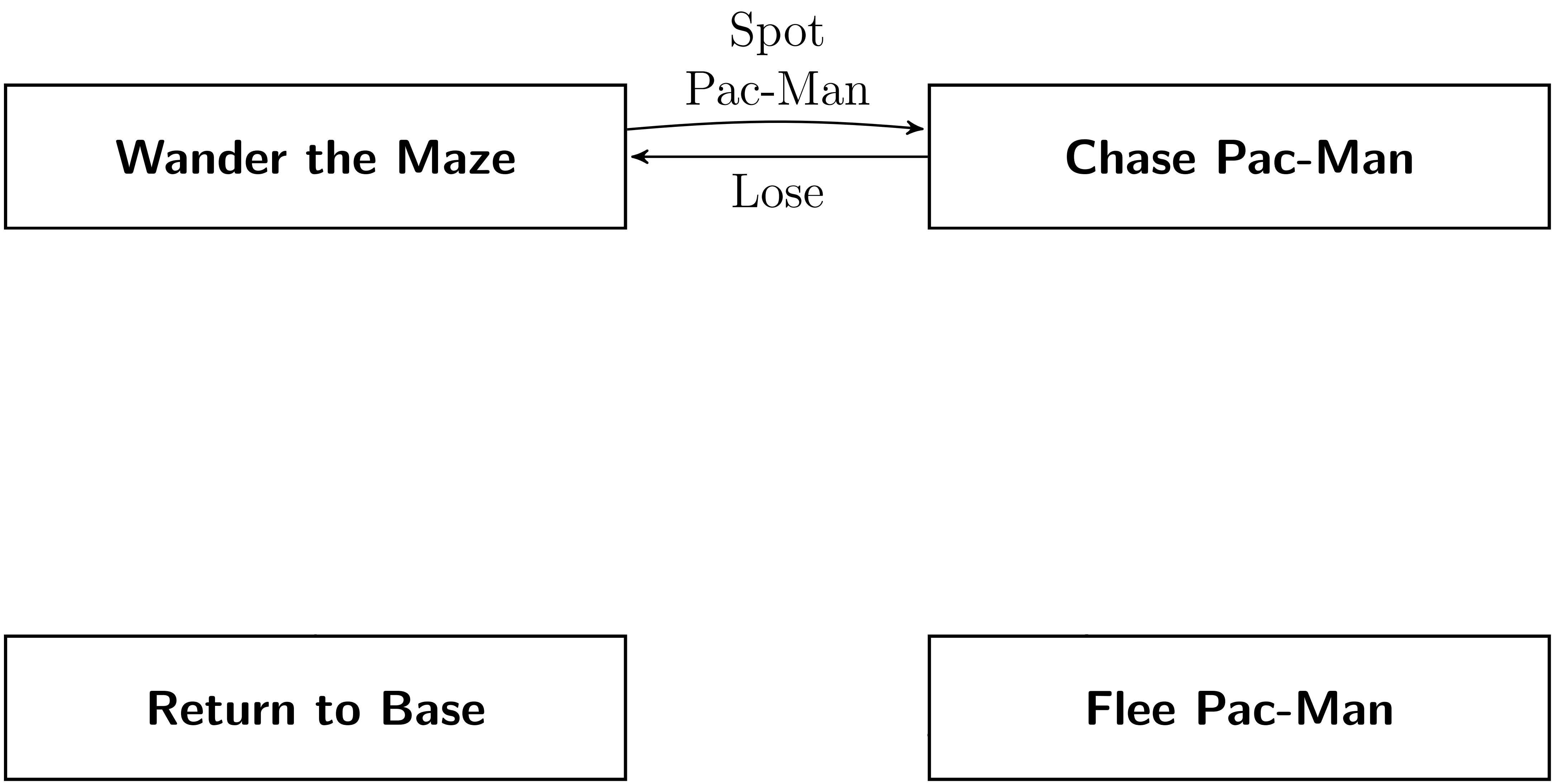
Wander the Maze

Chase Pac-Man

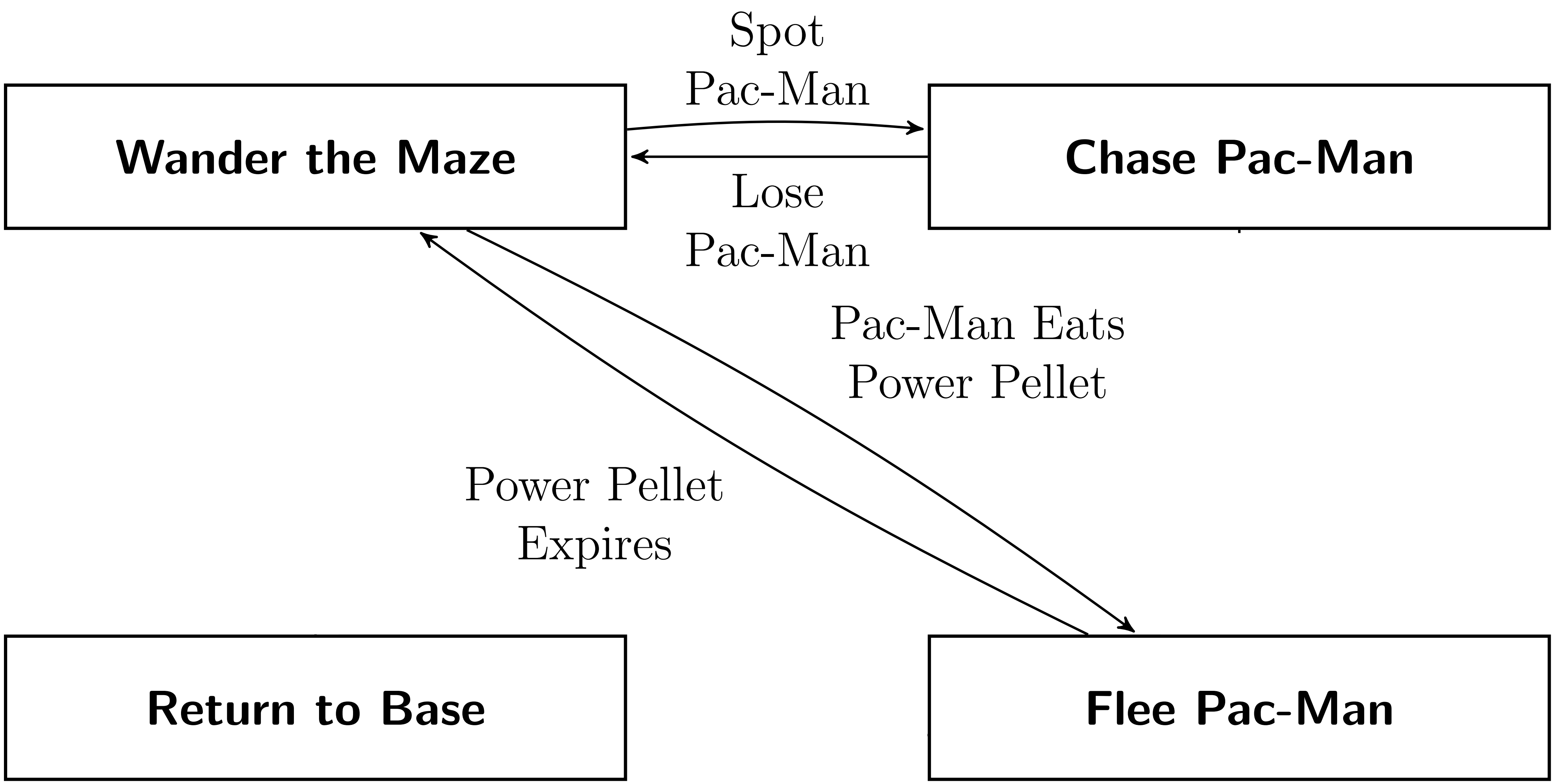
Return to Base

Flee Pac-Man

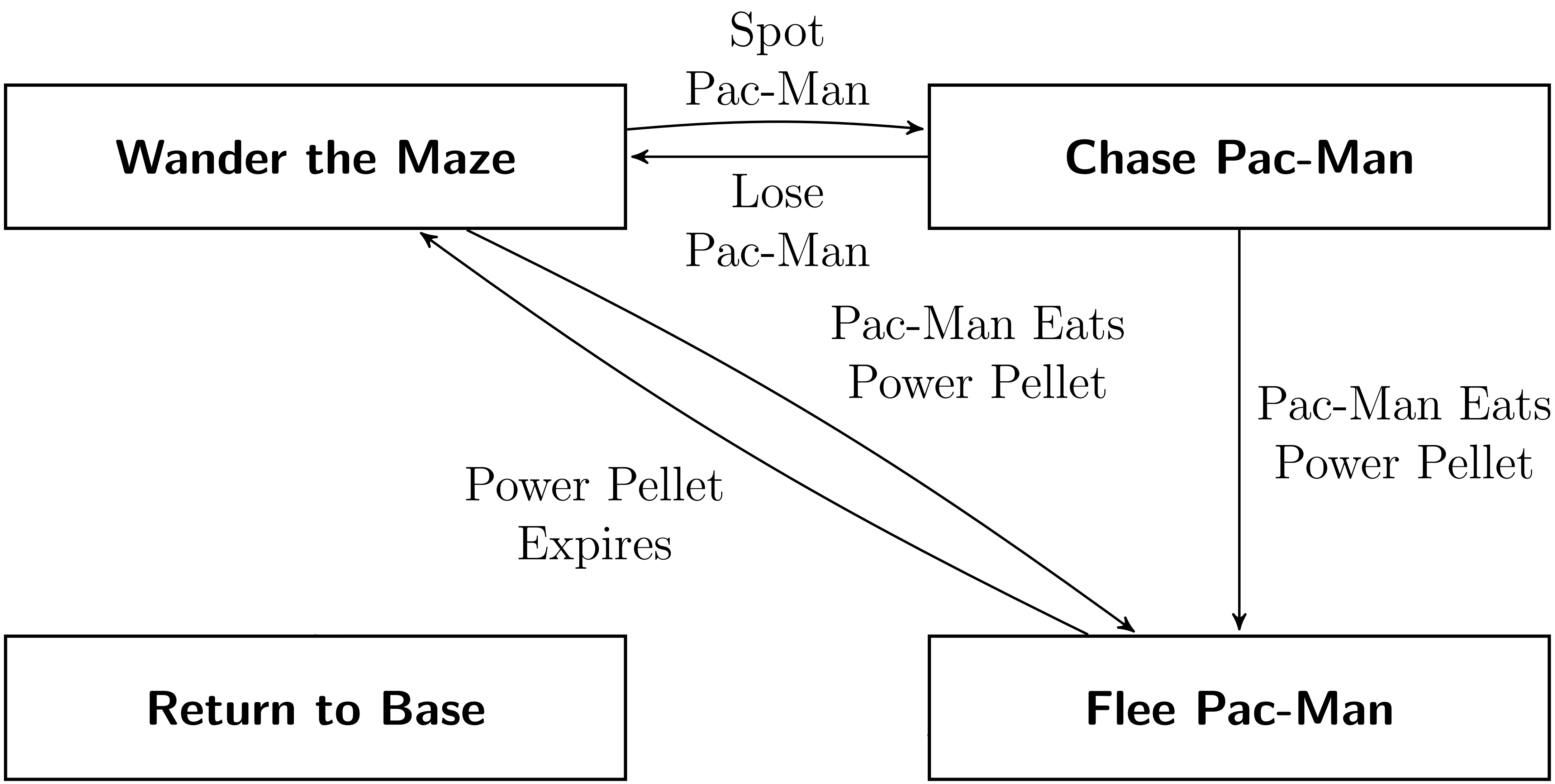
# Example: Pac-Man Ghosts



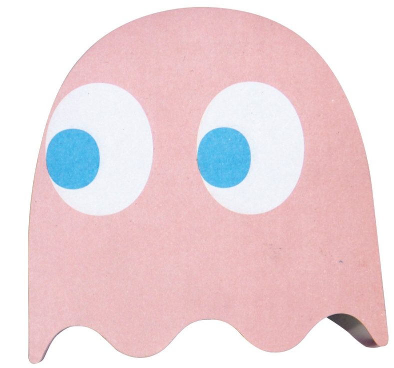
# Example: Pac-Man Ghosts

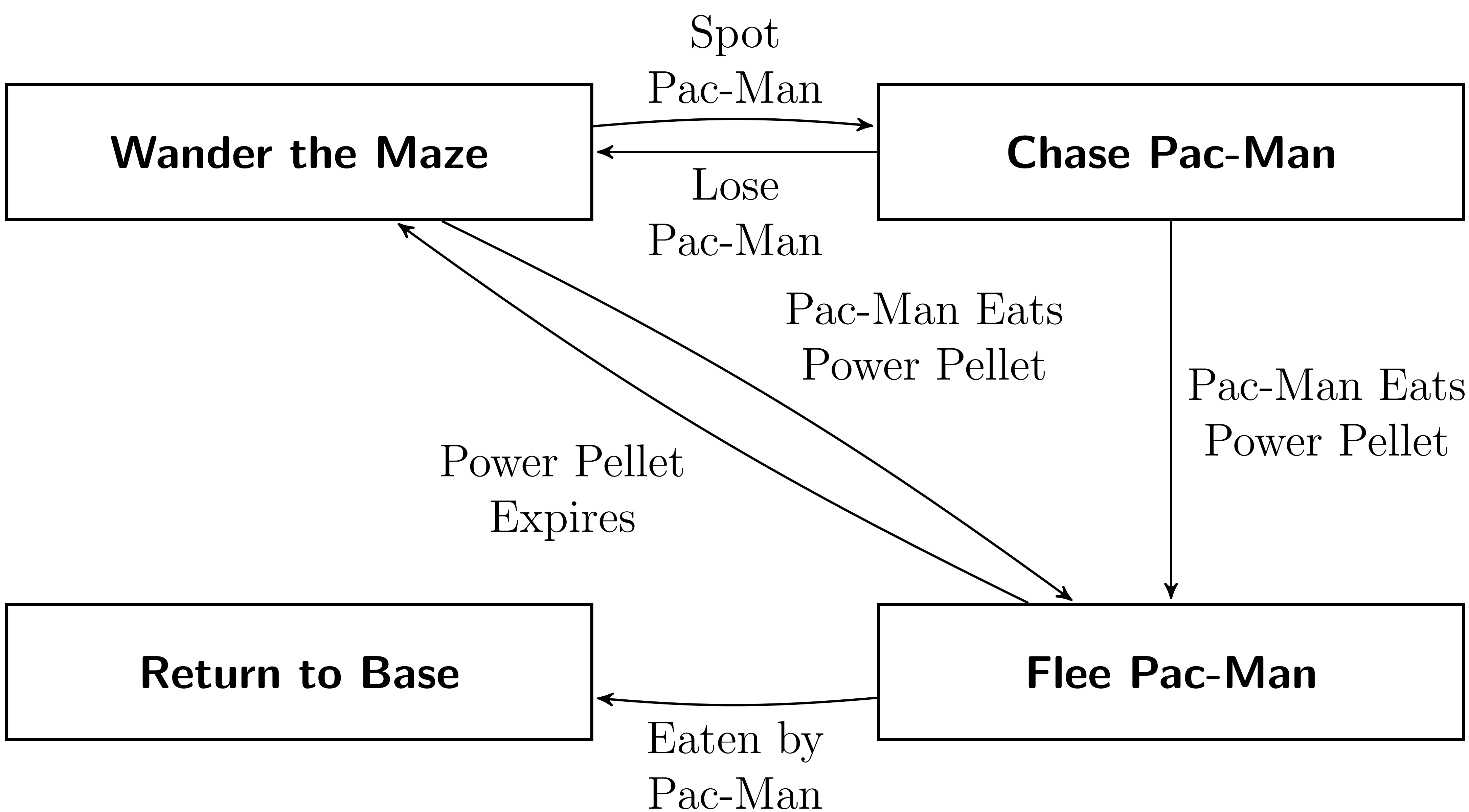


# Example: Pac-Man Ghosts

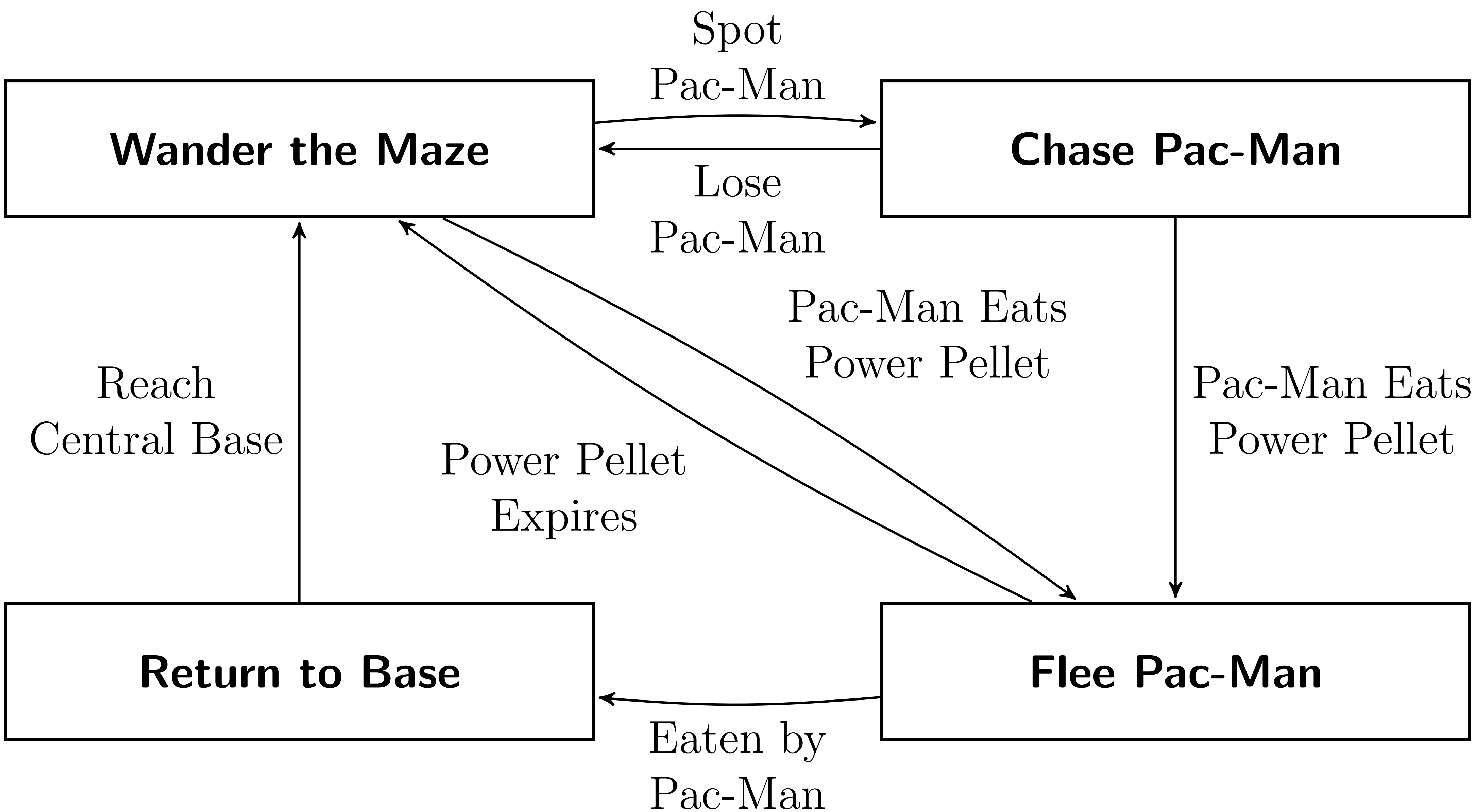


# Example: Pac-Man Ghosts

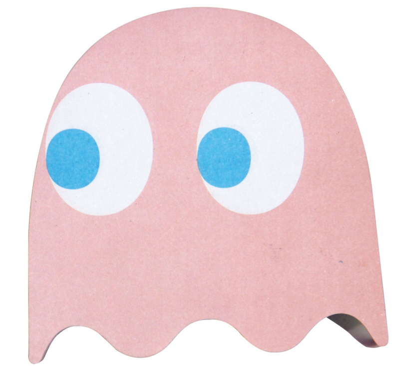


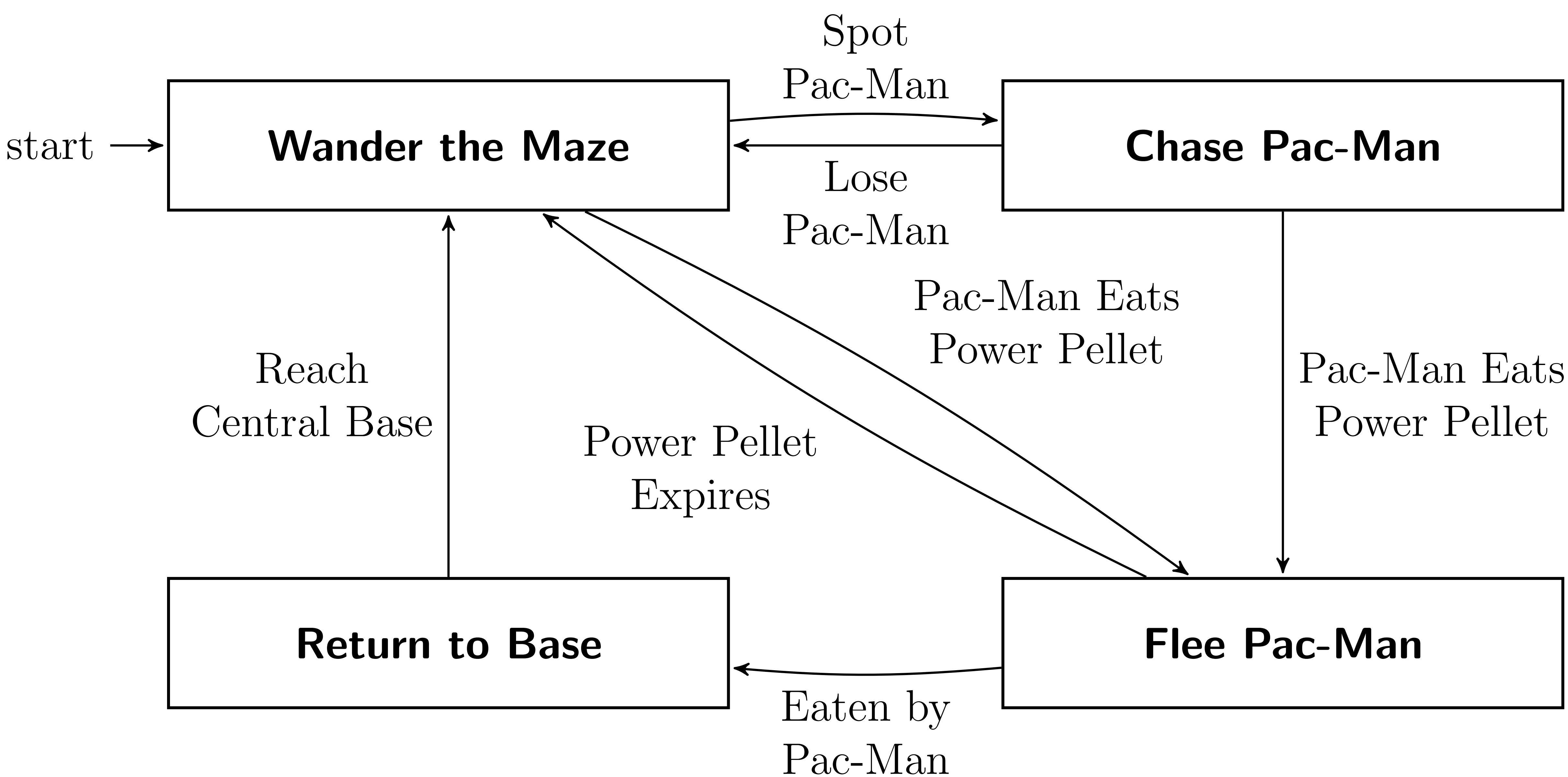


# Example: Pac-Man Ghosts



# Example:
# Pac-Man Ghosts





# Exercises

Without adding states, draw the automata for a SuperGhost that can’t be eaten.
It chases Pac-Man when the power pill is eaten, and it returns to base if Pac-Man eats a piece of fruit.

