# Methods for the specification and verification of business processes

MPB (6 cfu, 295AA)

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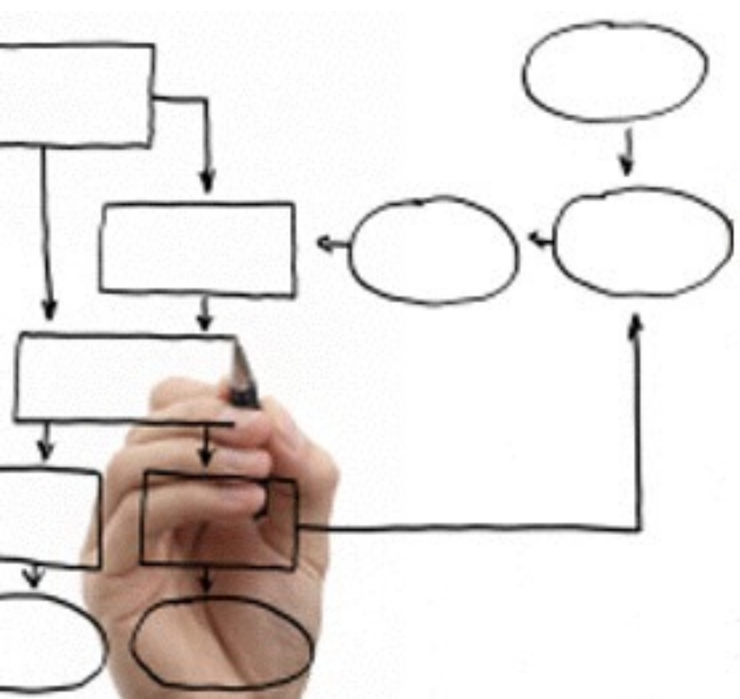


13 - Analysis of WF nets

# Object

We study suitable soundness properties of Workflow nets

# Structural analysis

No entry / exit point for a case

**no entry**: when should the case start?
**no exit**: when should the case end?
**ruled out by definition of workflow nets**

# Structural analysis

Multiple entry / exit point for a case

**multiple entry**: when should the case start?
**multiple exit**: when should the case end?
**ruled out by definition of workflow nets**

# Structural analysis

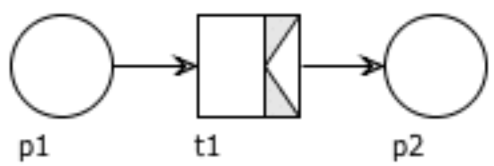
Tasks t without incoming and/or outgoing arcs

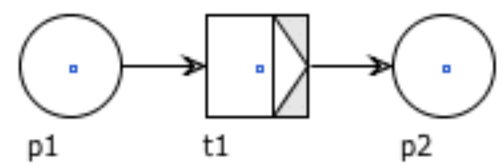
**no input**: when should t be carried out?
**no output**: t does not contribute to case completion
**ruled out by definition of workflow nets**

# Structural analysis

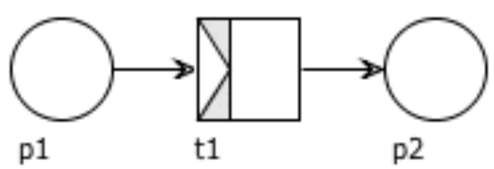
Wrong decoration of transitions

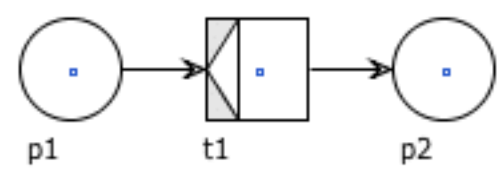
split with only one outgoing arc




join with only one incoming arc



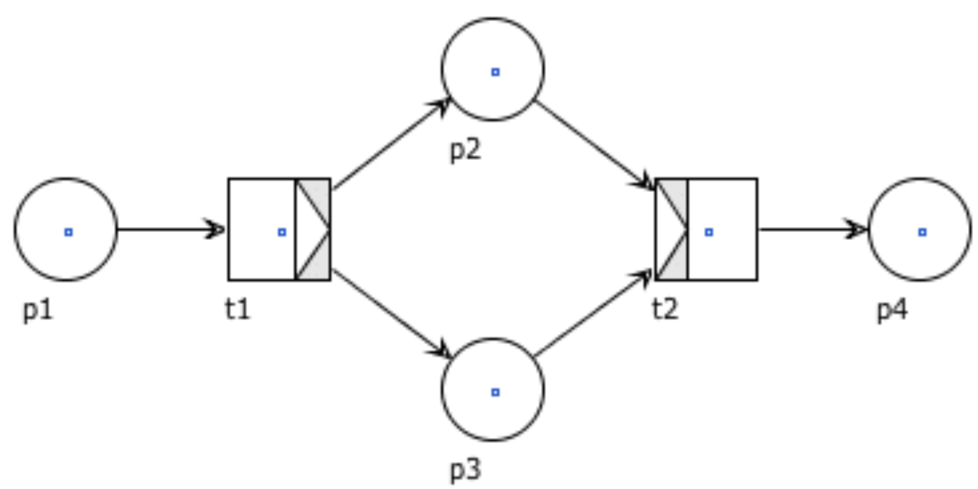




**left to designer responsibility**

# Activity analysis

Dead tasks

Tasks that can never be carried out
Each transitions lies on a path from i to o: not sufficient




**can arise in workflow nets**

# Token analysis

Some tokens left in the net after case completion

when a token is in the final place the case should end
**can arise in workflow nets**

# Activity analysis

Activities still take place after case completion

it can be a (worse) consequence of the previous flaw
**can arise in workflow nets**

# Token analysis

More than one token reach the end place

it can be a consequence of the above flaws

**can arise in workflow nets**

# Net analysis

Deadlock (stop before producing output)

a case blocks without coming to an end
**can arise in workflow nets**

# Net analysis

Livelock (divergence without producing output)

a case is trapped in a cycle with no opportunity to end
**can arise in workflow nets**

# Remark

**All the previous flaws are typical errors that can be detected without any knowledge about the actual content of the Business Process**

# Verification and validation

**Verification** aims to answer qualitative questions
Is there a deadlock possible?
Is it possible to successfully handle a specific case?
Will all cases terminate eventually?
Is it possible to execute a certain task?

**Validation** is concerned with
the relation between the model and the reality
How does a model fit log files?
Which model does fit better?

# Verification by simulation

**Test analysis**

Try and see if certain firing sequences are allowed by the workflow net

Using WoPeD:

Play (forward and backward) with net tokens

Record certain runs (to replay or explain)

Randomly select alternatives

**Problem**: how to make sure that all possible runs have been examined?

# Reachability analysis

**Verification by inspection**

All possible runs of a workflow net are represented in its Reachability Graph (if finite)

Using WoPeD:

Total number of states is evident

(a single run does not necessarily visit all nodes)

End states are evident (no outgoing arc)

Easy to check if dangerous or undesired states can arise

(e.g. the green-green state in the two-traffic-lights)

# Boundedness (for Nets)

**Proposition**:
The reachability graph of a net is finite

if and only if

the net is bounded

# Boundedness (for Nets)

**Proposition**:
A net is unbounded

if and only if

its reachability graph is not finite

# Coverability graph

A **coverability graph** is a finite over-approximation of the reachability graph

It allows for markings with infinitely many tokens in one place (called extended bags)

$$B : P \longrightarrow \mathbb{N} \cup \{\infty\}$$

# Discover unbounded places

Suppose

$$M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} M_2 \ldots \xrightarrow{t_i} M_i \ldots \xrightarrow{t_j} M_j$$

with $M_i \subset M_j$

Let $M = M_i$ and $M' = M_j$ and $L = M' - M$

By the monotonicity Lemma we have, for any $n \in \mathbb{N}$:
$M \rightarrow^* M + L \rightarrow^* M + 2L \rightarrow^* \ldots \rightarrow^* M + nL$

Hence all places $p$ marked by $L$ (i.e. if $L(p) > 0$) are unbounded

# Cover unbounded places

**Idea**:
When computing the RG, if $M'$ is found s.t.

$M_0 \to^* M \to^* M'$ with $M \subset M'$

Add the extended bag $B$ (instead of $M'$) to the graph

$$\text{where} \quad B(p) = \begin{cases} M'(p) & \text{if } M'(p) - M(p) = 0 \\ \infty & \text{otherwise} \end{cases}$$

# A few remarks

**Idea**: mark unbounded places by $\infty$

**Remind**: $M \subset M'$ means that $M \subseteq M' \wedge M \neq M'$, i.e.,
1. for any $p \in P$, $M'(p) \geq M(p)$
2. there exists at least one place $q \in P$ such that $M'(q) > M(q)$

**Remark**:
Requiring $M_0 \to^* M \to^* M'$ is different than requiring $M, M' \in [\, M_0 \,\rangle$

# Operations on extended bags

**Inclusion**: Let $B, B' : P \to \mathbb{N} \cup \{\infty\}$
We write $B \subseteq B'$ if for any $p$ we have
$B'(p) = \infty$ or $B(p), B'(p) \in \mathbb{N} \wedge B(p) \leq B'(p)$

**Sum**: Let $B, B' : P \to \mathbb{N} \cup \{\infty\}$

$$(B + B')(p) = \begin{cases} \infty & \text{if } B(p) = \infty \text{ or } B'(p) = \infty \\ B(p) + B'(p) & \text{if } B(p), B'(p) \in \mathbb{N} \end{cases}$$

**Difference**: Let $B : P \to \mathbb{N} \cup \{\infty\}$ and $M : P \to \mathbb{N}$ with $M \subseteq B$

$$(B - M)(p) = \begin{cases} \infty & \text{if } B(p) = \infty \\ B(p) - M(p) & \text{if } B(p) \in \mathbb{N} \end{cases}$$

# Compute a reachability graph

1. Initially N = { $M_0$ } and A = $\emptyset$
2. Take a bag B $\in$ N and a transition t $\in$ T such that
   1. B enables t and there is no arc labelled t leaving from B
3. Let B' = B - •t + t•
4. Add B' to N and (B,t,B') to A
5. Repeat steps 2,3,4 until no new arc can be added

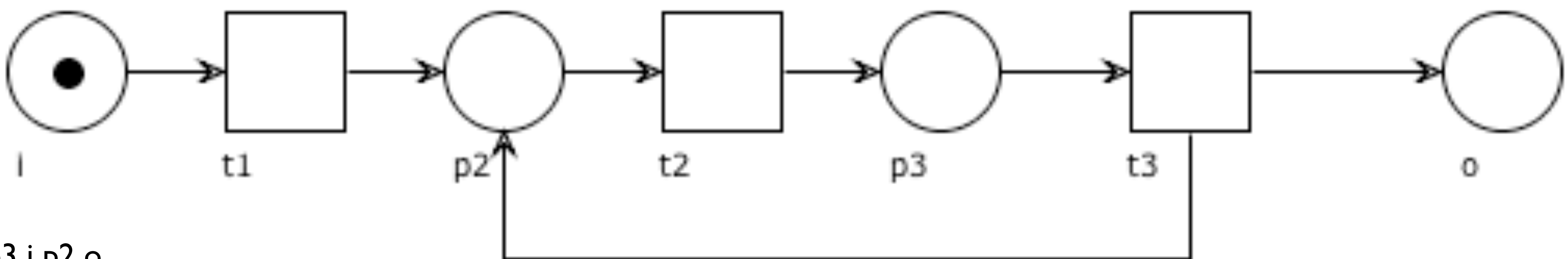
(all bags are finite in this case)

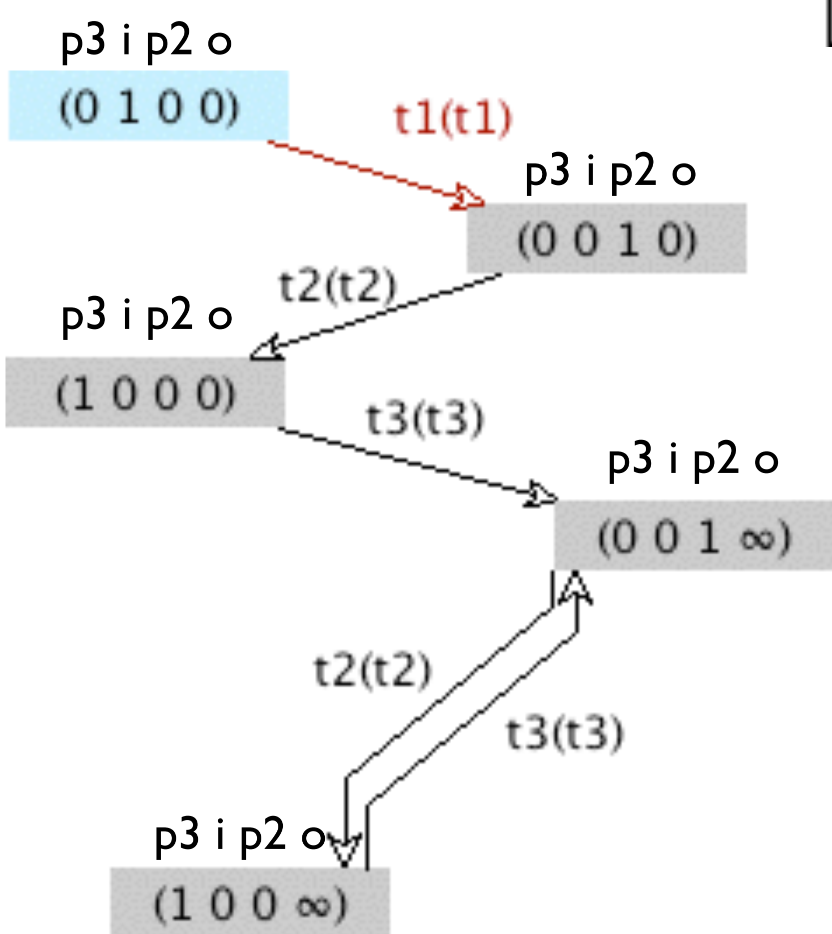
# Compute a coverability graph

1. Initially N = { $M_0$ } and A = ∅
2. Take a bag B ∈ N and a transition t ∈ T such that
    1. B enables t and there is no arc labelled t leaving from B
3. Let B' = B - •t + t•
4. Let $B_c$' such that for any p ∈ P
    1. $B_c$'(p) = ∞ if there is a node B'' ∈ N such that
        1. there is a direct path from B'' to B in the graph computed so far
        2. B'' ⊆ B',
        3. B''(p) < B'(p)
    2. $B_c$'(p) = B'(p) otherwise
5. Add $B_c$' to N and (B,t,$B_c$') to A
6. Repeat steps 2,3,4,5 until no new arc can be added

$$B'' \xrightarrow{\sigma} B \xrightarrow{t} B'$$

# Example


i
t1
p2
t2
p3
t3
o


p3 i p2 o
(0 1 0 0)
t1(t1)
p3 i p2 o
(0 0 1 0)
t2(t2)
p3 i p2 o
(1 0 0 0)
t3(t3)
p3 i p2 o
(0 0 1 ∞)
t2(t2)
t3(t3)
p3 i p2 o
(1 0 0 ∞)

# Properties of coverability graphs

A coverability graph is always finite,
but it is not always uniquely defined
(it depends on which B and t are selected at step 2)

Every firing sequence corresponds to a path in the CG
the converse is not necessarily true

Any path in a CG that visits only finite markings
corresponds to a valid firing sequence