Choose a favourite (video) game, and try drawing the state automata for one of the computer controlled characters in that game.

# From automata
# to Petri nets

# Some basis

Are you familiar with the following concepts?

Set notation

$$\emptyset \quad a \in A \quad A \subseteq B \quad A \times B \quad \wp(A)$$

Functions

$$f : A \to B$$

Predicate logic

$$\mathbf{tt} \quad \mathbf{ff} \quad P \wedge Q \quad P \vee Q \quad \neg P \quad P \to Q \quad \exists x.P(x) \quad \forall x.P(x)$$

Induction principle (base cases + inductive cases)

$$(\ P(0) \quad \wedge \quad \forall n.(\ P(n) \Rightarrow P(succ(n))\ )\ ) \ \Rightarrow\ \forall n.P(n)$$

# Kleene-star notation A*

Given a set $A$ we denote by $A^*$
the set of finite sequences of elements in $A$, i.e.:
$A^* = \{ a_1 \cdots a_n \mid n \geq 0 \wedge a_1, ..., a_n \in A \}$
We denote the empty sequence by $\epsilon \in A^*$

For example:
$A = \{ a, b \}$ $\qquad$ $A^* = \{ \epsilon, a, b, aa, ab, ba, bb, aaa, aab, ... \}$

# Inductive definitions

A natural number is either:

- $0$
- or the successor $n+1$ of a natural number $n$

A sequence over the alphabet $A$ is either:

- the empty sequence $\varepsilon$
- or the juxtaposition $wa$ of a sequence $w$ with an element $a$ of $A$

# Recursively defined functions

Let us define the exponential function

**base case:** for any $k>0$ we set
$exp(k,0) = 1$

**inductive case:** for any $k>0$, $n \geq 0$ we set
$exp(k,n+1) = exp(k,n) \times k$

# Recursively defined functions

Let us define the exponential function

**base case:** for any $k>0$ we set

$exp(k,0) = 1$

**inductive case:** for any $k>0$, $n \geq 0$ we set

$exp(k,n+1) = \boxed{exp(k,n)} \times k$

# Recursively defined functions

Let us define the exponential function

**base case:** for any $k>0$ we set
$exp(k,0) = 1$

**inductive case:** for any $k>0$, $n \geq 0$ we set
$exp(k,\boxed{n+1}) = exp(k,\boxed{n})$ x $k$

# DFA

A **Deterministic Finite Automaton** (**DFA**) is a tuple $A = (Q, \Sigma, \delta, q_0, F)$, where

- $Q$ is a finite set of states;
- $\Sigma$ is a finite set of input symbols;
- $\delta : Q \times \Sigma \to Q$ is the transition function;
- $q_0 \in Q$ is the initial state (also called start state);
- $F \subseteq Q$ is the set of final states (also accepting states)

# Extended transition function (destination function)

Given $A = (Q, \Sigma, \delta, q_0, F)$, we define $\widehat{\delta} : Q \times \Sigma^* \to Q$ by induction:

**base case:** For any $q \in Q$ we let

$$\widehat{\delta}(q, \epsilon) = q$$

**inductive case:** For any $q \in Q, a \in \Sigma, w \in \Sigma^*$ we let

$$\widehat{\delta}(q, wa) = \delta(\ \widehat{\delta}(q, w)\ ,\ a\ )$$

($\widehat{\delta}(q, w)$ returns the state reached from $q$ by observing $w$)

# Extended transition function (destination function)

Given $A = (Q, \Sigma, \delta, q_0, F)$, we define $\widehat{\delta} : Q \times \Sigma^* \to Q$ by induction:

**base case:** For any $q \in Q$ we let

$$\widehat{\delta}(q, \epsilon) = q$$

**inductive case:** For any $q \in Q, a \in \Sigma, w \in \Sigma^*$ we let

$$\widehat{\delta}(q, wa) = \delta(\boxed{\widehat{\delta}(q, w)}\,,\ a\ )$$

($\widehat{\delta}(q, w)$ returns the state reached from $q$ by observing $w$)

# Extended transition function (destination function)

Given $A = (Q, \Sigma, \delta, q_0, F)$, we define $\widehat{\delta} : Q \times \Sigma^* \to Q$ by induction:

**base case:** For any $q \in Q$ we let

$$\widehat{\delta}(q, \epsilon) = q$$

**inductive case:** For any $q \in Q, a \in \Sigma, w \in \Sigma^*$ we let

$$\widehat{\delta}(q, \boxed{wa}) = \delta(\ \widehat{\delta}(q, \boxed{w})\ ,\ a\ )$$

($\widehat{\delta}(q, w)$ returns the state reached from $q$ by observing $w$)

# String processing

Given $A = (Q, \Sigma, \delta, q_0, F)$ and $w \in \Sigma^*$ we say that $A$ **accept** $w$ iff

$$\widehat{\delta}(q_0, w) \in F$$

The **language** of $A = (Q, \Sigma, \delta, q_0, F)$ is

$$L(A) = \{ \ w \ \mid \ \widehat{\delta}(q_0, w) \in F \ \}$$

# Transition diagram

We represent $A = (Q, \Sigma, \delta, q_0, F)$ as a graph s.t.

- $Q$ is the set of nodes;
- $\{\, q \xrightarrow{a} q' \mid q' = \delta(q, a) \,\}$ is the set of arcs.

Plus some graphical conventions:

- there is one special arrow $Start$ with $\xrightarrow{Start} q_0$
- nodes in $F$ are marked by double circles;
- nodes in $Q \setminus F$ are marked by single circles.

# String processing as paths

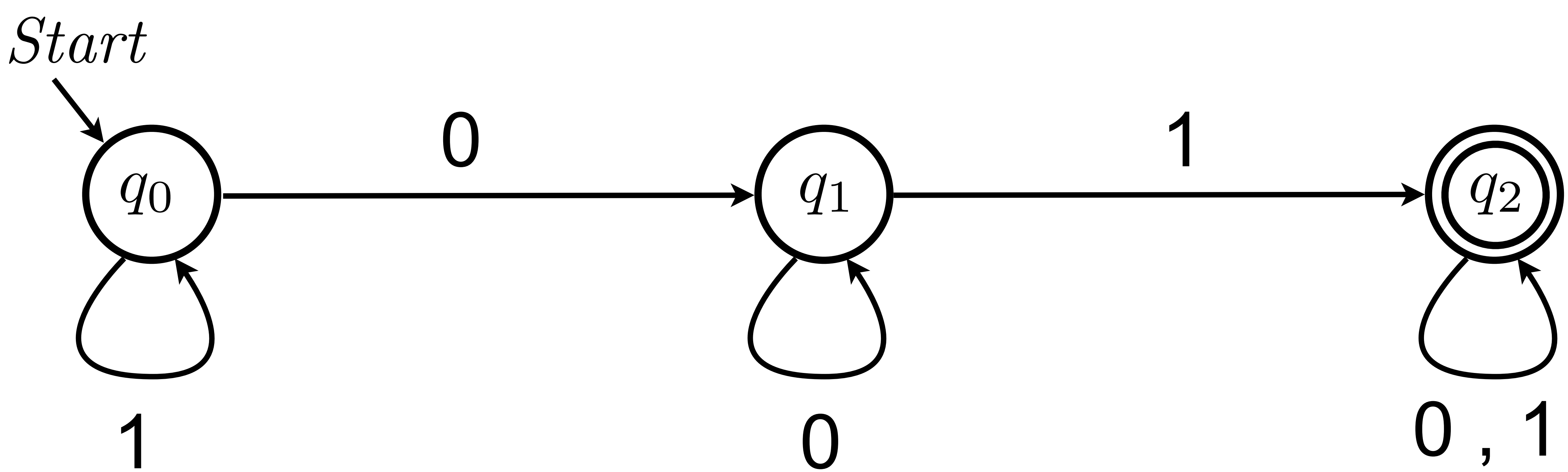
A DFA accepts a string $w$, if there is a path in its transition diagram such that:

it starts from the initial state

it ends in one final state

the sequence of labels in the path is exactly $w$

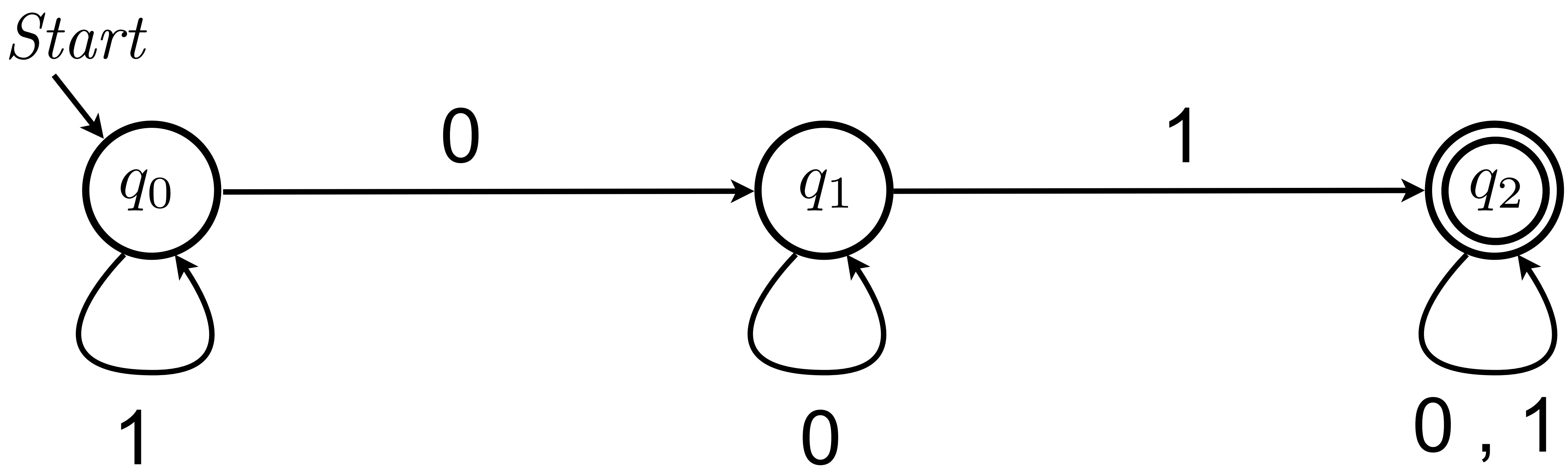
# DFA: example




| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 0 |

# DFA: example

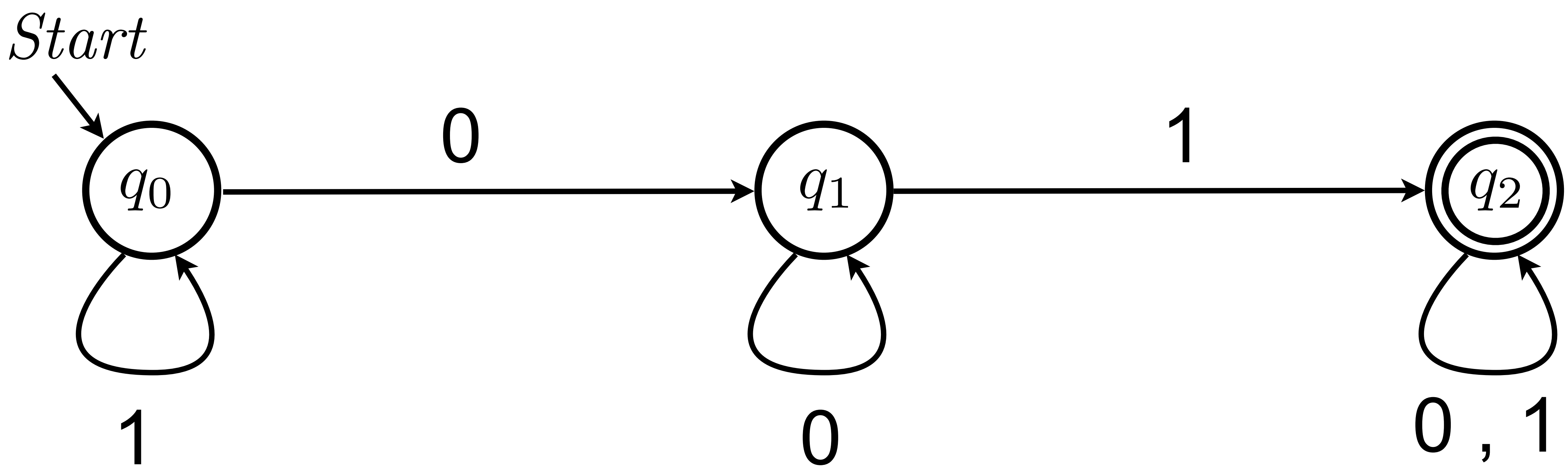



$q_0$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 0 |

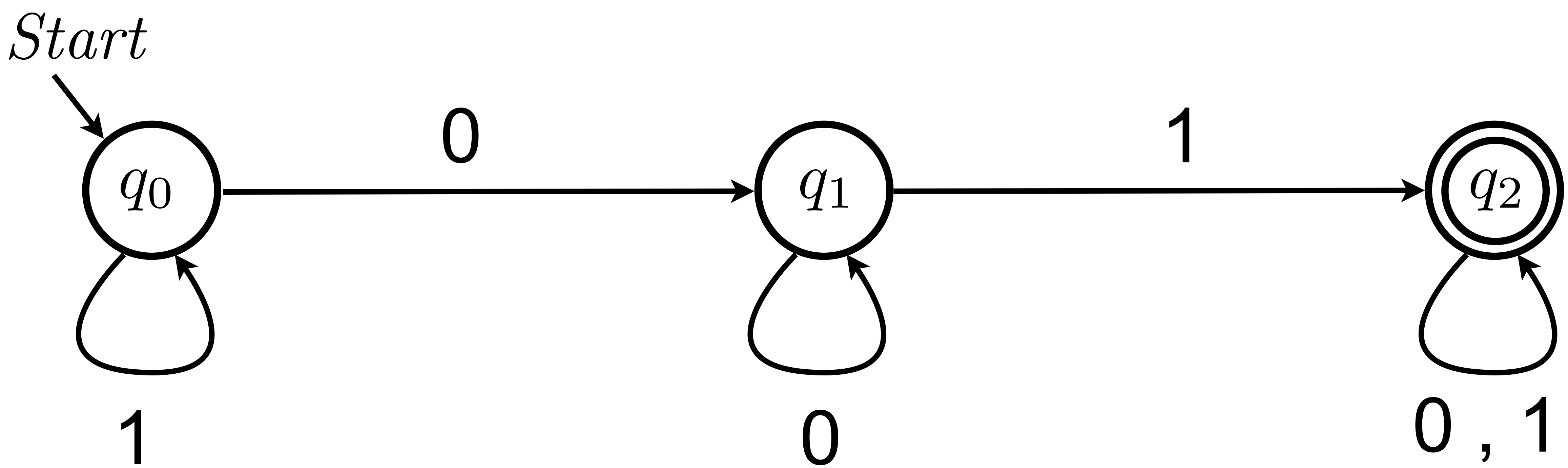
# DFA: example




| | | | | | |
|---|---|---|---|---|---|
| $q_0$ 1 | $q_0$ 1 | 1 | 0 | 0 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 0 |

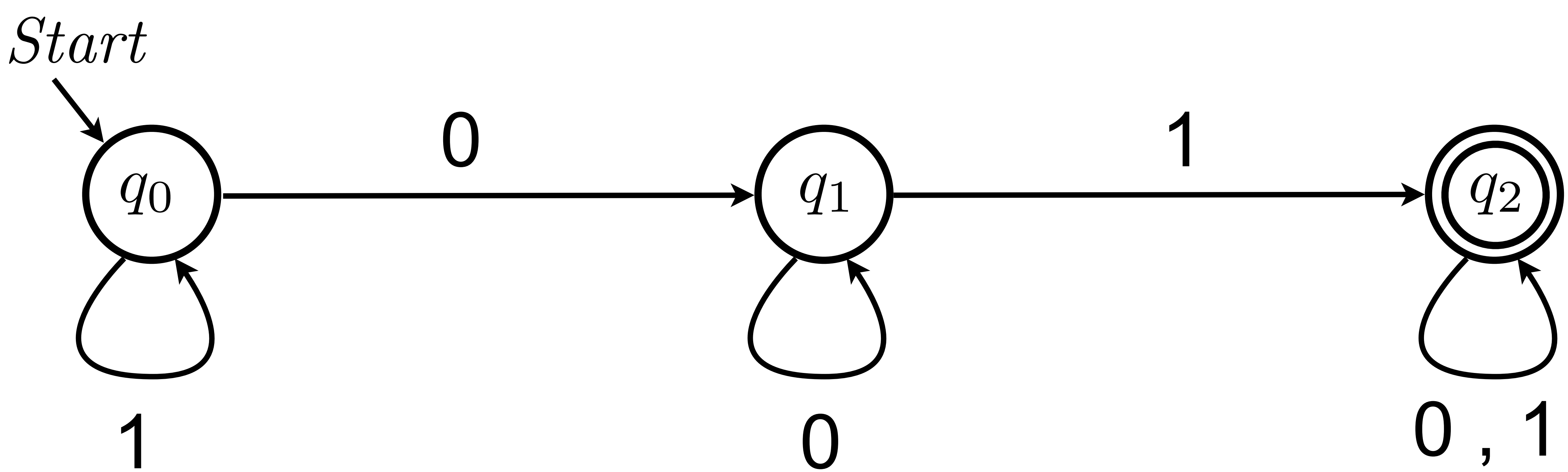
# DFA: example




| | | | | | |
|---|---|---|---|---|---|
| $q_0$ 1 | $q_0$ 1 | $q_0$ 1 | 0 | 0 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 0 |

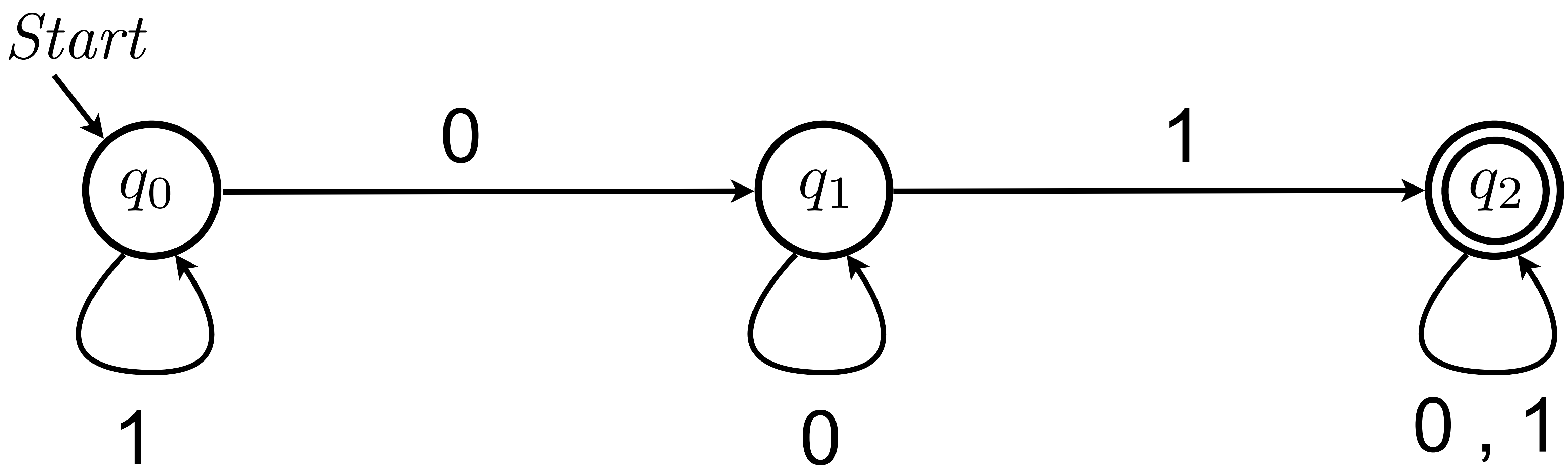
# DFA: example




| | | | | | |
|---|---|---|---|---|---|
| $q_0$ 1 | $q_0$ 1 | $q_0$ 1 | $q_0$ 0 | 0 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 0 |