If the RG is finite, then it coincides with the CG

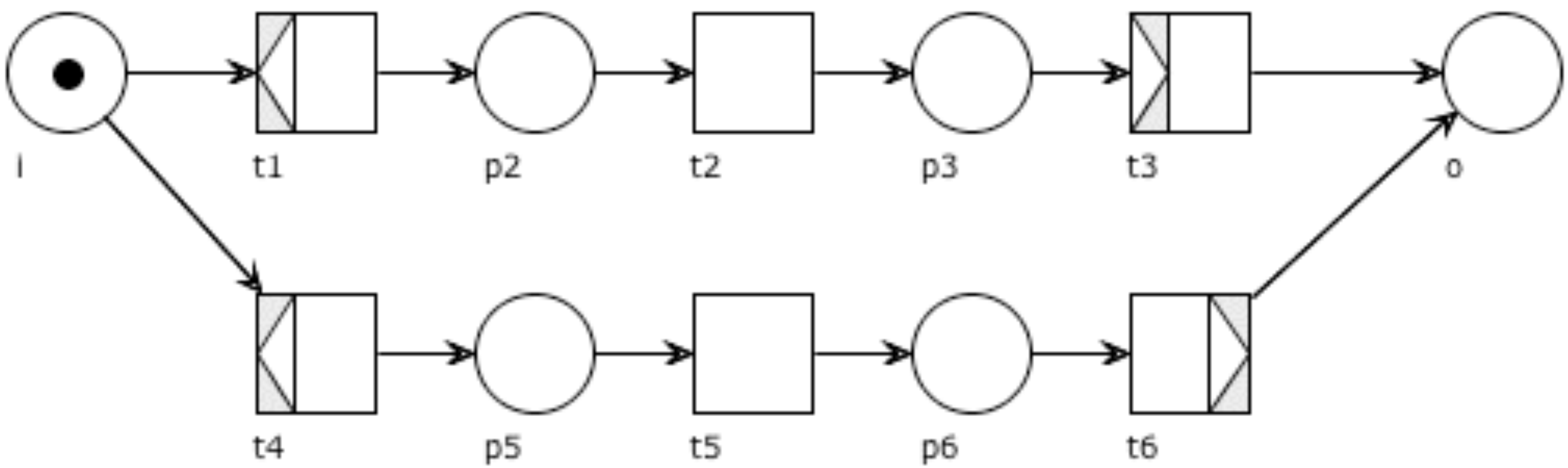
# Reachability analysis by coverability

All possible behaviours of a workflow net are represented exactly in the Reachability Graph (if finite)

We use Coverability Graph when necessary (RG not finite)

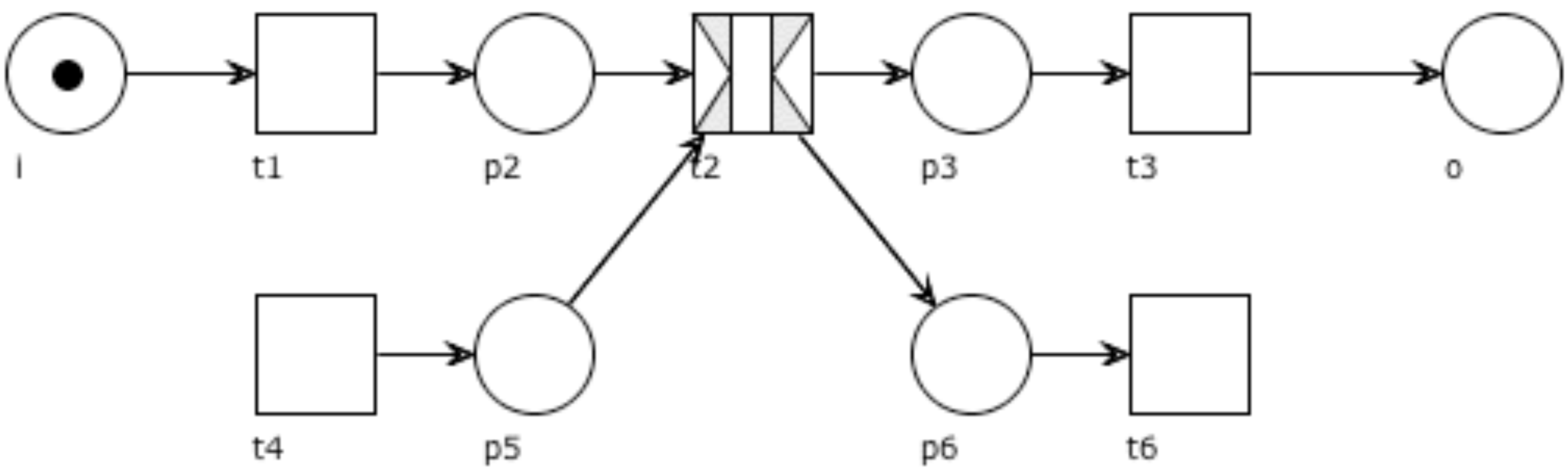
# Question time

Do you see any problem in the workflow net below?



# Question time

Do you see any problem in the workflow net below?



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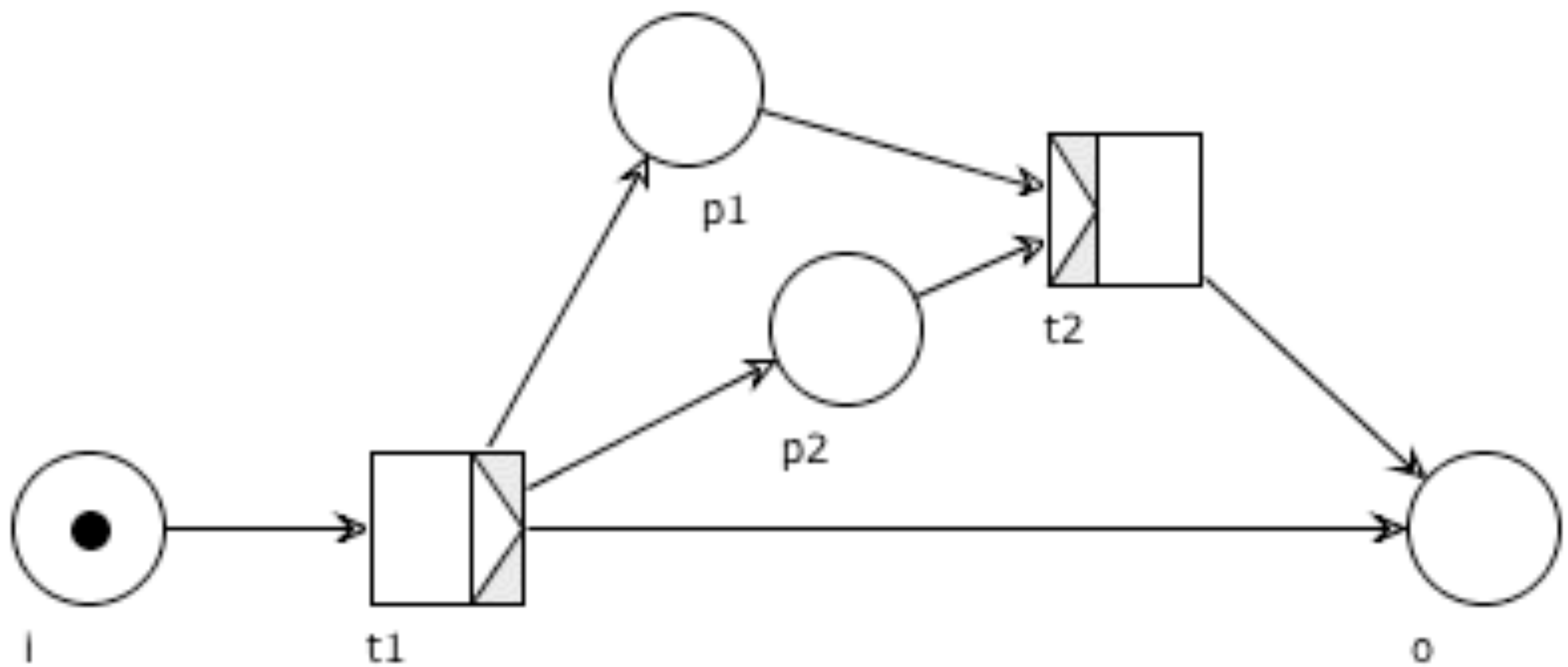
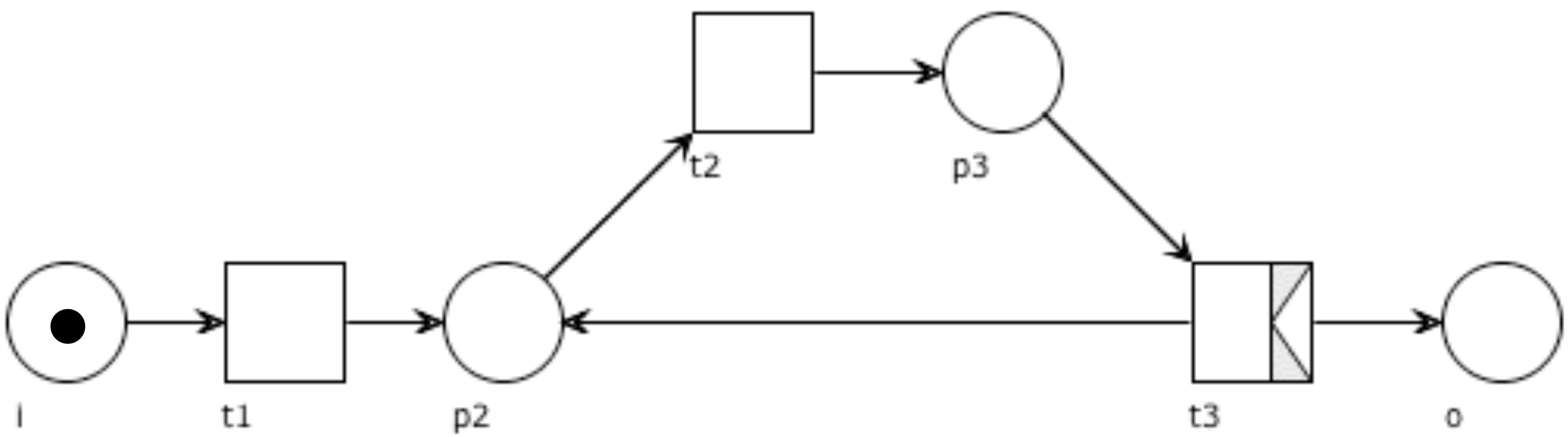