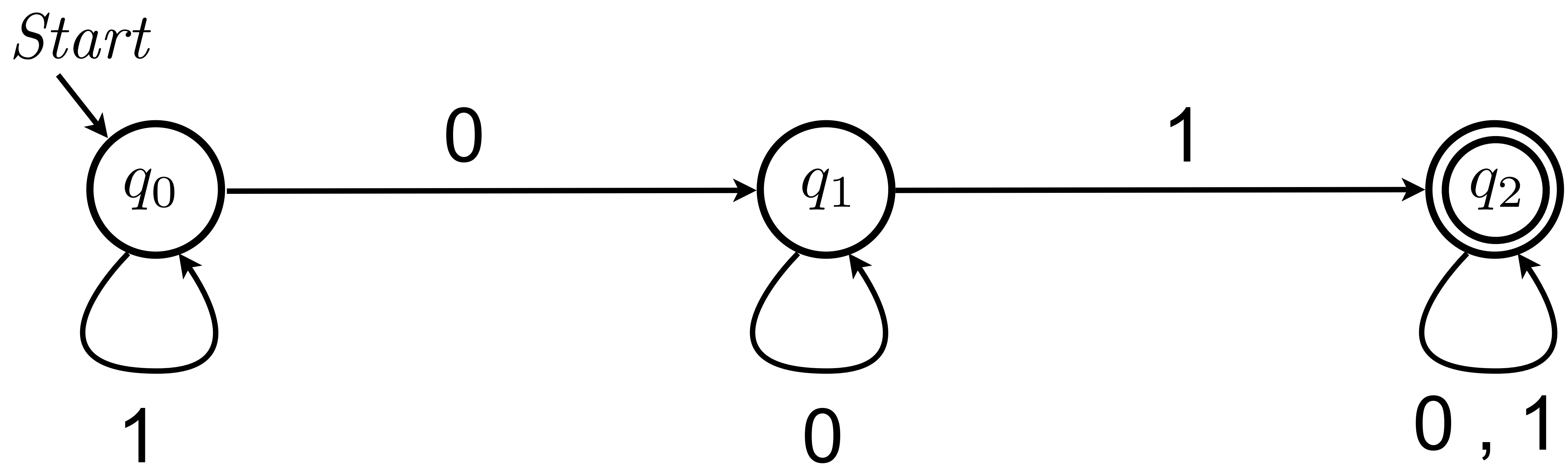
# DFA: example




| | | | | | |
|---|---|---|---|---|---|
| $q_0$ 1 | $q_0$ 1 | $q_0$ 1 | $q_0$ 0 | $q_1$ 0 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 0 |

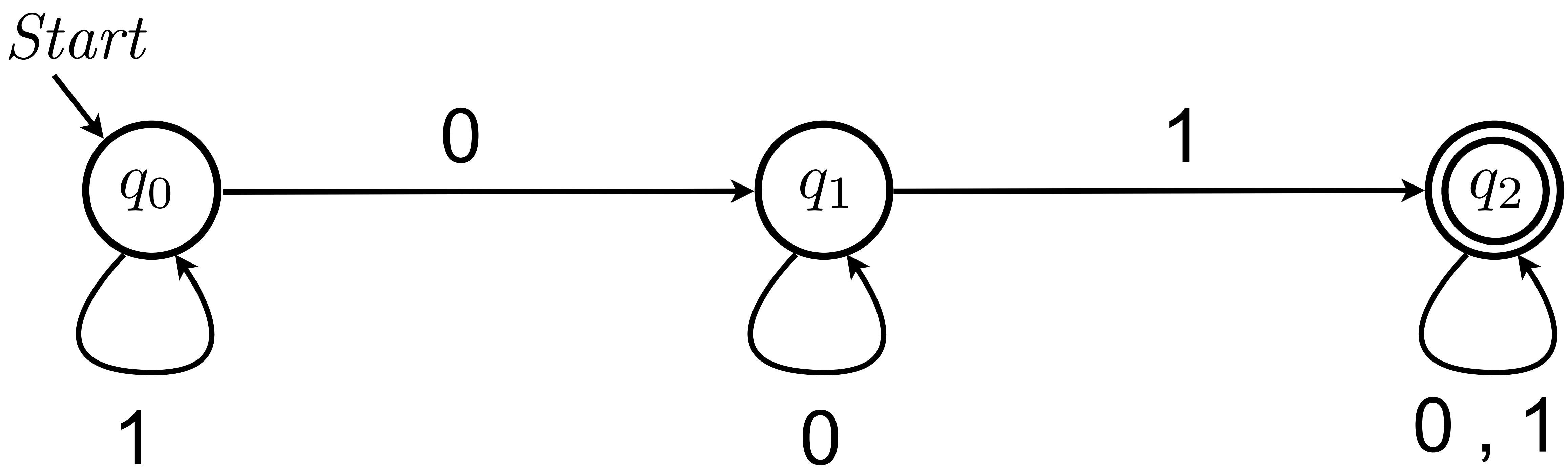
# DFA: example




| | | | | | |
|---|---|---|---|---|---|
| $q_0$ 1 | $q_0$ 1 | $q_0$ 1 | $q_0$ 0 | $q_1$ 0 | $q_1$ 0 |

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 0 |

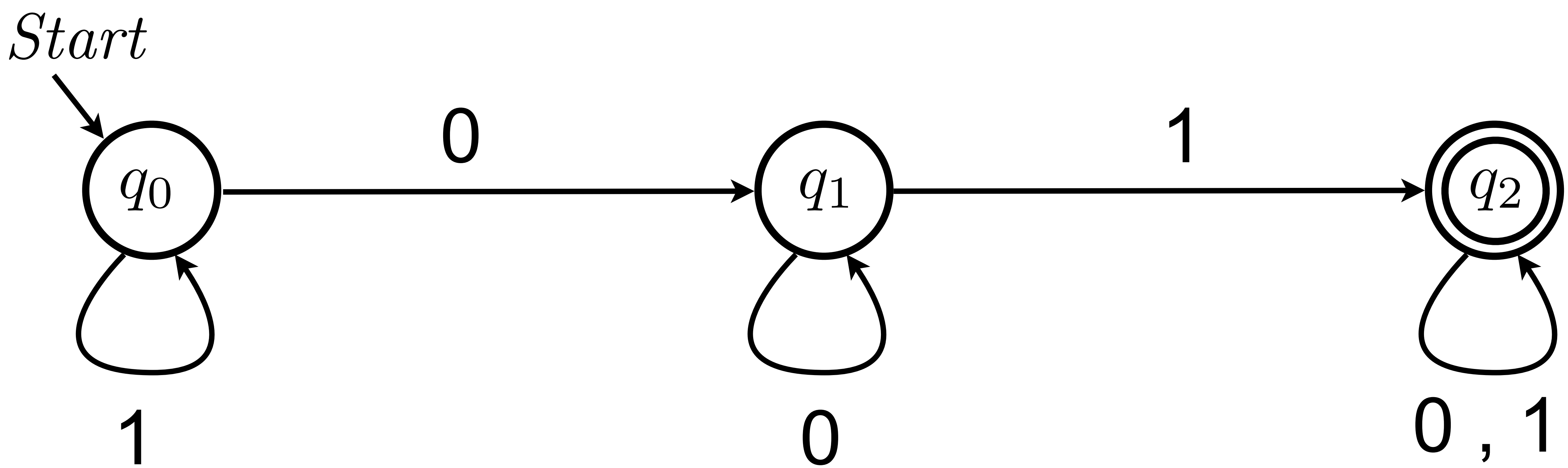
# DFA: example




| | | | | | |
|---|---|---|---|---|---|
| $q_0$ 1 | $q_0$ 1 | $q_0$ 1 | $q_0$ 0 | $q_1$ 0 | $q_1$ 0 $q_1$ |

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 0 |

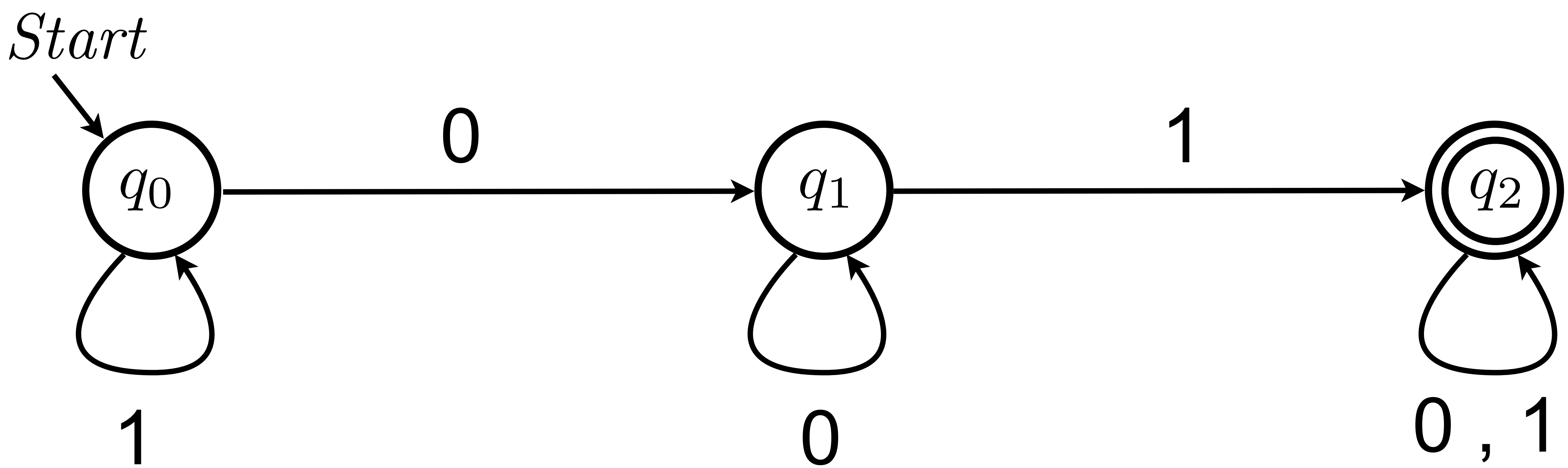
# DFA: example




$q_0$ | 1 | $q_0$ | 1 | $q_0$ | 1 | $q_0$ | 0 | $q_1$ | 0 | $q_1$ | 0 | $q_1 \notin F$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 0 |

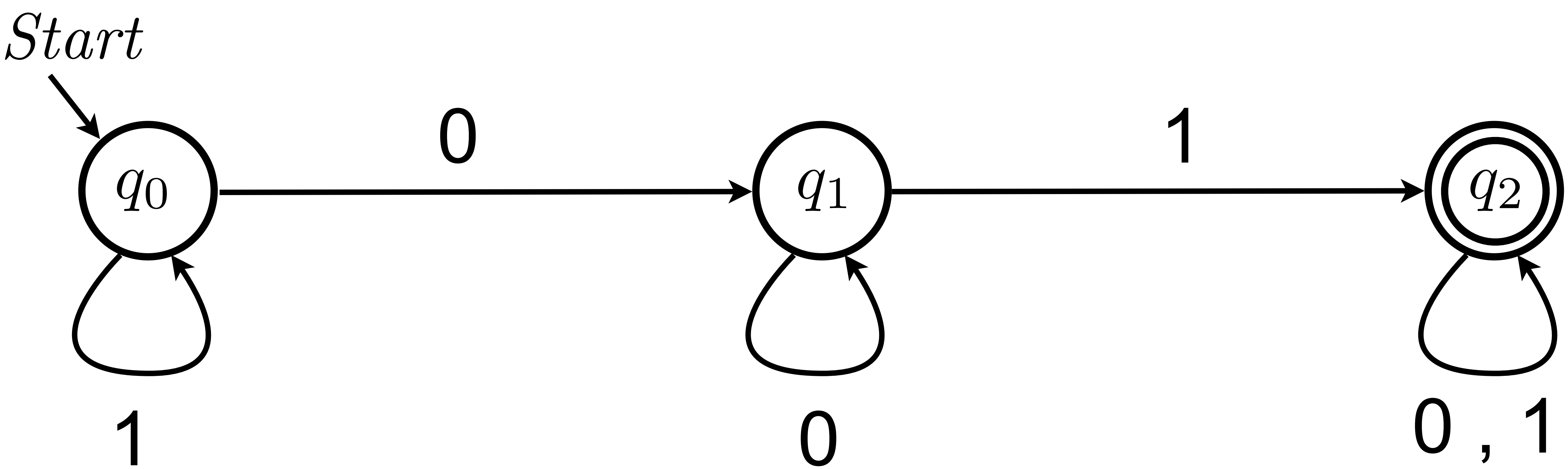
# DFA: example




| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $q_0$ | 1 | $q_0$ | 1 | $q_0$ | 1 | $q_0$ | 0 | $q_1$ | 0 | $q_1$ | 0 | $q_1 \notin F$ |

| | | | | | | |
|---|---|---|---|---|---|---|
| $q_0$ | 1 | 0 | 0 | 1 | 1 | 0 |

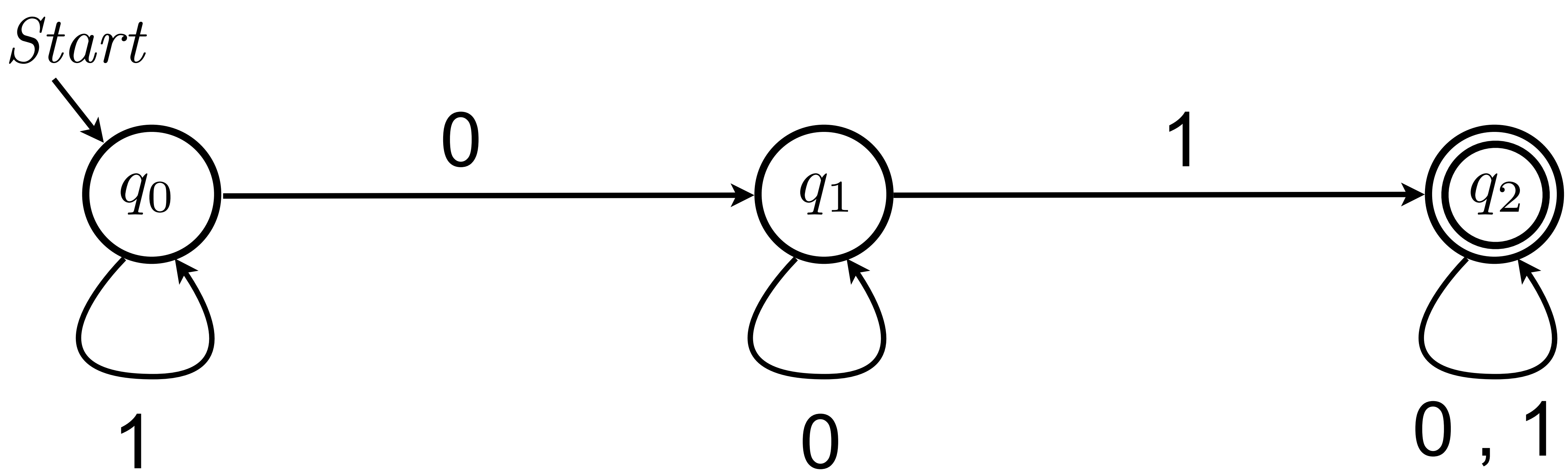
# DFA: example




$q_0$ 1 $q_0$ 1 $q_0$ 1 $q_0$ 0 $q_1$ 0 $q_1$ 0 $q_1 \notin F$

$q_0$ 1 $q_0$ 0 0 1 1 0

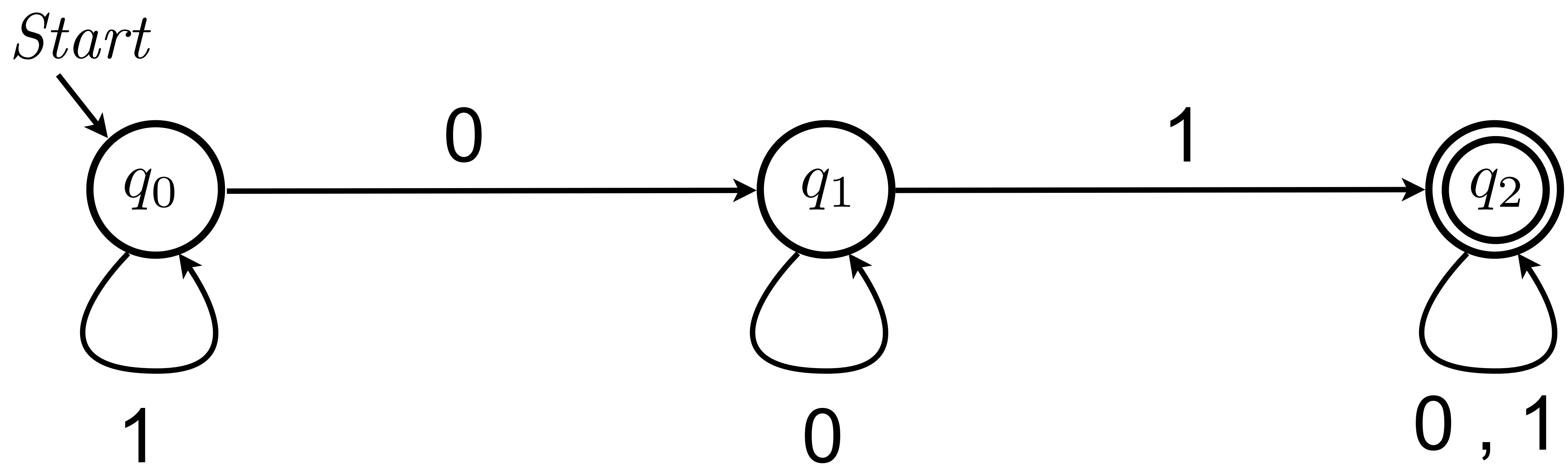
# DFA: example




| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $q_0$ | 1 | $q_0$ | 1 | $q_0$ | 1 | $q_0$ | 0 | $q_1$ | 0 | $q_1$ | 0 | $q_1 \notin F$ |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $q_0$ | 1 | $q_0$ | 0 | $q_1$ | 0 | | 1 | | 1 | | 0 |

# DFA: example



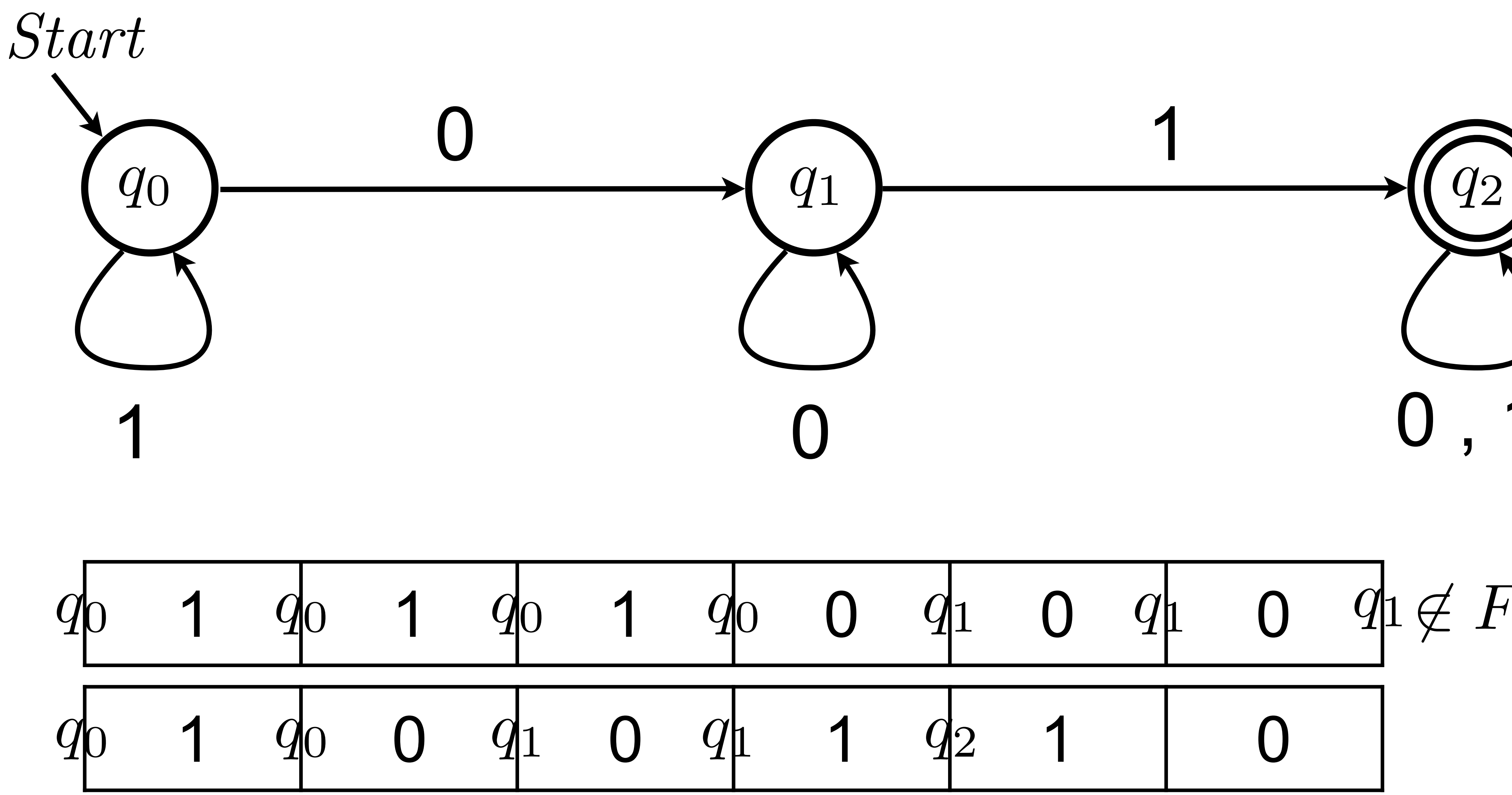

$q_0$ | 1 | $q_0$ | 1 | $q_0$ | 1 | $q_0$ | 0 | $q_1$ | 0 | $q_1$ | 0 | $q_1 \notin F$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 |