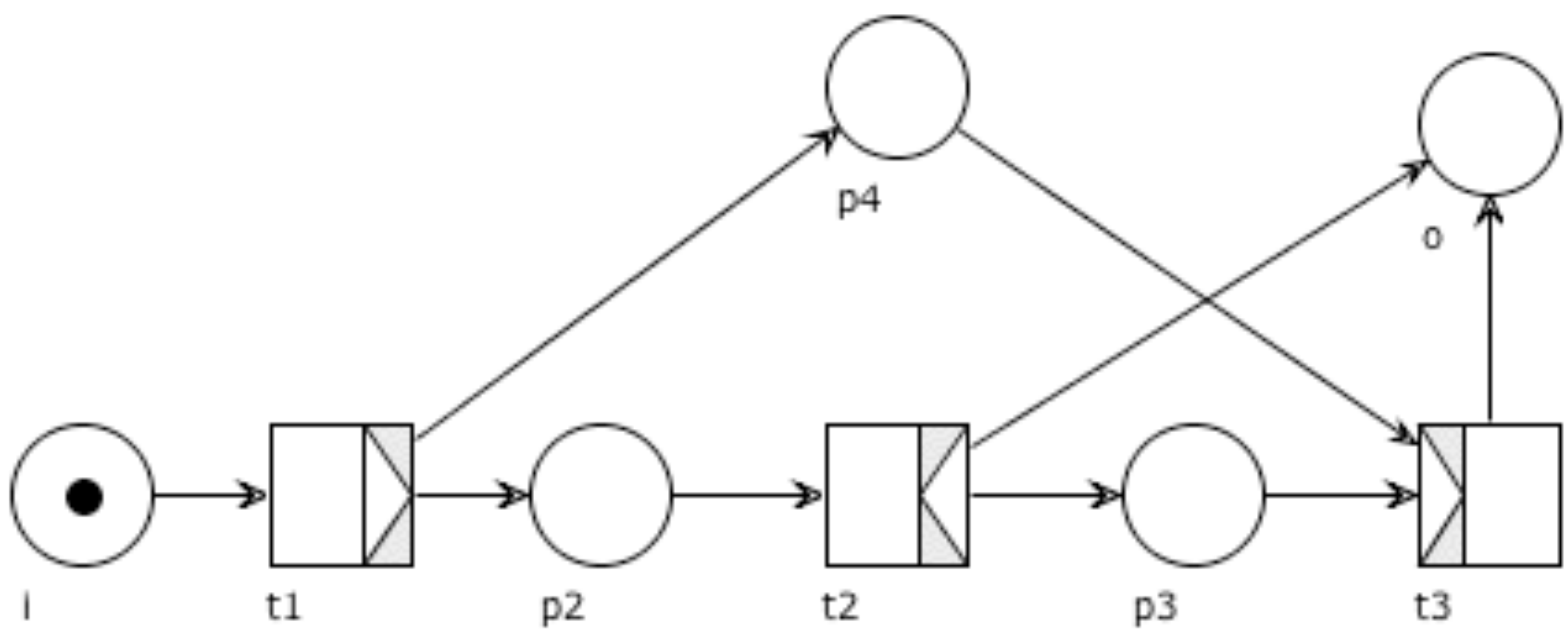
# Question time

Do you see any problem in the workflow net below?



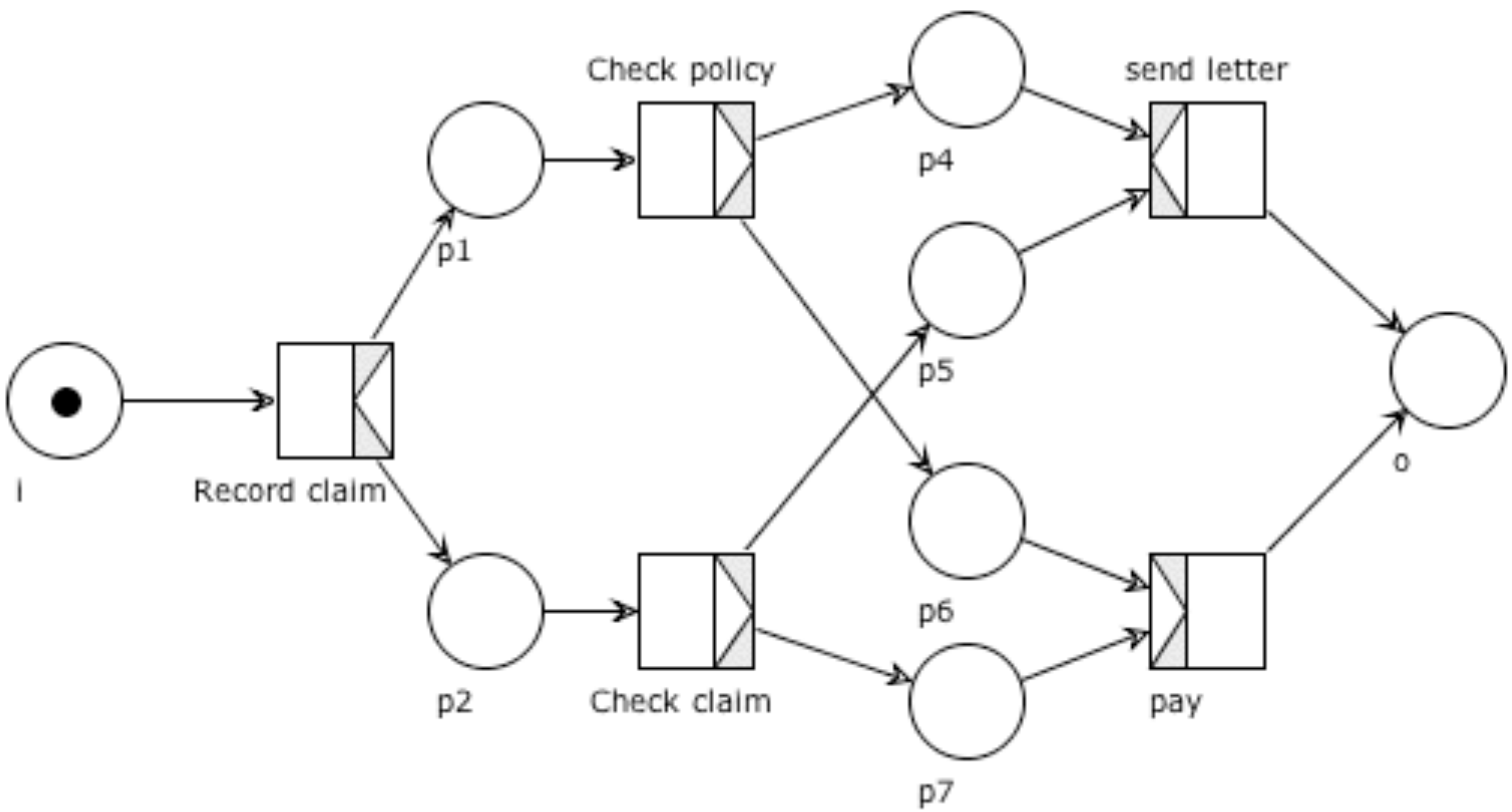
# Exercise

Do you see any problem in the workflow net below?



# Exercise

Which problem(s) in the workflow net below?
How would you redesign the business process?



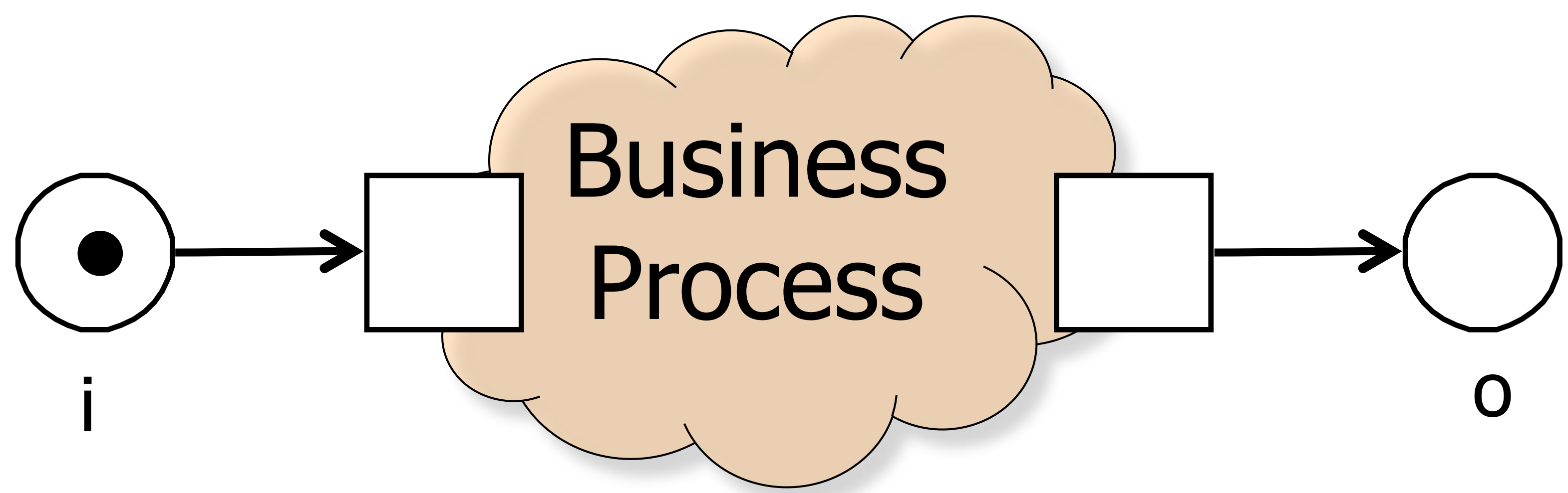
# Soundness

# Soundness of Business Processes

A process is called **sound** if

1. it contains no unnecessary tasks
2. every case is always completed in full
3. no pending items are left after case completion

# Soundness of Business Processes



# Soundness of Workflow nets

A workflow net is called **sound** if

1. for each transition $t$,

   there is a marking $M$ (reachable from $i$) that enables $t$

2. for each token put in place $i$,

   one token eventually appears in the place $o$

3. when a token is in place $o$, all other places are empty

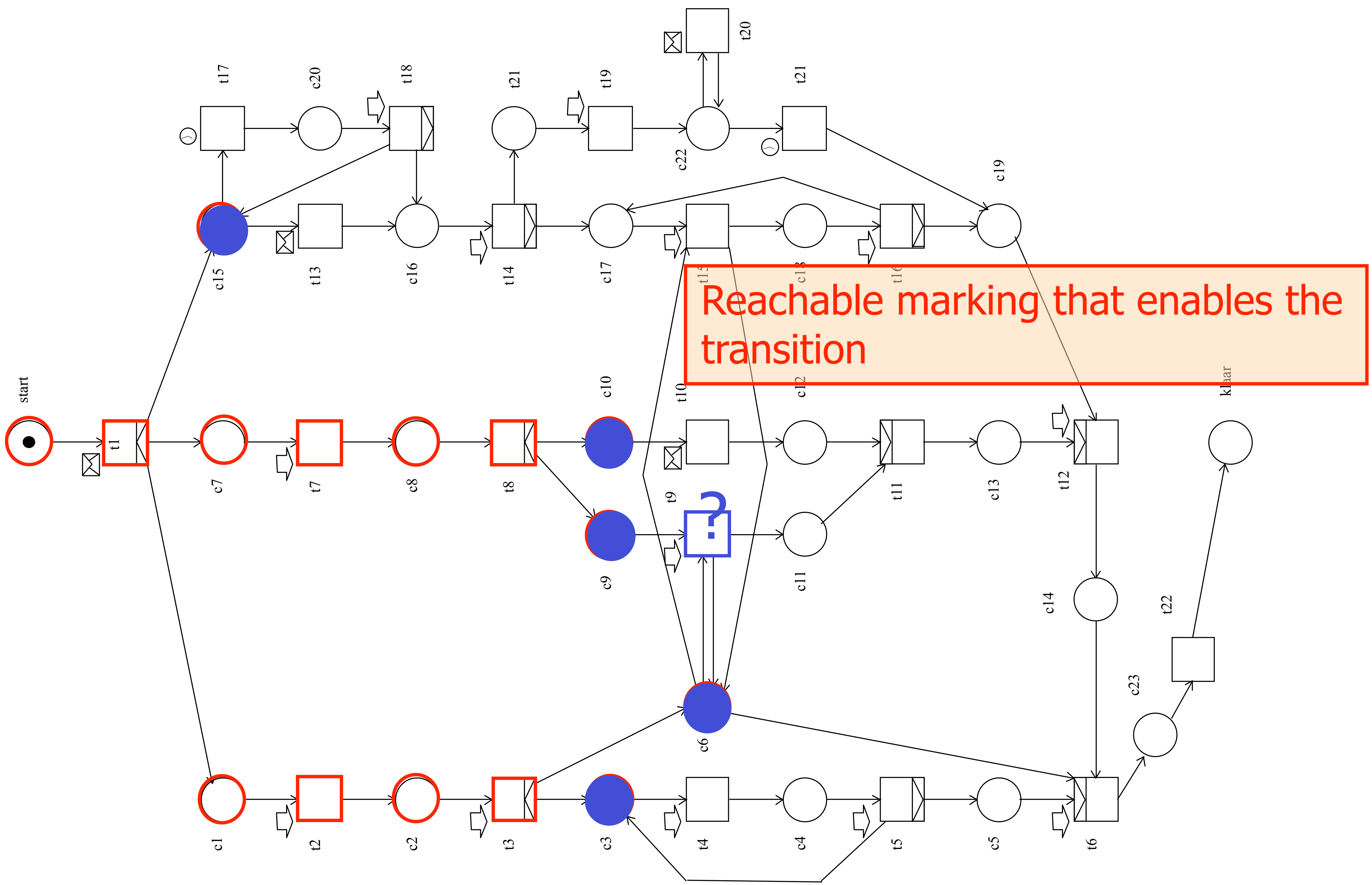
# Fairness assumption

**Remark**:
Condition 2 does not mean that iteration must be forbidden or bound

It says that from any reachable marking $M$
there must be possible to reach $o$ in some steps