$q_0$ | 1 | $q_0$ | 0 | $q_1$ | 0 | $q_1$ | 1 | 1 | 0

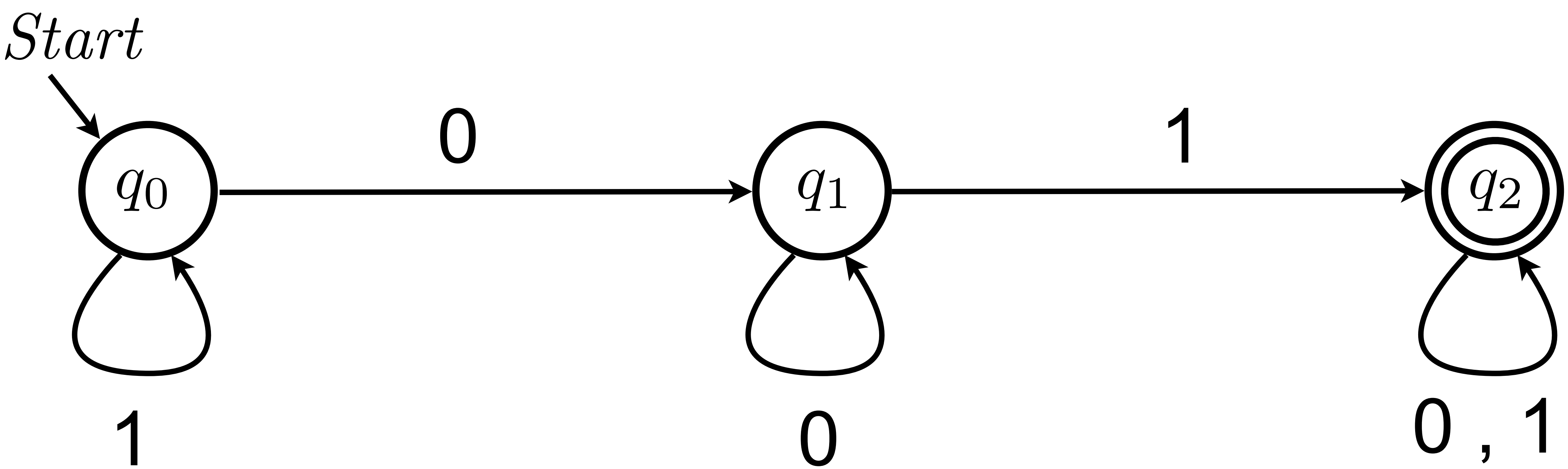
| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 0 |

# DFA: example



$Start$
$q_0$
$q_1$
$q_2$
0
1
1
0
0 , 1
$q_0$ 1 $q_0$ 1 $q_0$ 1 $q_0$ 0 $q_1$ 0 $q_1$ 0 $q_1 \notin F$
$q_0$ 1 $q_0$ 0 $q_1$ 0 $q_1$ 1 $q_2$ 1 0

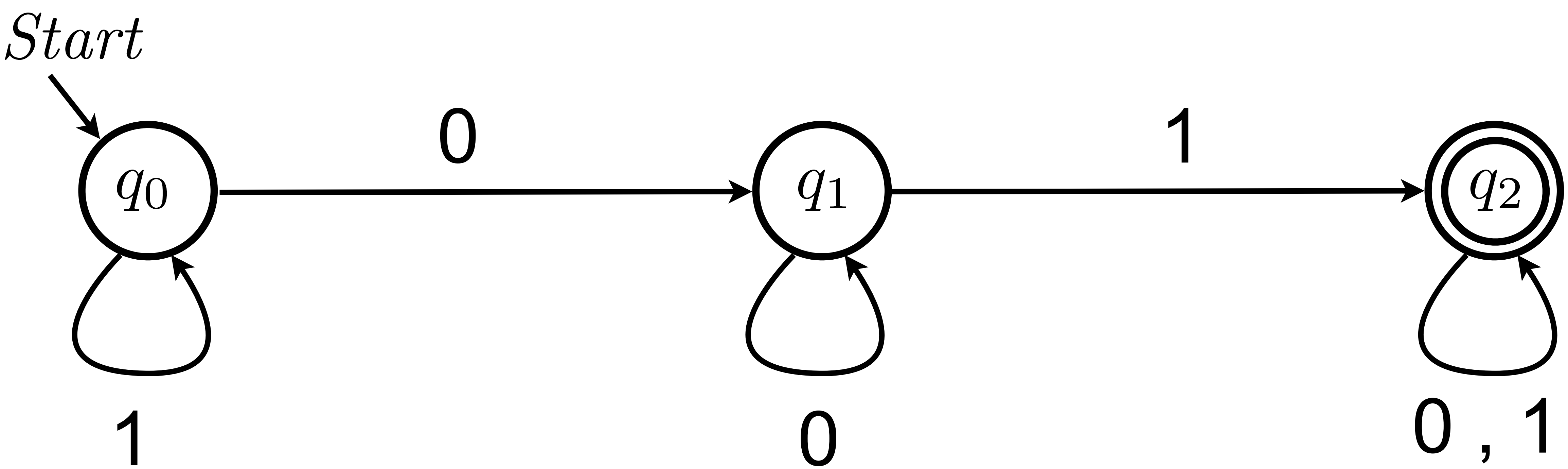
# DFA: example




| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $q_0$ | 1 | $q_0$ | 1 | $q_0$ | 1 | $q_0$ | 0 | $q_1$ | 0 | $q_1$ | 0 | $q_1 \notin F$ |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $q_0$ | 1 | $q_0$ | 0 | $q_1$ | 0 | $q_1$ | 1 | $q_2$ | 1 | $q_2$ | 0 |

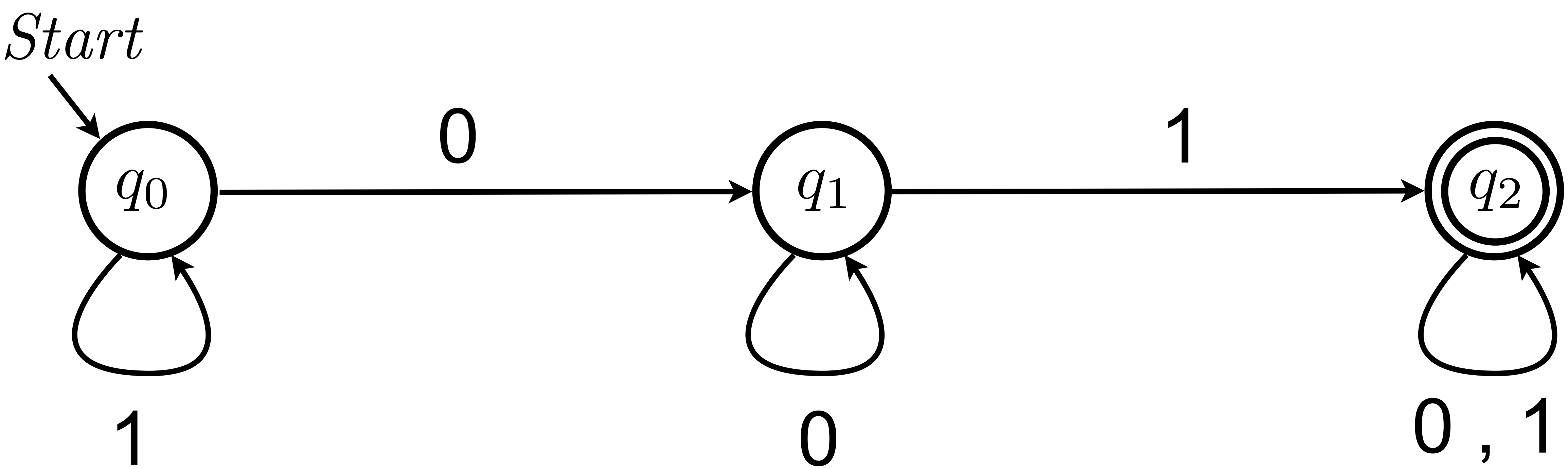
# DFA: example




$$q_0 \; 1 \; q_0 \; 1 \; q_0 \; 1 \; q_0 \; 0 \; q_1 \; 0 \; q_1 \; 0 \; q_1 \notin F$$

$$q_0 \; 1 \; q_0 \; 0 \; q_1 \; 0 \; q_1 \; 1 \; q_2 \; 1 \; q_2 \; 0 \; q_2$$

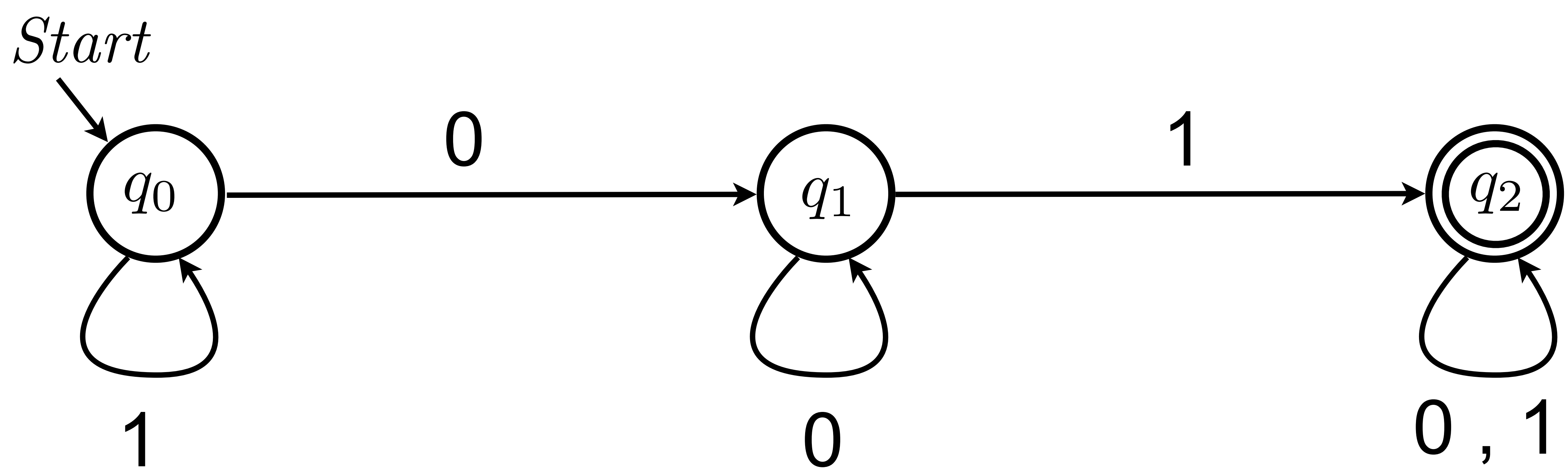
# DFA: example




$$q_0 \;\; 1 \;\; q_0 \;\; 1 \;\; q_0 \;\; 1 \;\; q_0 \;\; 0 \;\; q_1 \;\; 0 \;\; q_1 \;\; 0 \;\; q_1 \notin F$$

$$q_0 \;\; 1 \;\; q_0 \;\; 0 \;\; q_1 \;\; 0 \;\; q_1 \;\; 1 \;\; q_2 \;\; 1 \;\; q_2 \;\; 0 \;\; q_2 \in F$$

# DFA: question time




Does it accept 100 ?
Does it accept 011 ?
Does it accept 1010010 ?
What is L(A) ?

# Transition table

Conventional tabular representation

its rows are in correspondence with states

its columns are in correspondence with input symbols

its entries are the states reached after the transition

Plus some decoration

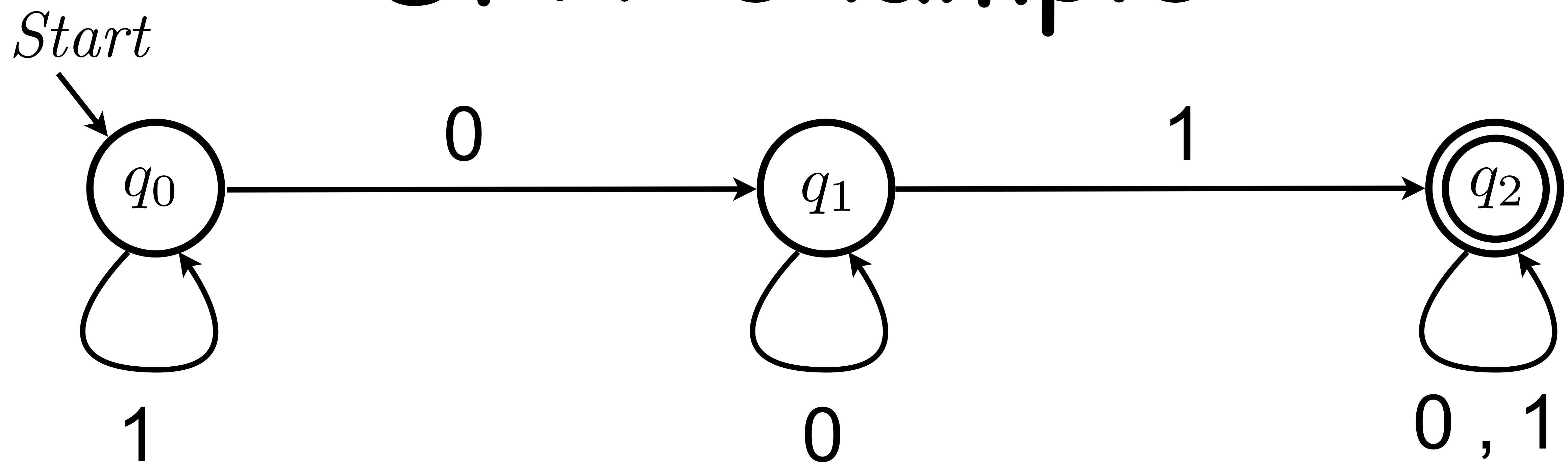
start state decorated with an arrow

all final states decorated with *

# Transition table

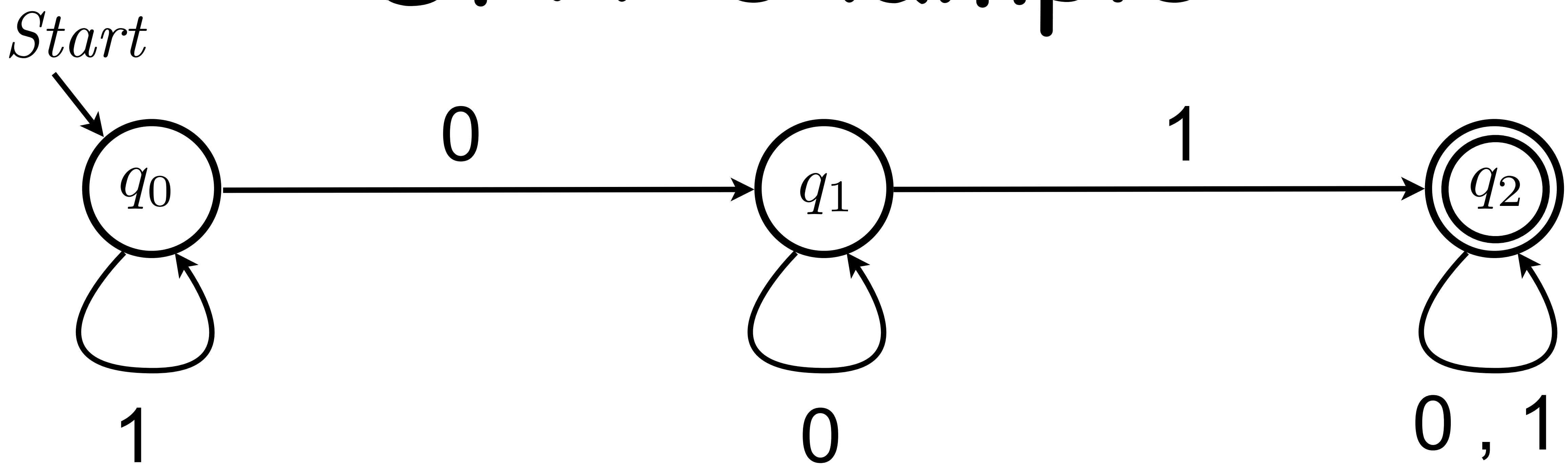
| | | | | | $\mathbf{a}$ | | | |
|---|---|---|---|---|---|---|---|---|
| $\rightarrow$ | | | | | | | | |
| | | | | | | | | |
| | $\mathbf{q}$ | | | | $\delta(q, a)$ | | | |
| $*$ | | | | | | | | |
| $*$ | | | | | | | | |
| | | | | | | | | |

# DFA: example



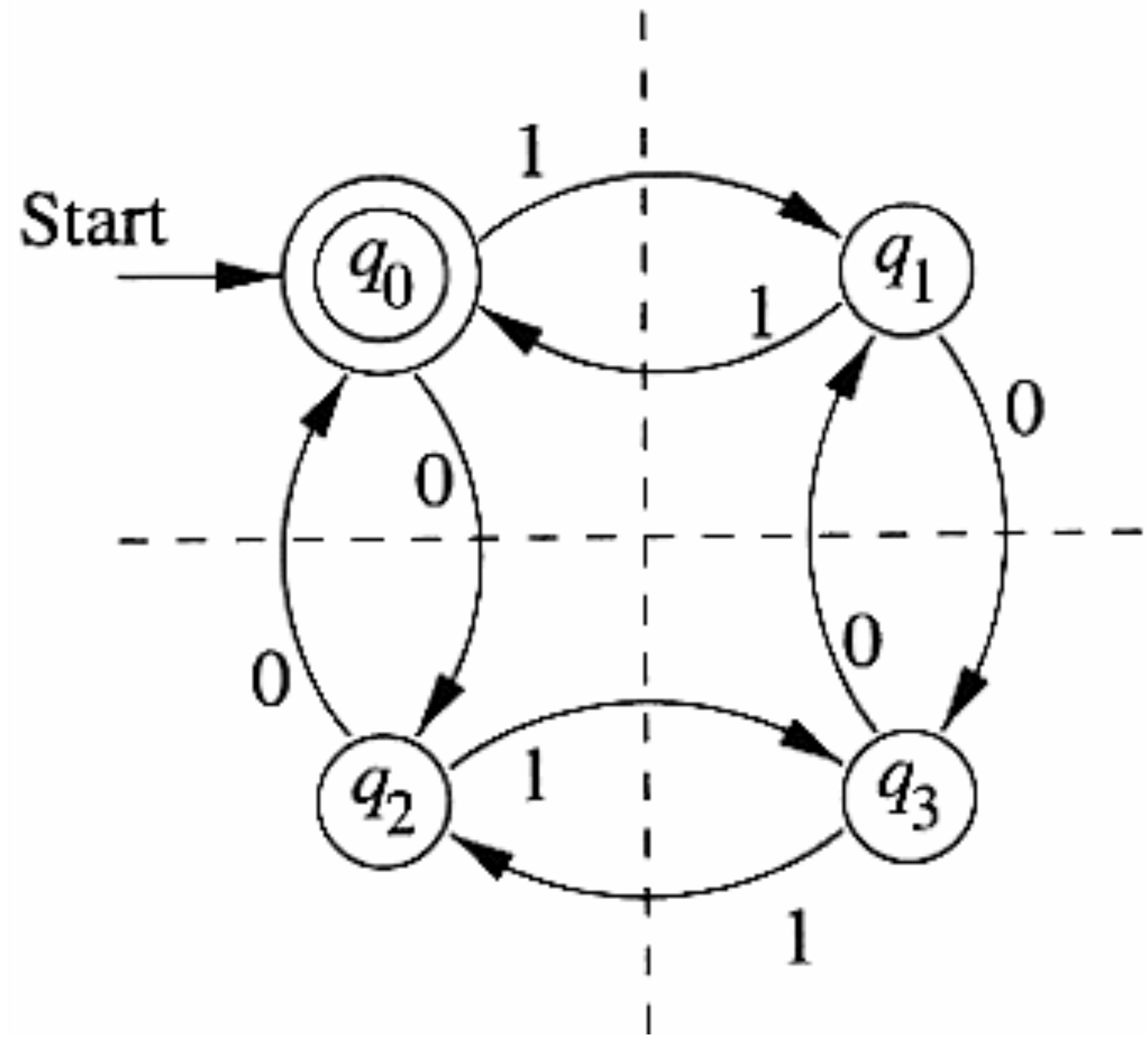

| | $\mathbf{0}$ | $\mathbf{1}$ |
|---|---|---|
| $\rightarrow \mathbf{q_0}$ | | |
| $\mathbf{q_1}$ | | |
| $\star\ \mathbf{q_2}$ | | |

# DFA: example




| | $\mathbf{0}$ | $\mathbf{1}$ |
|---|---|---|
| $\rightarrow \mathbf{q_0}$ | $q_1$ | $q_0$ |
| $\mathbf{q_1}$ | $q_1$ | $q_2$ |
| $\star\ \mathbf{q_2}$ | $q_2$ | $q_2$ |

# DFA: exercise




Does it accept 100 ? Does it accept 1010 ?
Write its transition table. What is L(A) ?

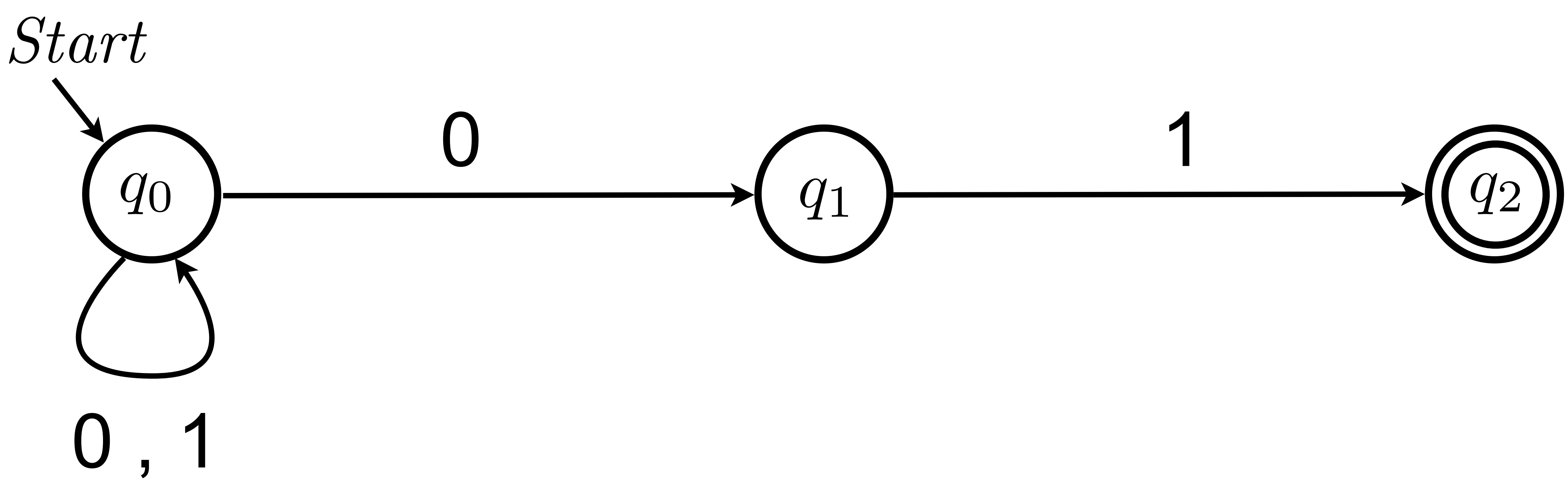
# NFA

A **Non-deterministic Finite Automaton** (**NFA**) is a tuple $A = (Q, \Sigma, \delta, q_0, F)$, where

- $Q$ is a finite set of states;
- $\Sigma$ is a finite set of input symbols;

  powerset of Q = set of sets over Q
- $\delta : Q \times \Sigma \to \wp(Q)$ is the transition function;
- $q_0 \in Q$ is the initial state (also called start state);
- $F \subseteq Q$ is the set of final states (also accepting states)

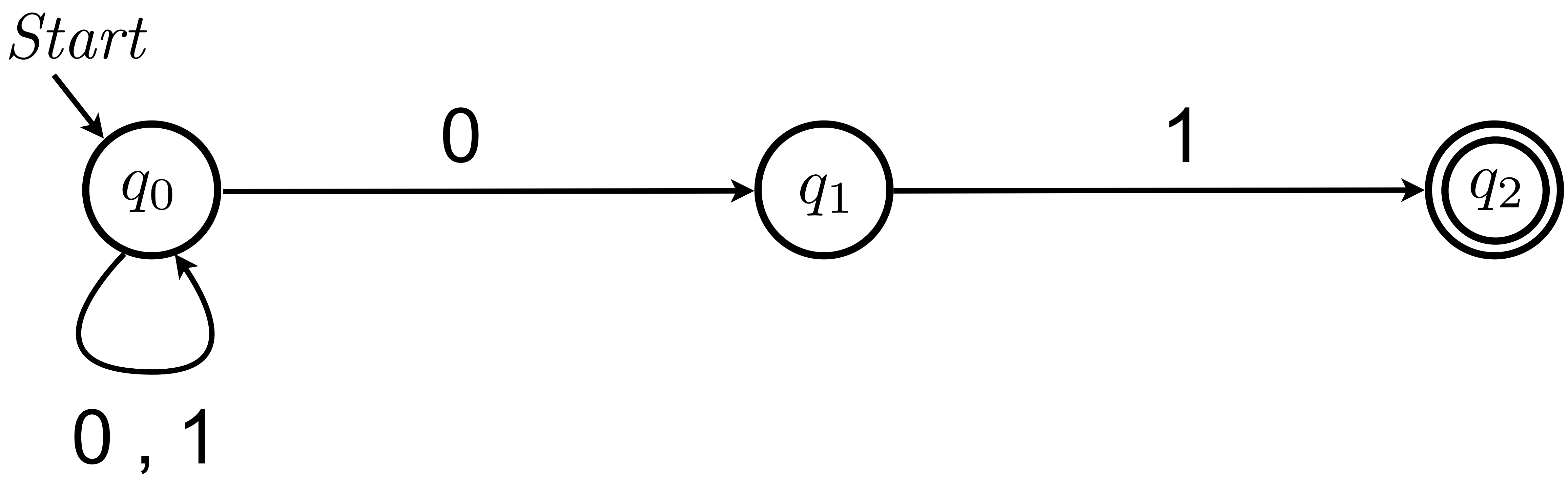
# NFA: example




Can you explain why it is not a DFA?