**Fairness assumption**:
A task cannot be postponed indefinitely

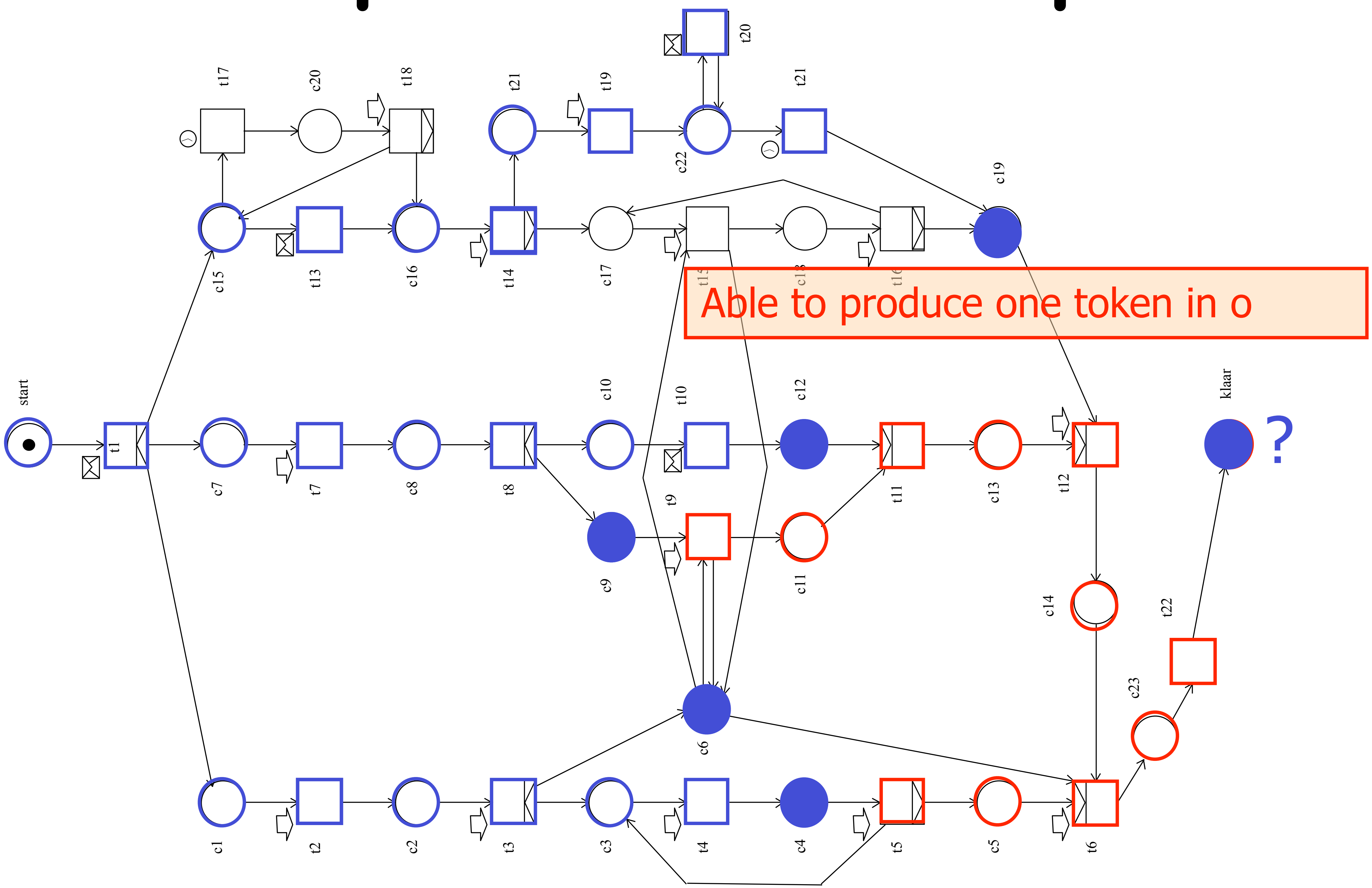
# 1: no dead tasks



# 1: no dead tasks

The check must be repeated for each task

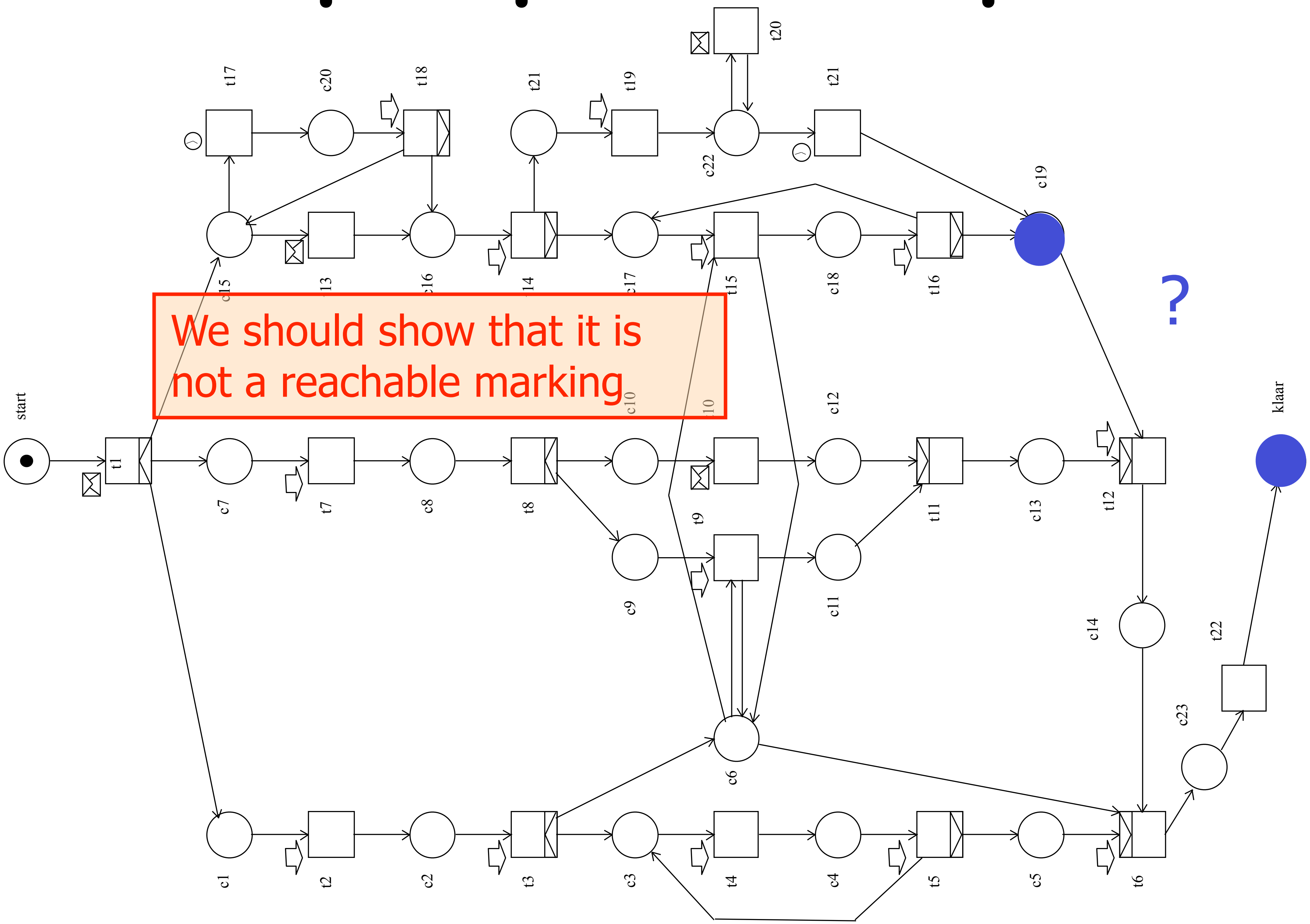
# 2: option to complete



# 2: option to complete

The check must be repeated for each reachable marking

# 3: proper completion




We should show that it is not a reachable marking

# 3: proper completion

The check must be repeated for each marking M such that M(o) = 1

# Soundness, Formally

A workflow net is called **sound** if

**no dead task** no transition is dead

$$\forall t \in T.\ \exists M \in [\, i \,\rangle.\ M \xrightarrow{t}$$

**option to complete** place $o$ is eventually marked

$$\forall M \in [\, i \,\rangle.\ \exists M' \in [\, M \,\rangle.\ M'(o) \geq 1$$

**proper completion** when $o$ is marked, no other token is left

$$\forall M \in [\, i \,\rangle.\ M(o) \geq 1 \Rightarrow M = o$$

# Dead, live or non-live

A remark about terminology:

t is **dead**: its firing is always ruled out

t is **live**: its firing can never be ruled out

t is **non-live** = its firing is possibly ruled out

# Brute-force analysis

First, check if the Petri net is a workflow net
easy "syntactic" check

Second, check soundness (more involved)
build the Reachability Graph
**to check 1**: for each transition t there must be an arc in the RG that is labelled with t
**to check 2&3**: the RG must have only one final state (sink) and it must consists of one token in o

# Some Pragmatic Considerations

All checks can better be done automatically
(computer aided)

but nevertheless RG construction...
1. can be computationally expensive for large nets
(because of state explosion)
2. provides little support in repairing unsound processes
3. can be infinite (CG can be used, but it is not exact)