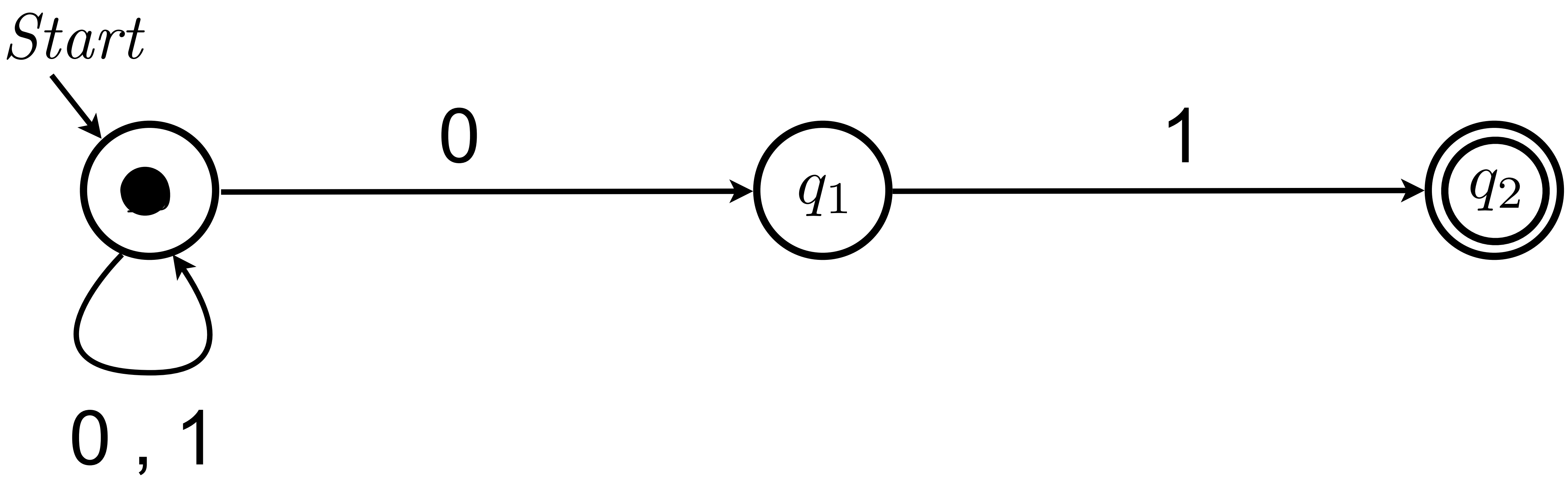
# Reshaping

# Step 1: get a token



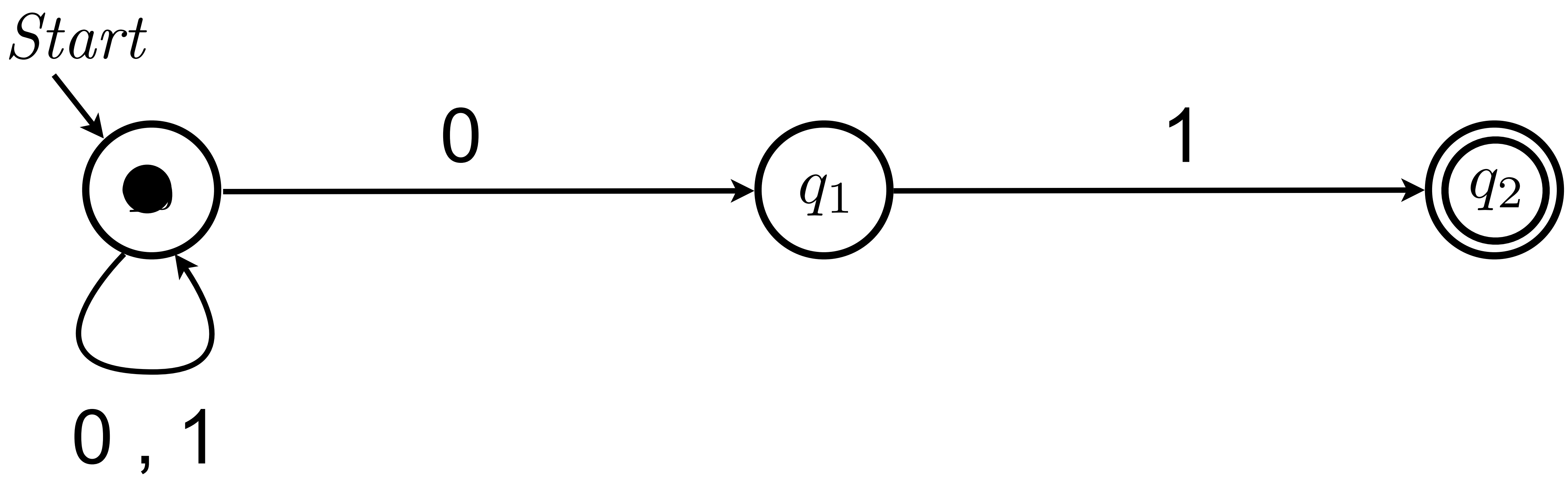

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 1 |

# Step 1: get a token



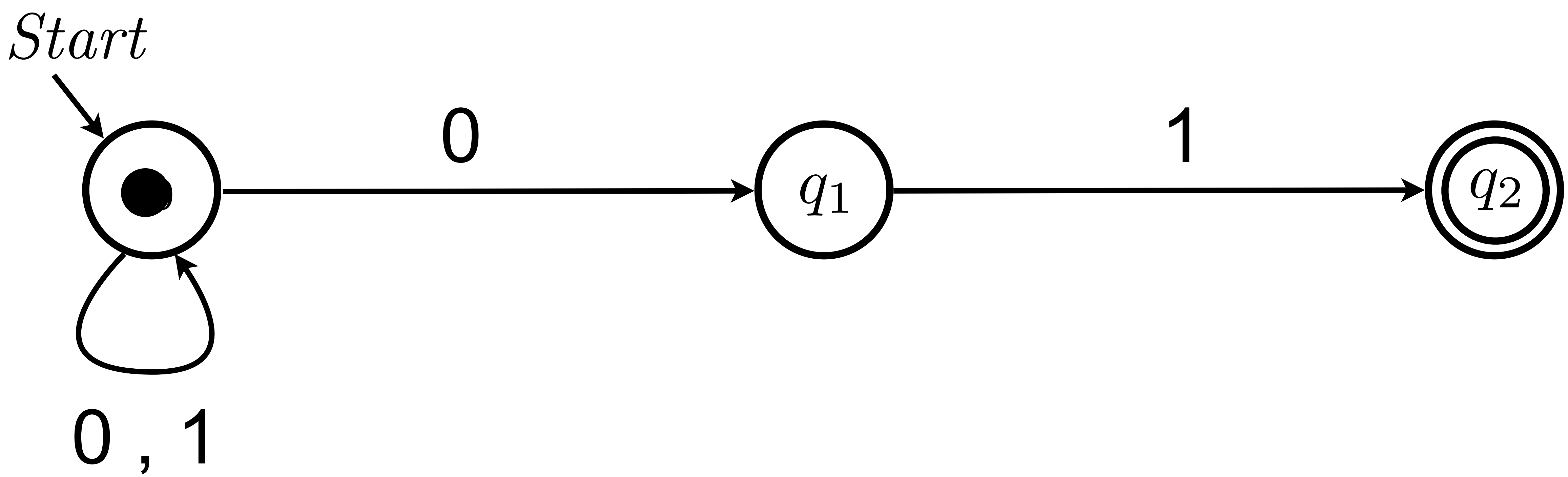

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 1 |

# Step 1: get a token



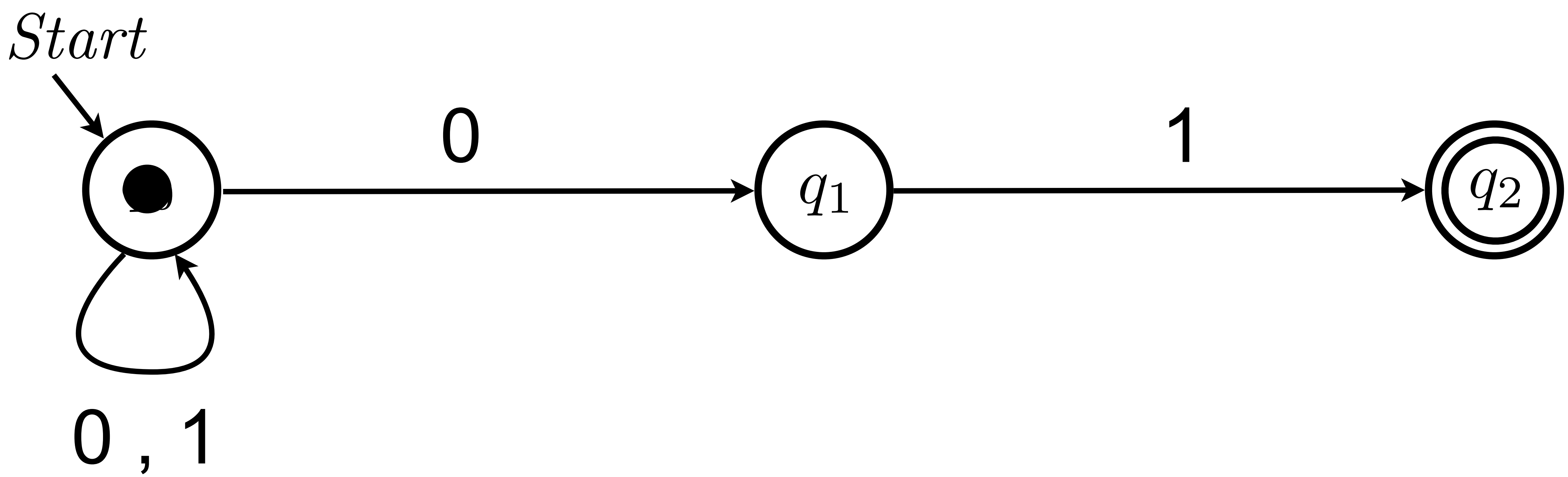

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 1 |

# Step 1: get a token



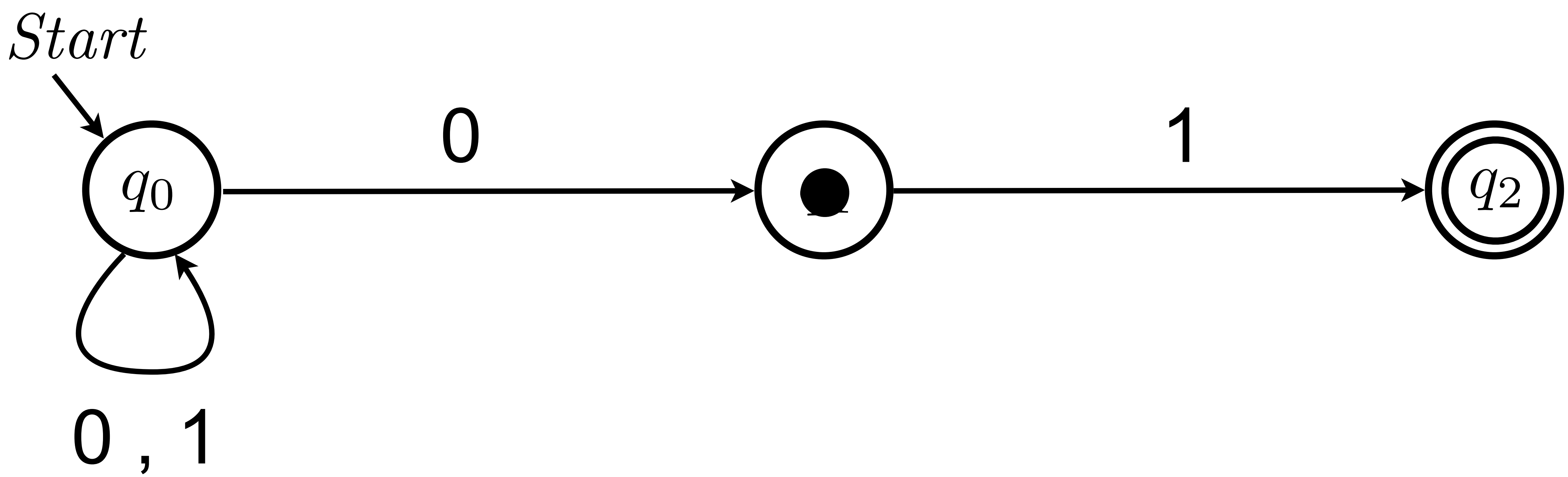

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 1 |

# Step 1: get a token



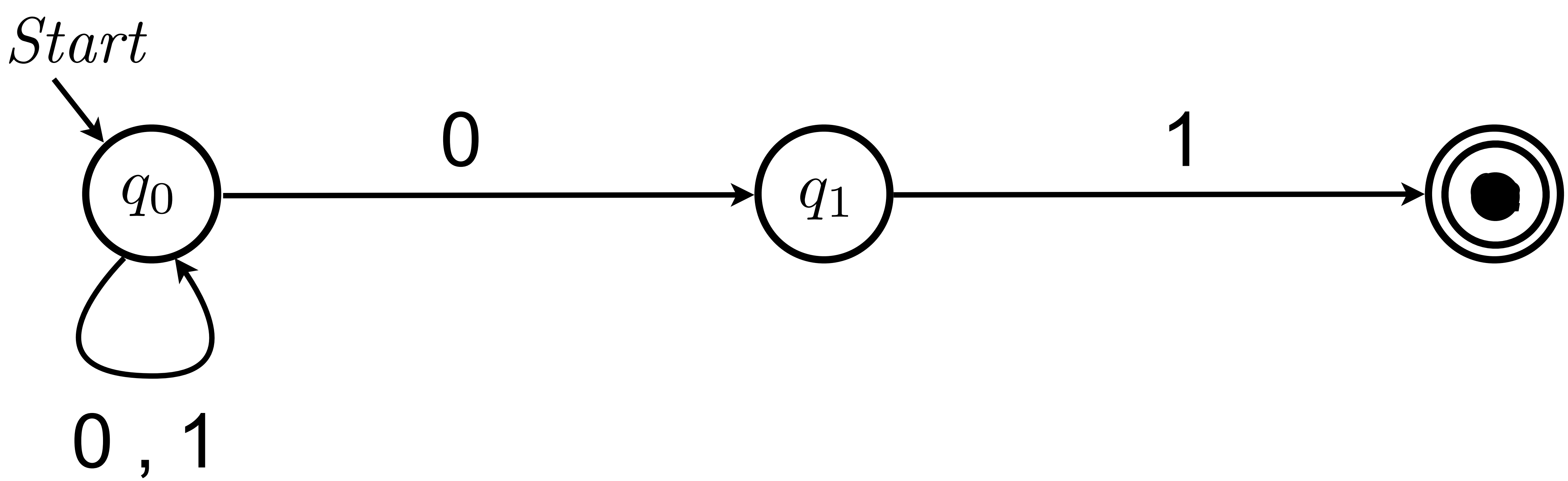

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 1 |

# Step 1: get a token



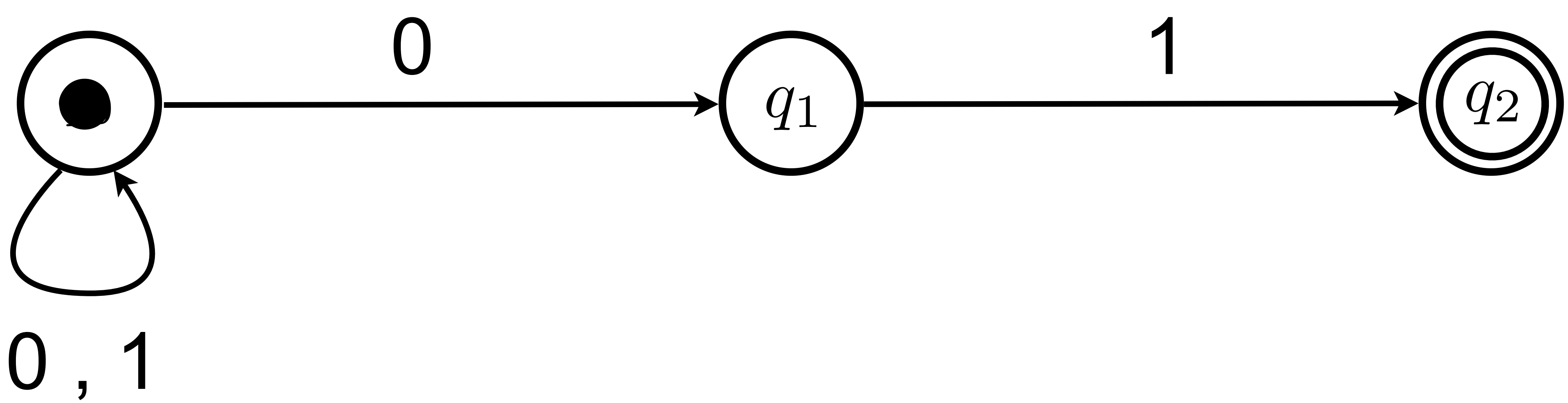

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 1 |

# Step 1: get a token

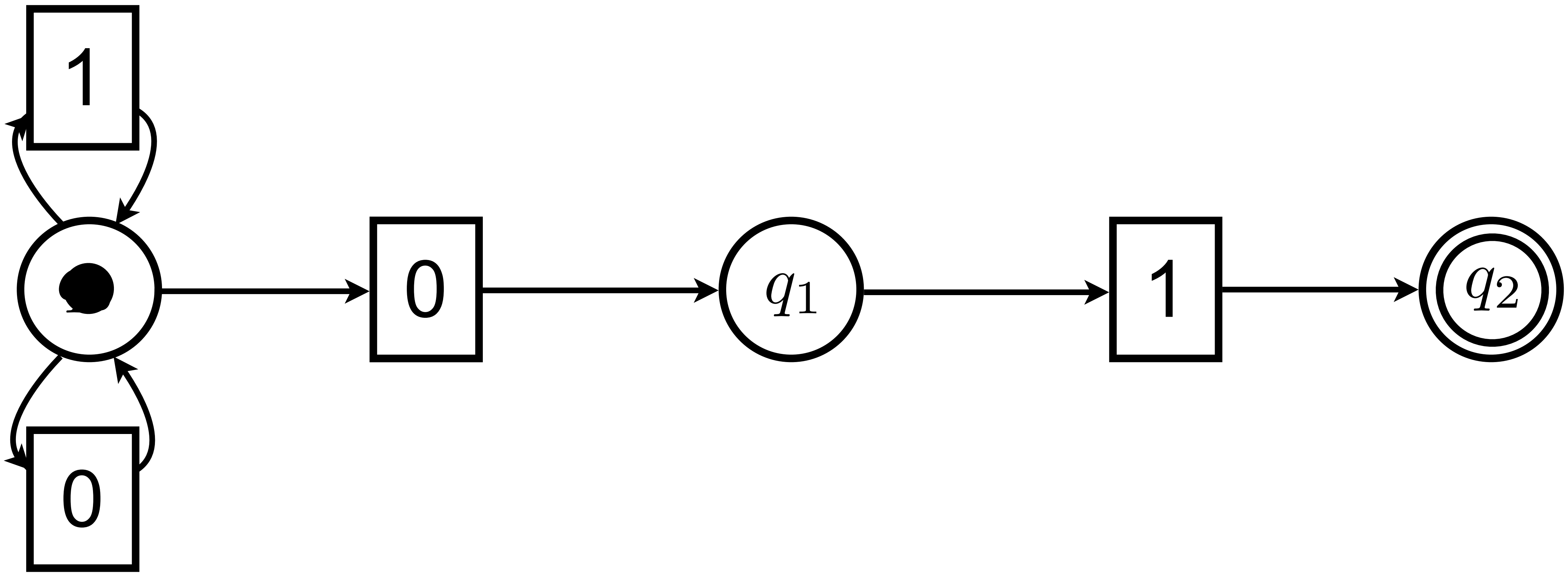



| | | | | |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 1 |

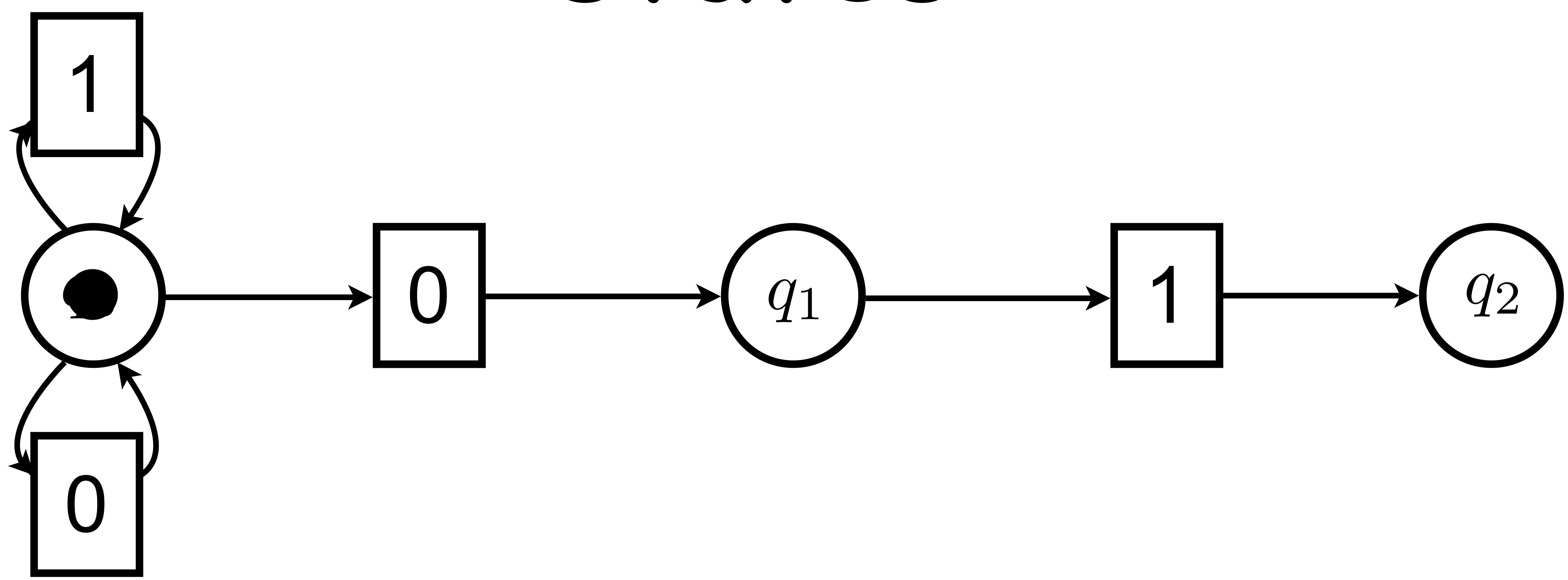
# Step 2: forget initial state decoration



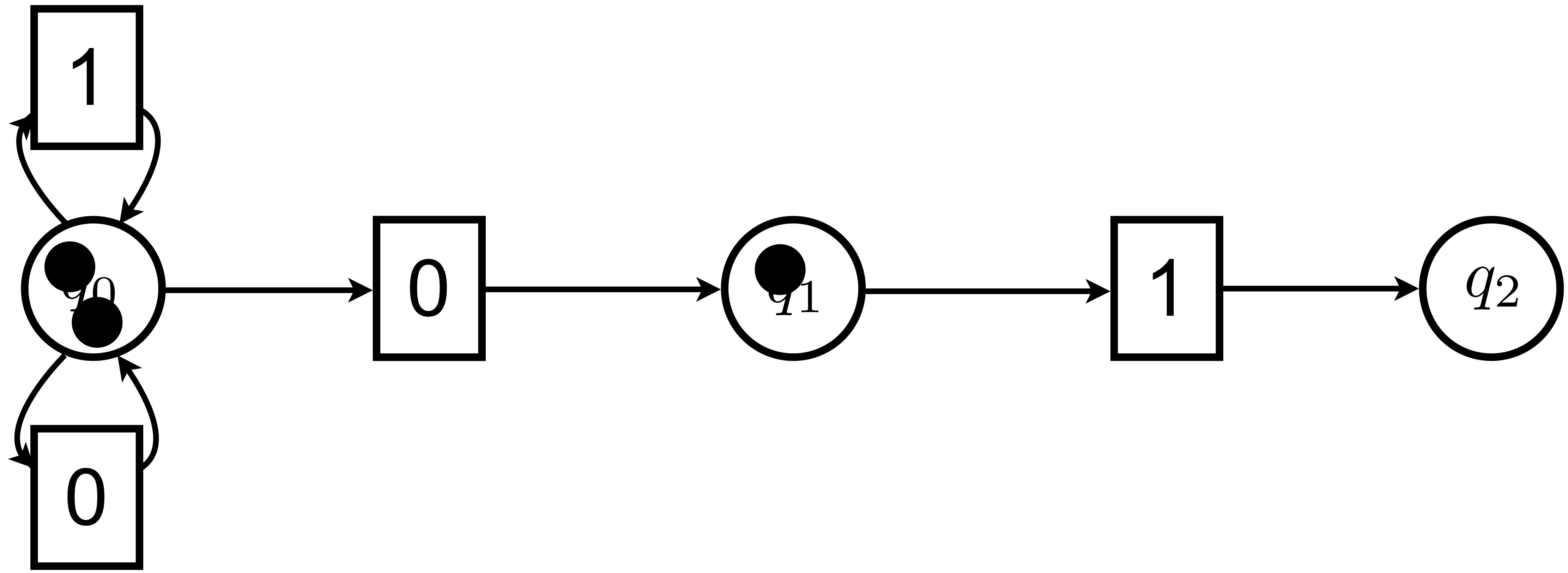
# Step 3: transitions as boxes



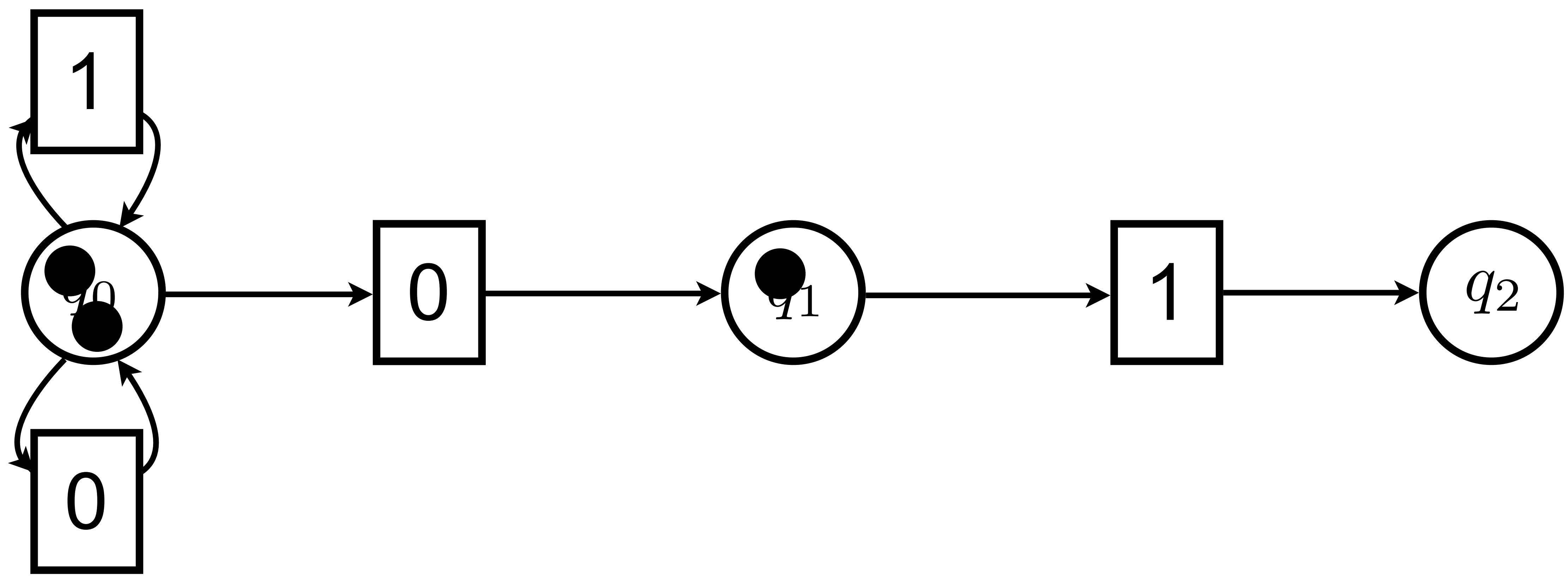
# Step 4: forget final states



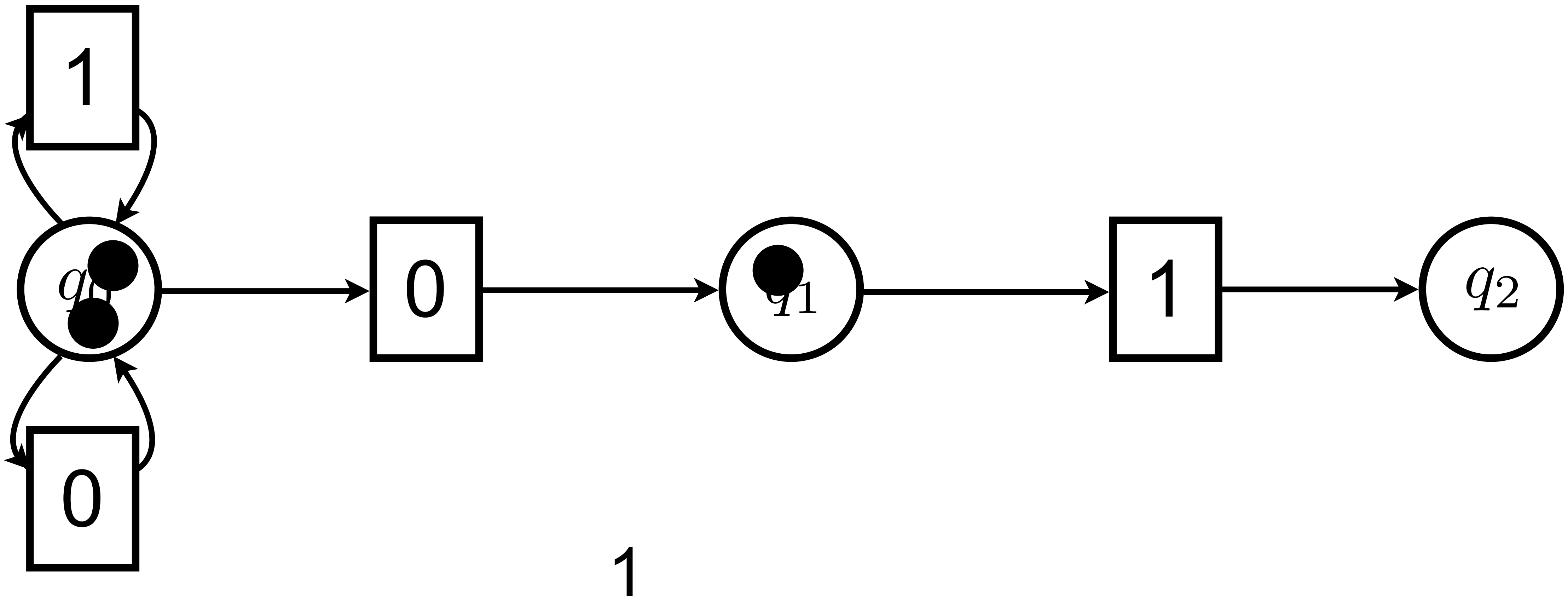
# Step 5: allow for more tokens



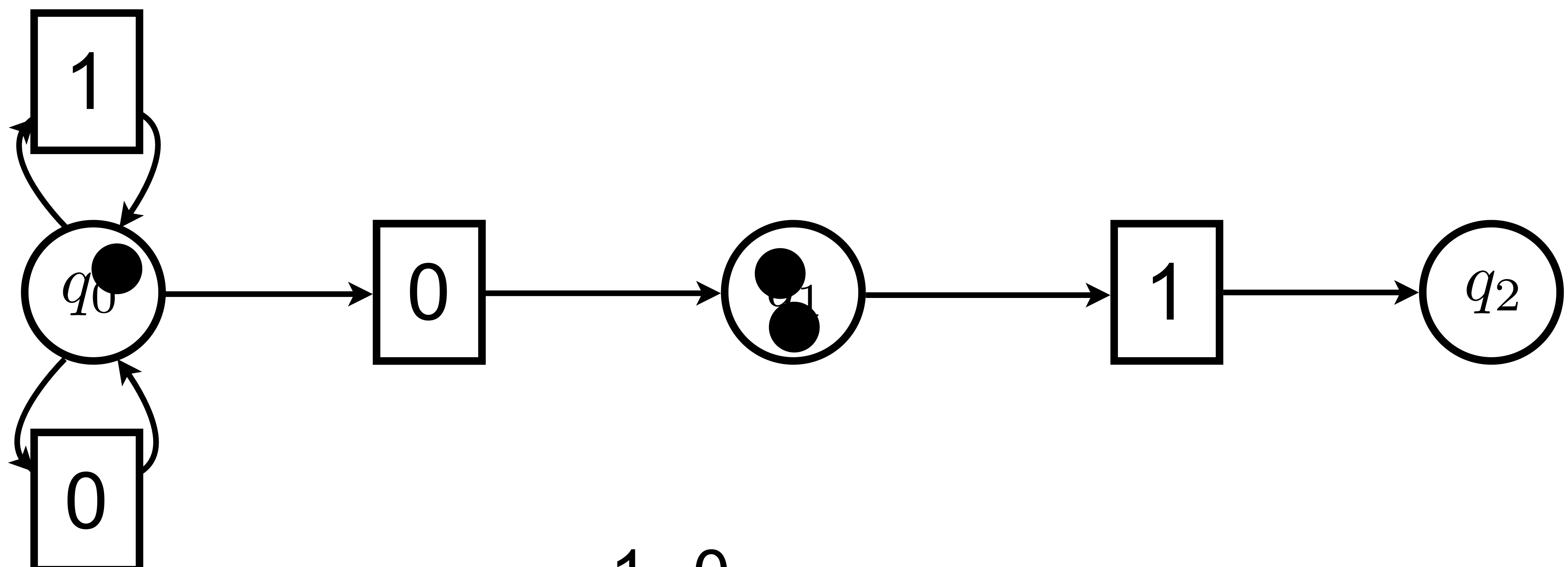
# Example: token game



# Example: token game

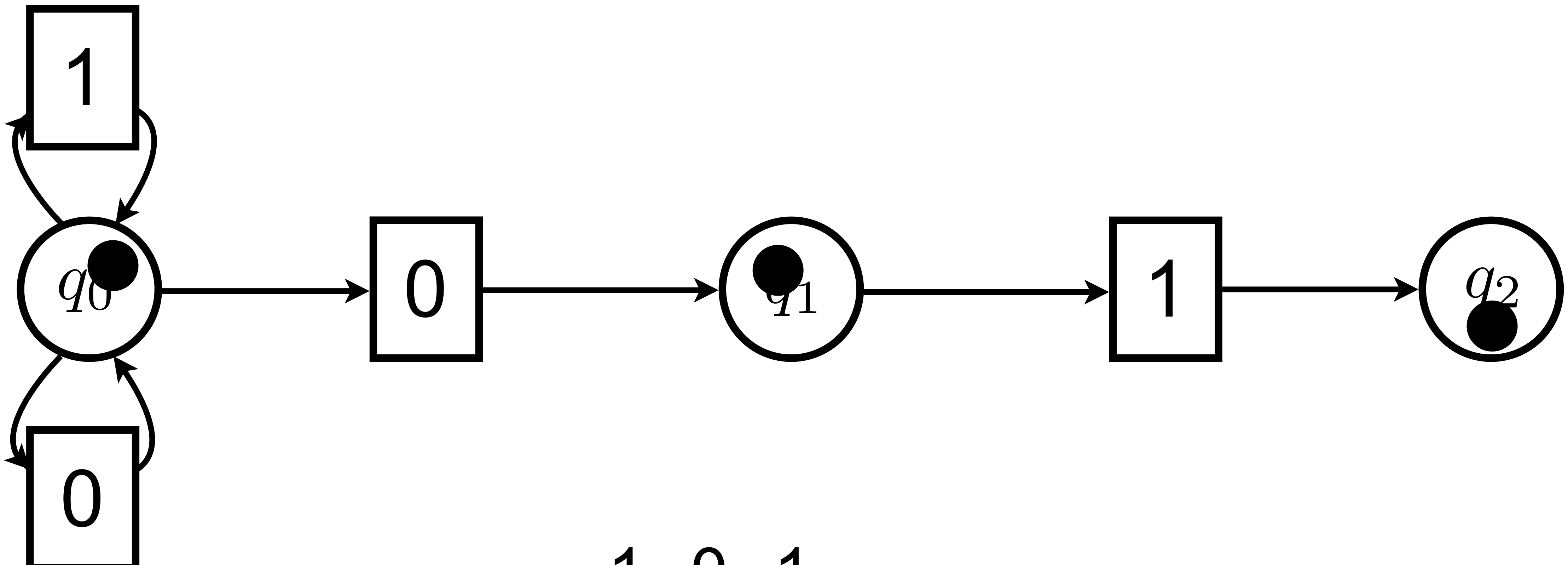


# Example: token game



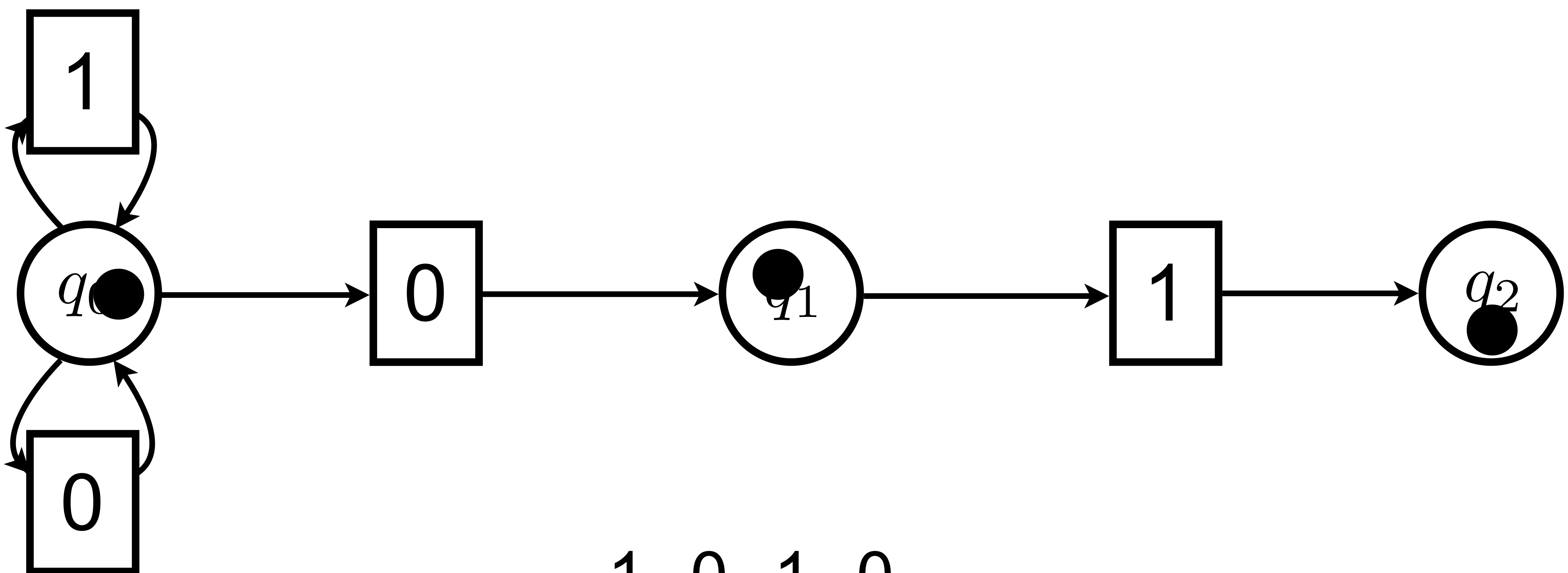

1 0

# Example: token game



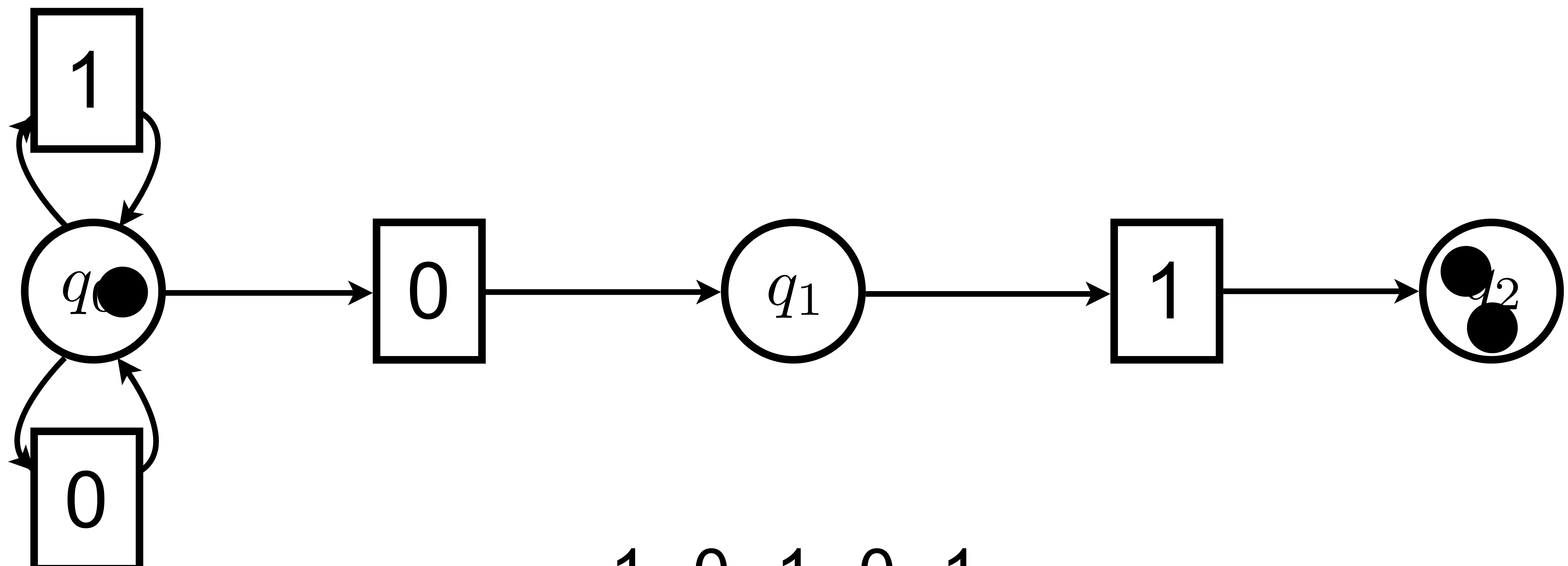

1 0 1

# Example: token game



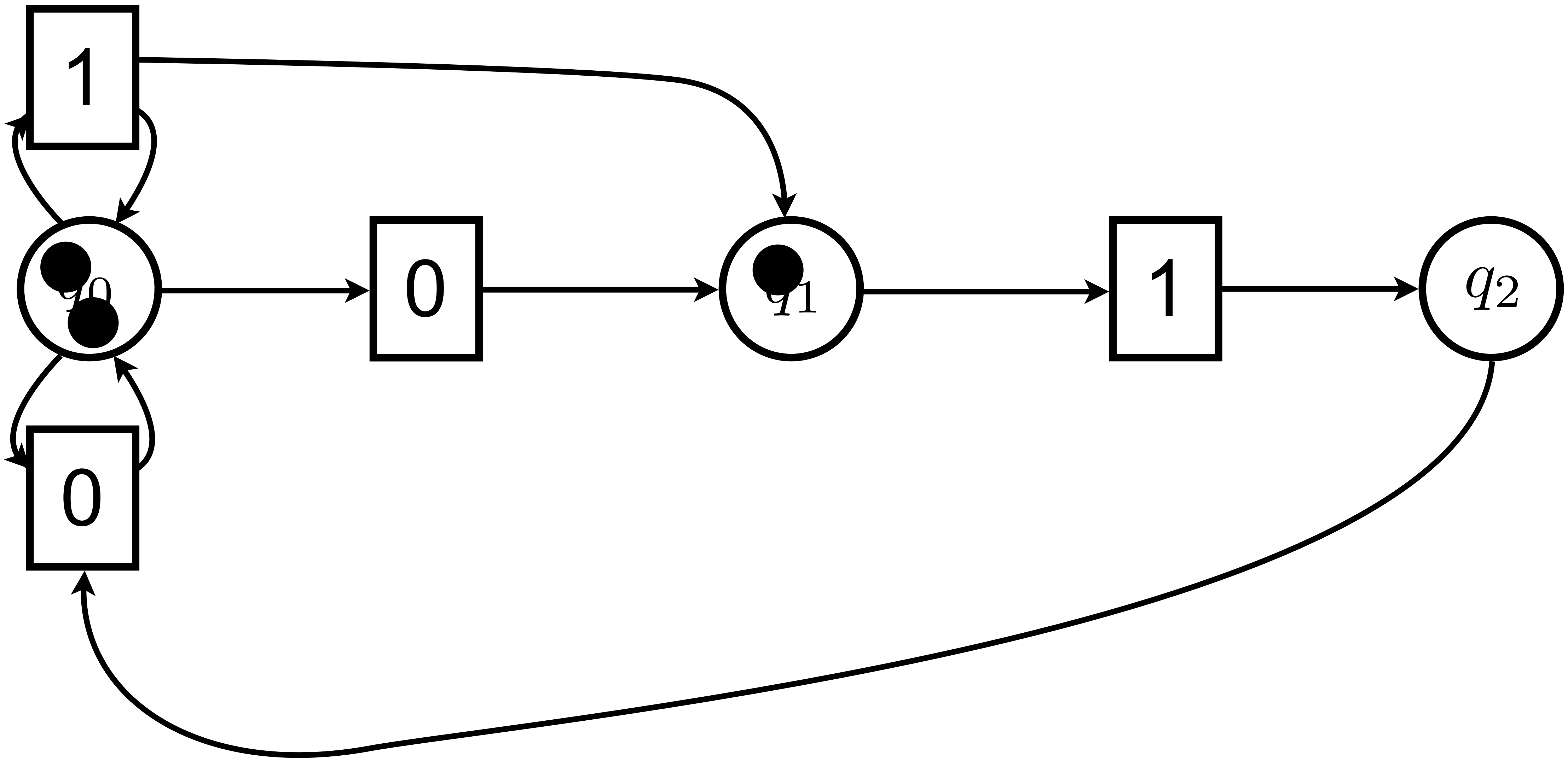

1 0 1 0

# Example: token game

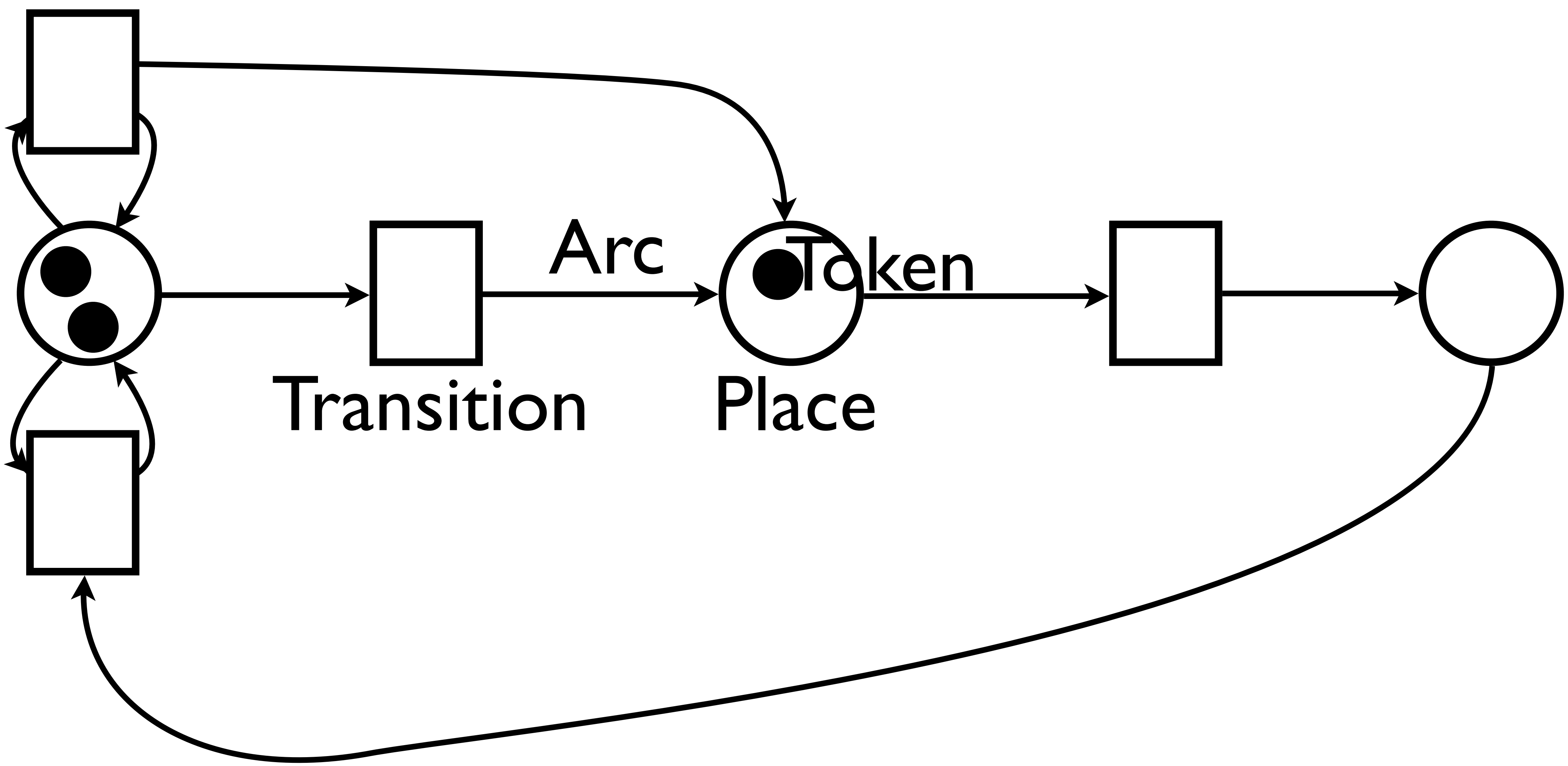



1 0 1 0 1

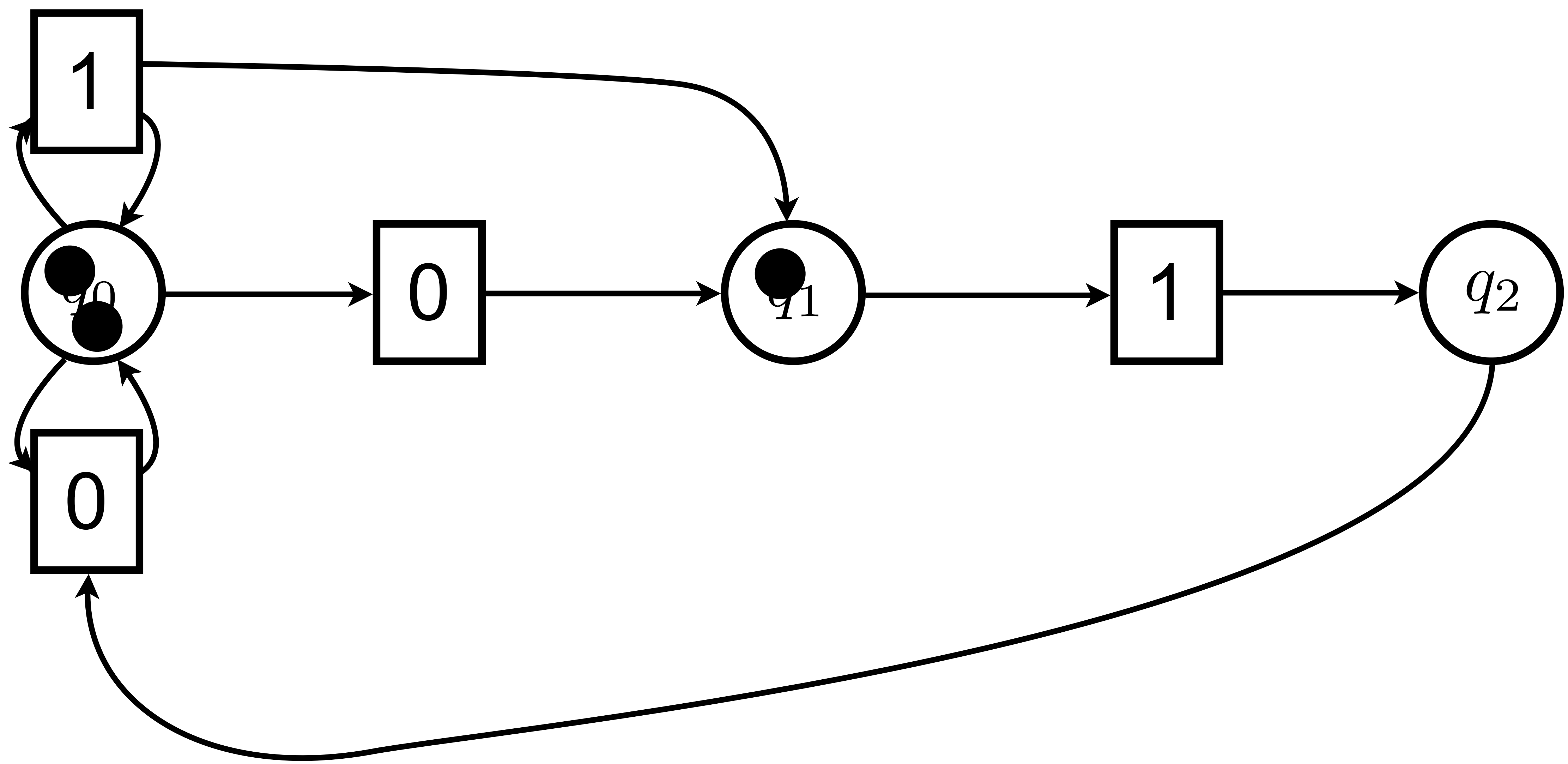
# Step 6: allow for more arcs



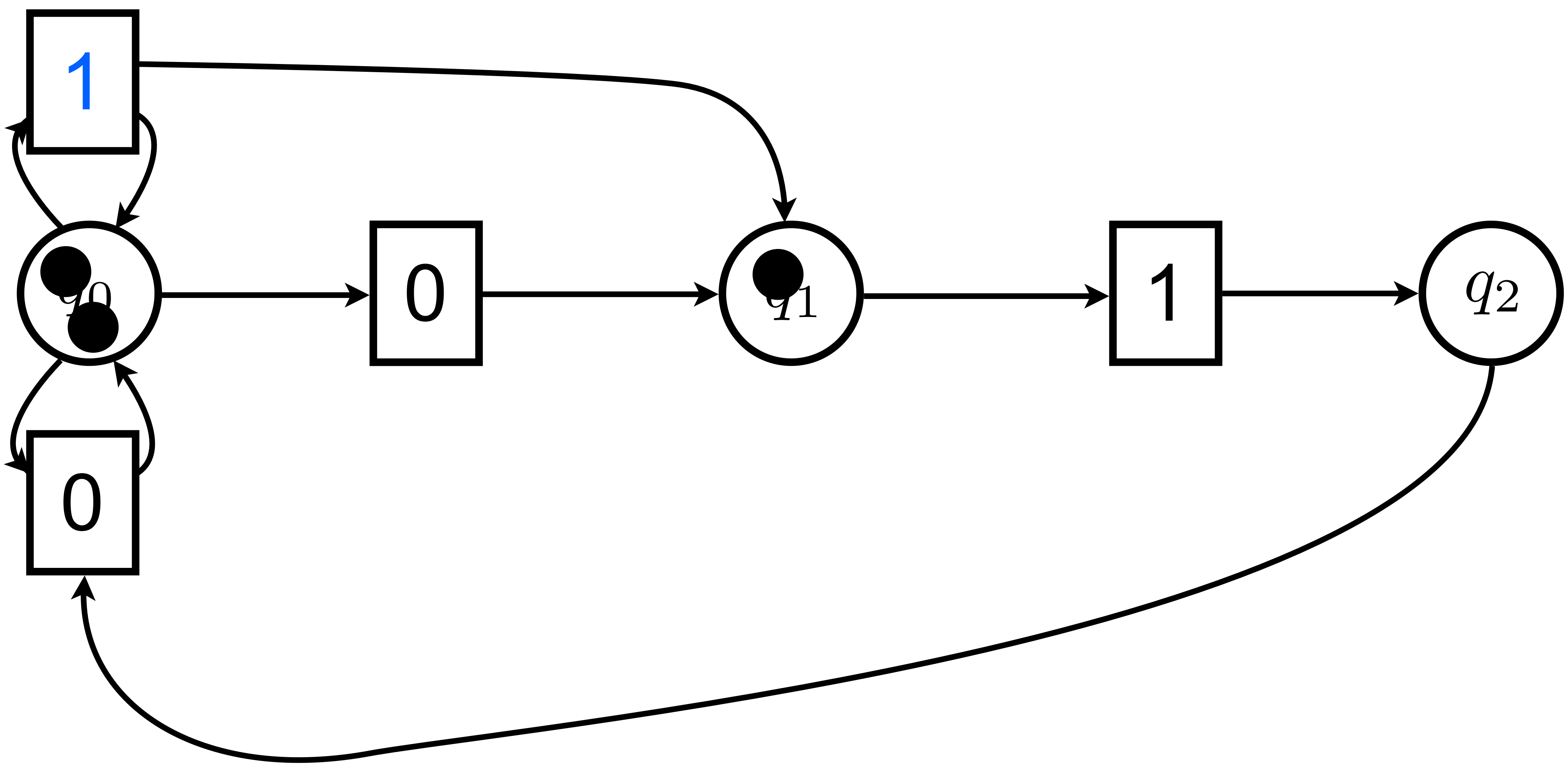
# Terminology



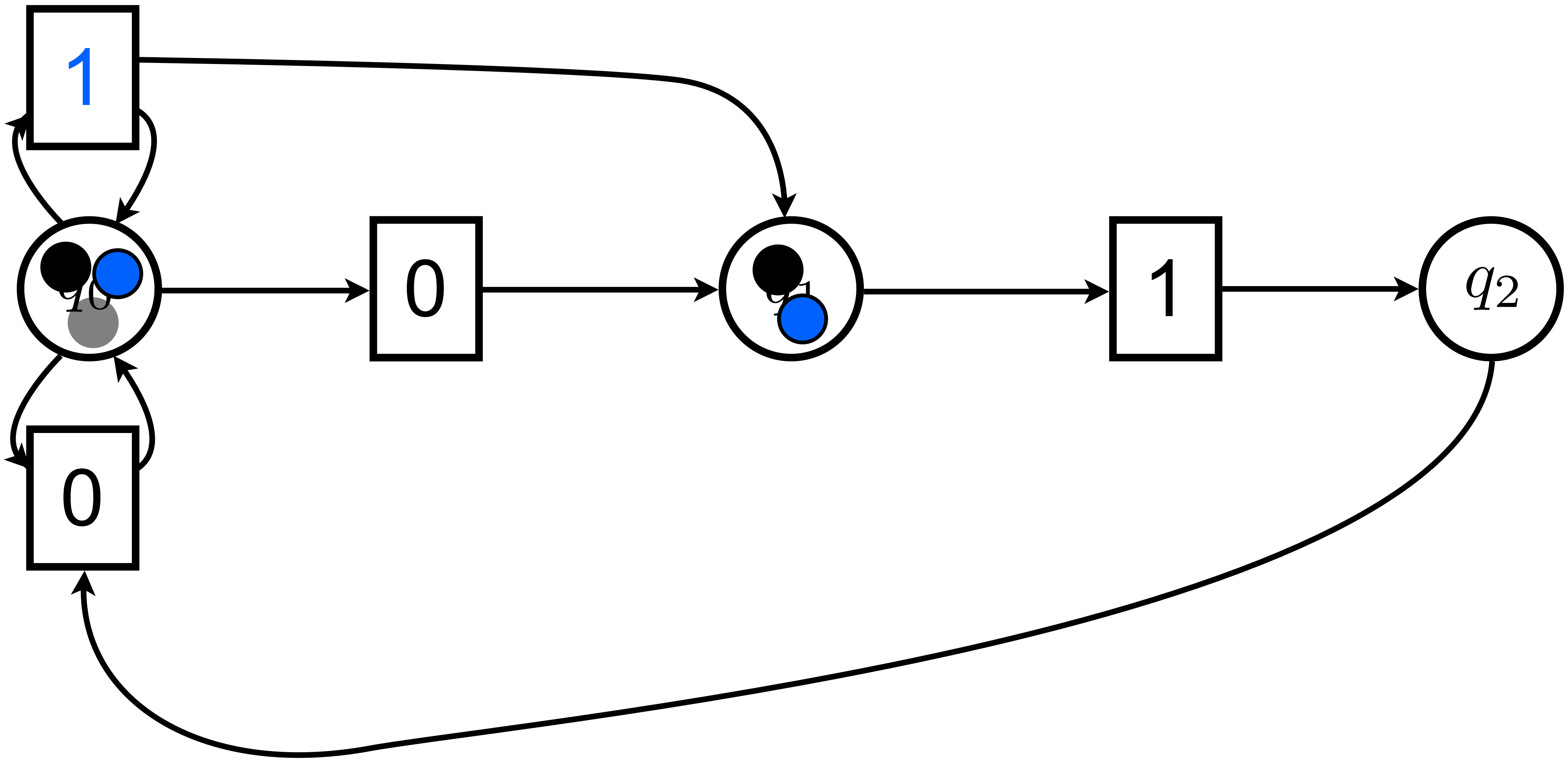
# Example: token game



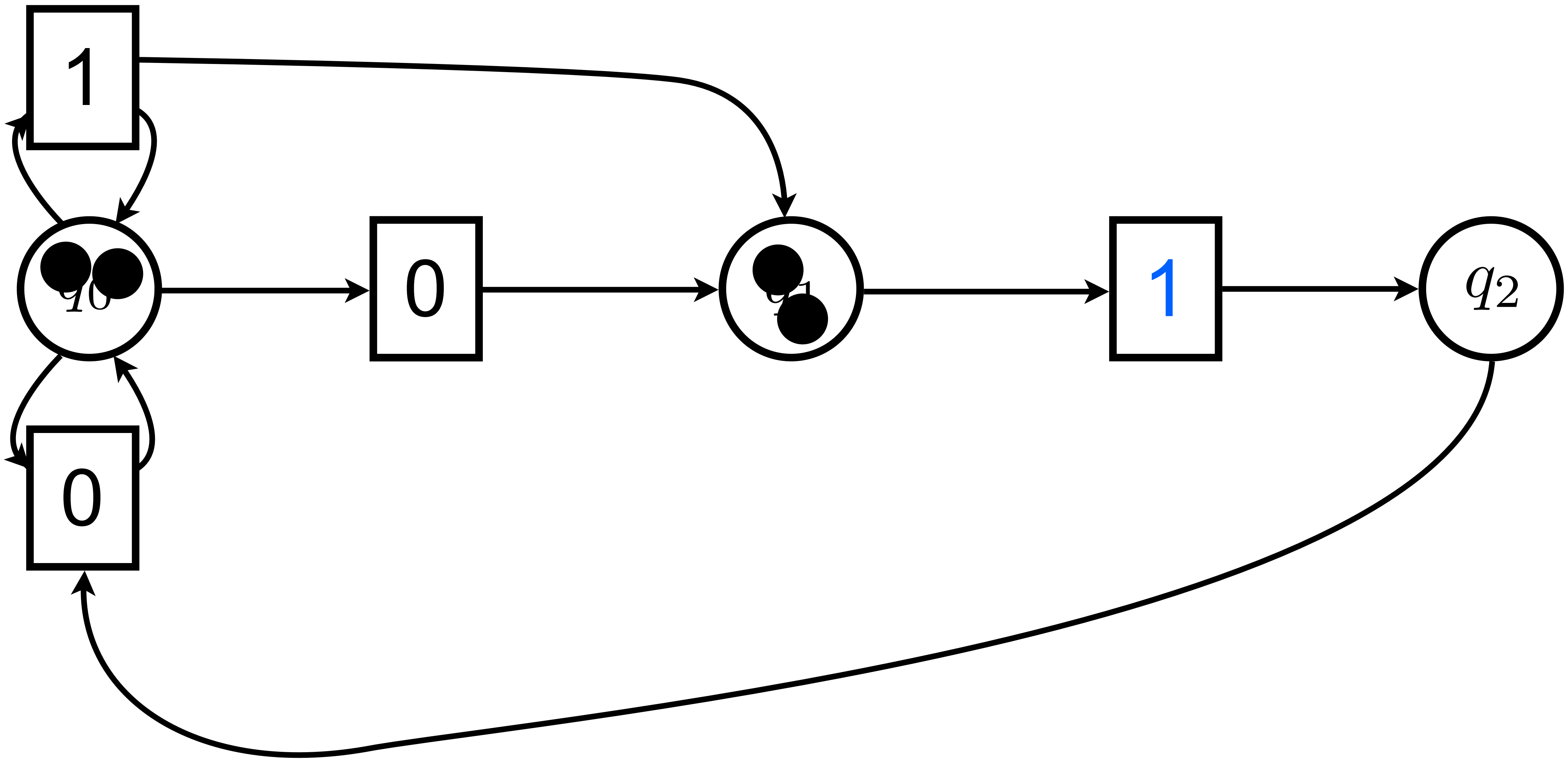
# Example: token game



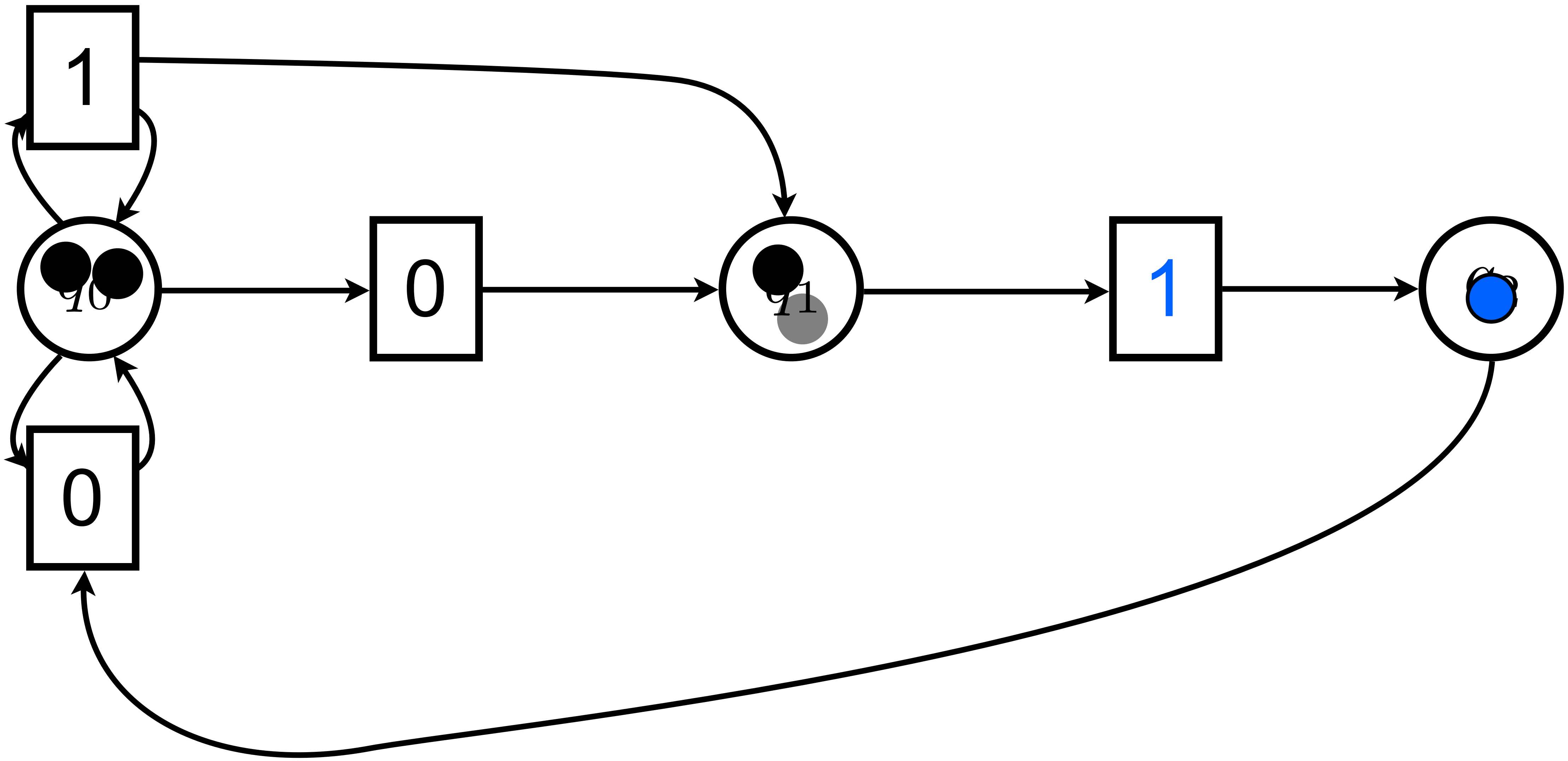
# Example: token game



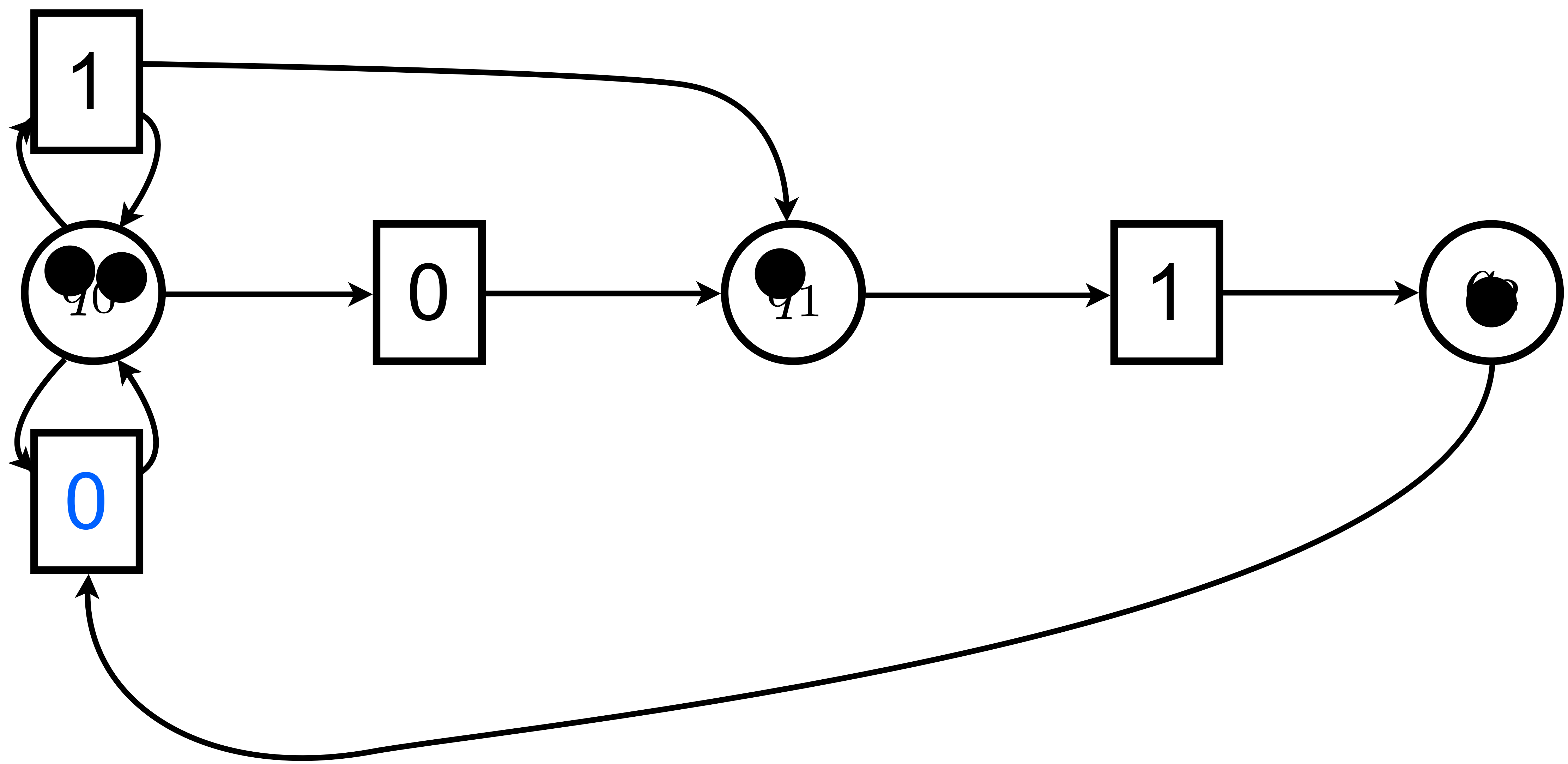
# Example: token game



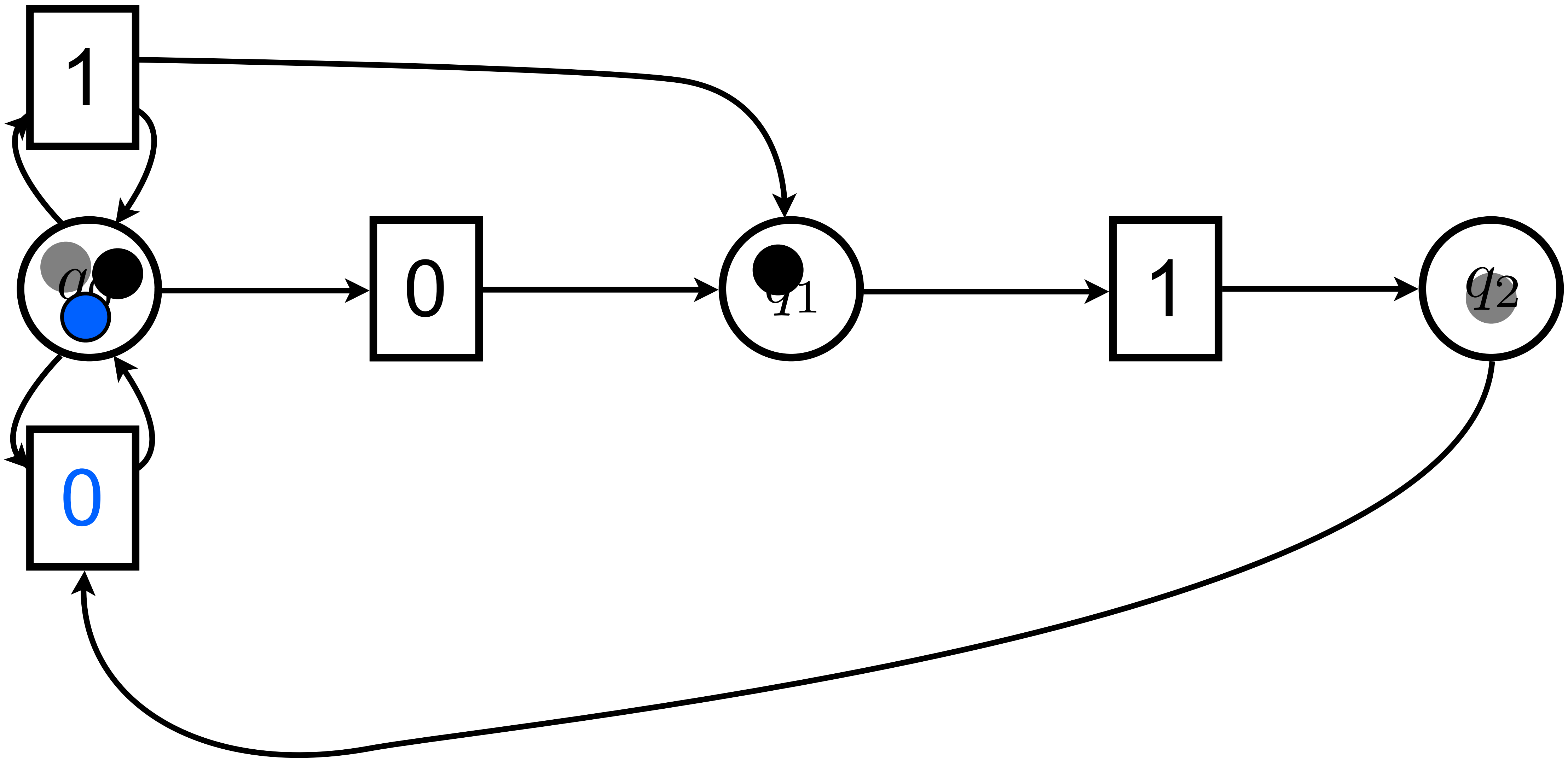
# Example: token game



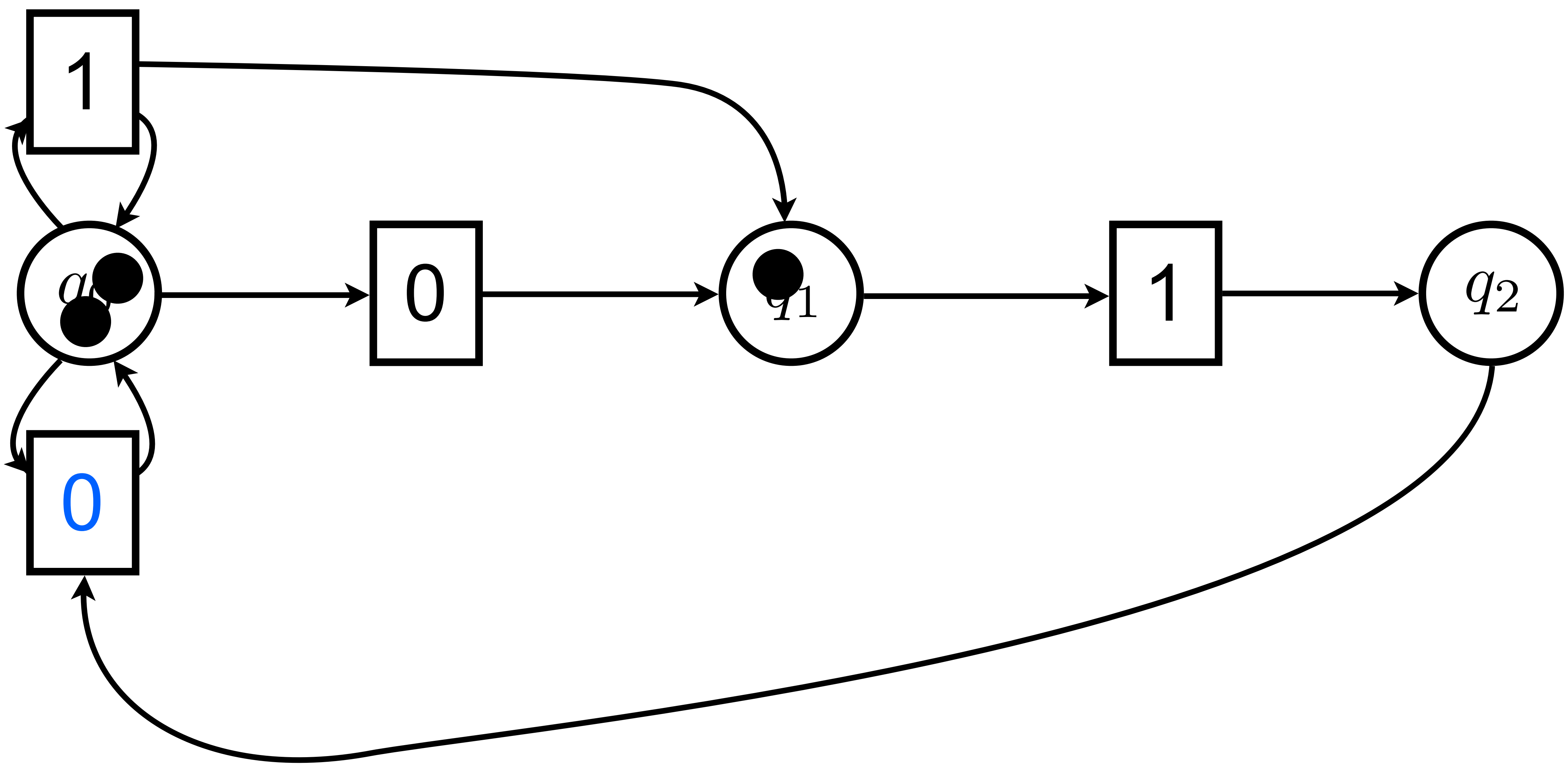
# Example: token game



# Example: token game



# Example: token game



# Some hints

Nets are **bipartite graphs**:
arcs never connect two places
arcs never connect two transitions

Static structure for dynamic systems:
places, transitions, arcs do not change
tokens move around places

**Places are passive** components
**Transitions are active** components:
tokens do not flow!
(they are removed or freshly created)