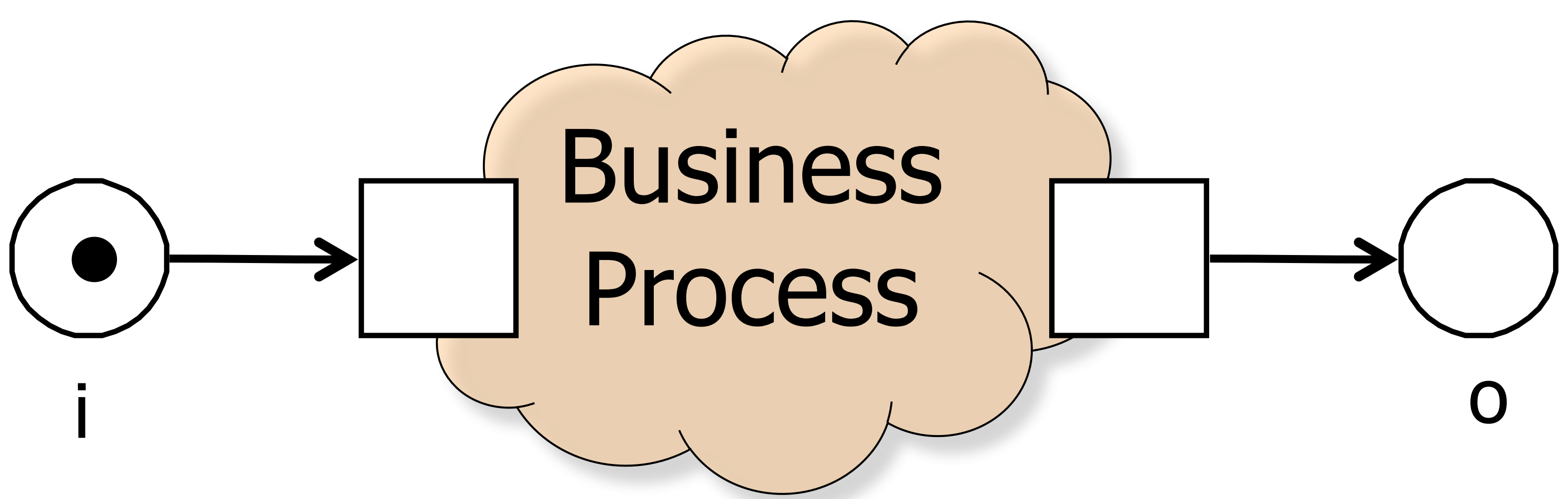
# N*

# Advanced support

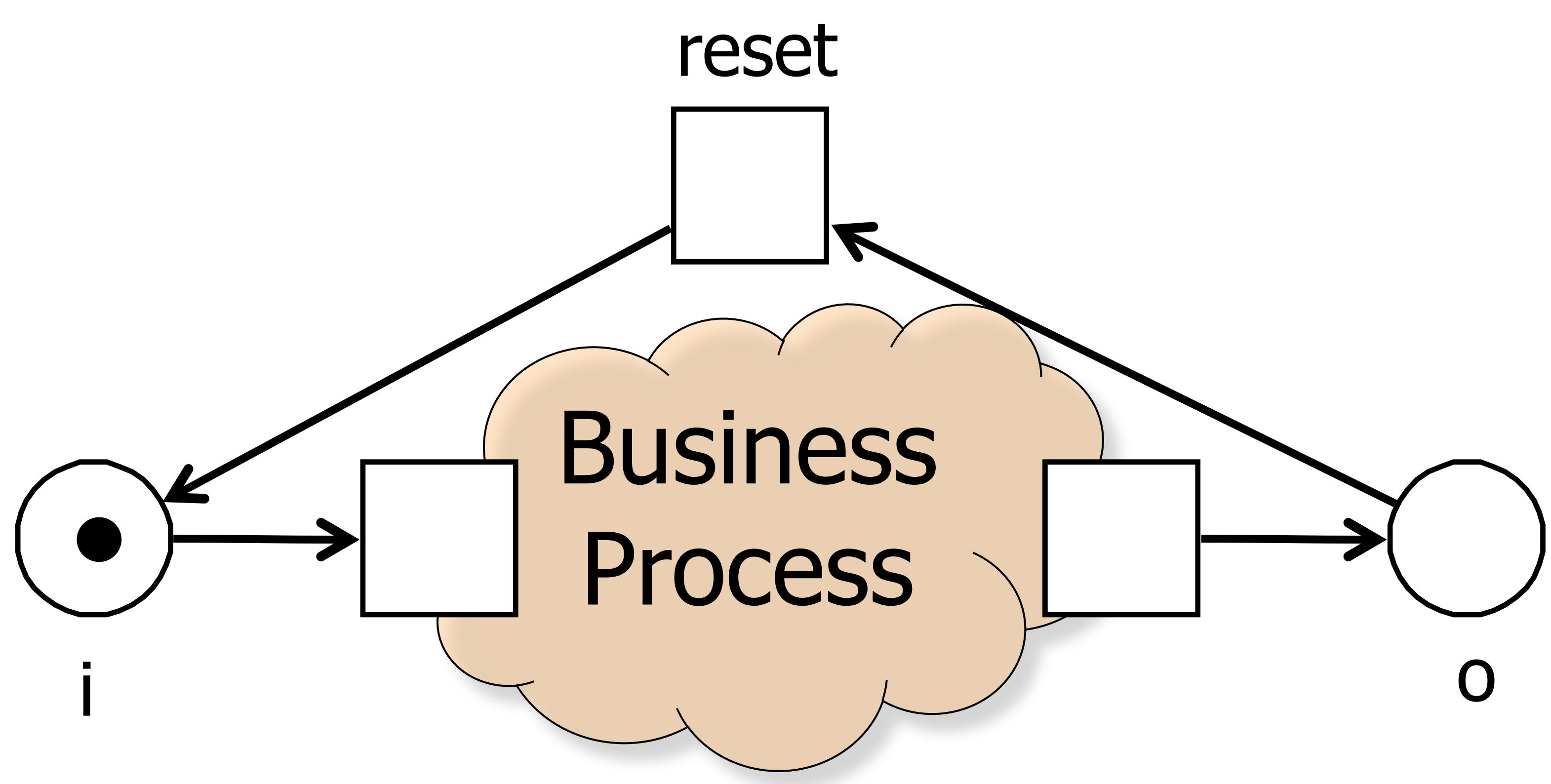
Translate soundness to other well-known properties that can be checked more efficiently:

boundedness and liveness

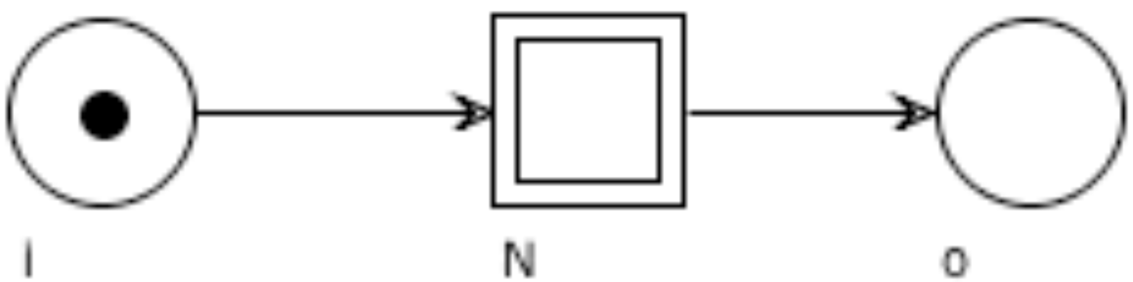
# Play once

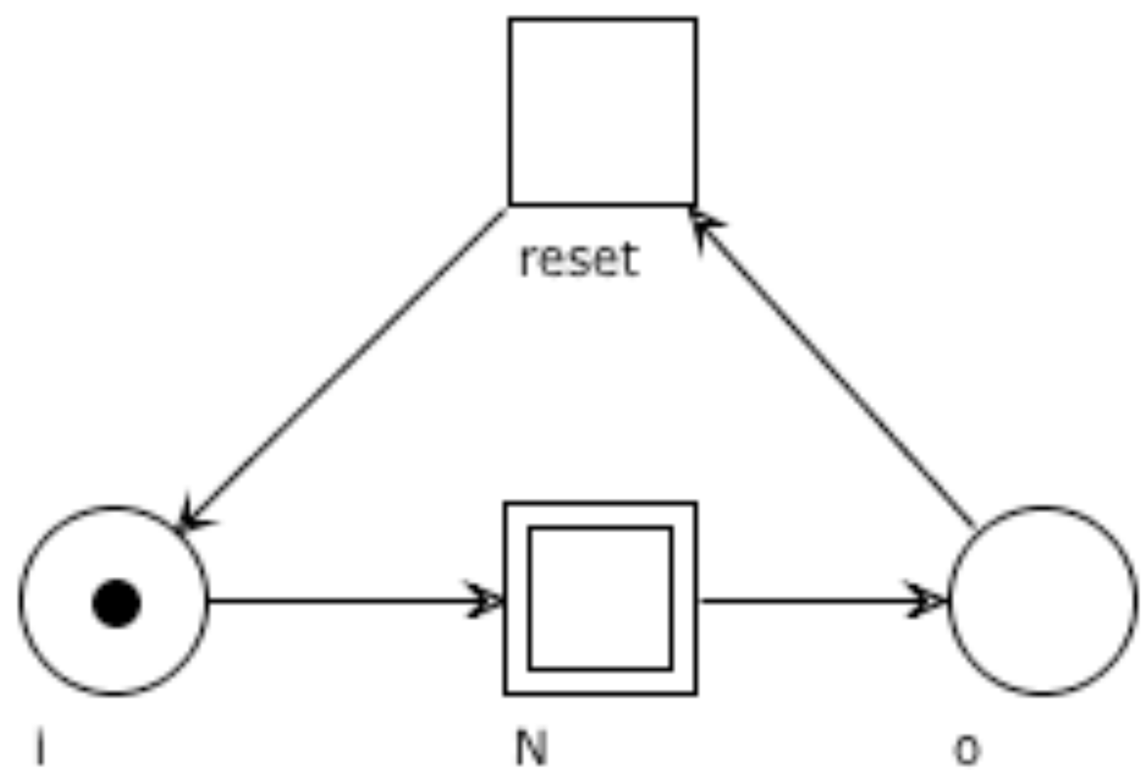


# Play Twice



# From N to N*



# Strong connectedness of N*

Let us denote by $N : i \to o$ a workflow net with entry place $i$ and exit place $o$.

Let $N^*$ be the net obtained by adding the "$reset$" transition to $N$ $reset : o \to i$.

**Proposition**:
$N^*$ is strongly connected.

Take two nodes of $(x, y) \in F_{N^*}$,
we want to build a path from $y$ to $x$

If $x, y \neq reset$, then
$y$ lies on a path $i \to^* y \to^* o$, because $N$ is a workflow net,
$x$ lies on a path $i \to^* x \to^* o$, because $N$ is a workflow net,
we combine the paths $y \to^* o \to reset \to i \to^* x$

# Strong connectedness of N*

Let us denote by $N : i \rightarrow o$ a workflow net with entry place $i$ and exit place $o$.

Let $N^*$ be the net obtained by adding the "$reset$" transition to $N$
$reset : o \rightarrow i$.

**Proposition**:
$N^*$ is strongly connected.

Take two nodes of $(x, y) \in F_{N^*}$,
we want to build a path from $y$ to $x$

If $x = o, y = reset$, then
take any path $i \rightarrow^* o$,
we build the path $reset \rightarrow i \rightarrow^* o$

# Strong connectedness of N*

Let us denote by $N : i \to o$ a workflow net with entry place $i$ and exit place $o$.

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**Proposition**:
$N^*$ is strongly connected.

Take two nodes of $(x, y) \in F_{N^*}$,
we want to build a path from $y$ to $x$

If $x = reset, y = i$, then
take any path $i \to^* o$,
we build the path $i \to^* o \to reset$

# MAIN THEOREM

Let us denote by $N : i \to o$ a workflow net
with entry place $i$ and exit place $o$

Let $N^*$ be the net obtained by adding the "$reset$" transition to $N$
$reset : o \to i$

**Theorem**:
$N$ is sound iff $N^*$ is live and bounded

# Proof of MAIN THEOREM (1)

$N^*$ **live and bounded implies** $N$ **sound**:

Since $N^*$ is **live**: for each $t \in T$ there is $M \in [\,i\,\rangle$. $M \xrightarrow{t}$

Take any $M \in [\,i\,\rangle$ enabling $reset : o \to i$, hence $M \supseteq o$

Let $M \xrightarrow{reset} M'$. Then $M' \in [\,i\,\rangle$ and $M' \supseteq i$

Since $N^*$ is bound, it must be $M' = i$ (and $M = o$)
Otherwise all places marked by $M' - i = M - o$ would be unbounded

Hence $N^*$ just allows multiple runs of $N$:
"option to complete" and "proper completion" hold (see above)
"no dead task" holds because $N^*$ is live

# A technical lemma

**Lemma**:
If $N$ is sound, $M$ is reachable in $N$ iff $M$ is reachable in $N^*$

$\Rightarrow$) straightforward

$\Leftarrow$) Let $i \xrightarrow{\sigma} M$ in $N^*$ for $\sigma = t_1 t_2 ... t_n$
We proceed by induction on the number $r$ of instances of $reset$ in $\sigma$
If $r = 0$, then $reset$ does not occur in $\sigma$ and $M$ is reachable in $N$
If $r > 0$, let $k$ be the least index such that $t_k = reset$
Let $\sigma = \sigma' t_k \sigma''$ with $\sigma' = t_1 t_2 ... t_{k-1}$ fireable in $N$
Since $N$ is sound: $i \xrightarrow{\sigma'} o$ and $i \xrightarrow{\sigma''} M$
Since $\sigma''$ contains $r - 1$ instances of $reset$:
by inductive hypothesis $M$ is reachable in $N$

# Proof of MAIN THEOREM (2)

$N$ **sound implies** $N^*$ **bounded** :
We proceed by contradiction, assuming $N^*$ is unbounded

Since $N^*$ is unbounded:
$\exists M, M'$ such that $i \rightarrow^* M \rightarrow^* M'$ with $M \subset M'$
Let $L = M' - M \neq \emptyset$

Since $N$ is sound:
$\exists \sigma \in T^*$ such that $M \stackrel{\sigma}{\rightarrow} o$

By the monotonicity Lemma: $M' \stackrel{\sigma}{\rightarrow} o + L$ and thus $o + L \in [\, i\,\rangle$
Which is absurd, because $N$ is sound

# Proof of MAIN THEOREM (3)

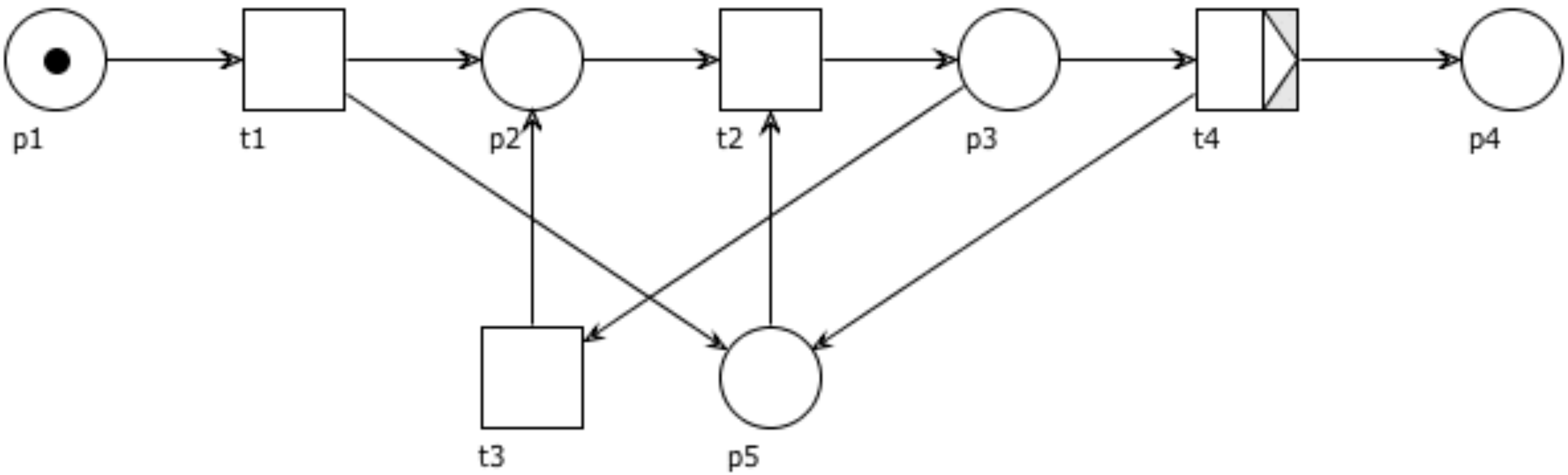
$N$ **sound implies** $N^*$ **live**:
Take any transition $t$ and let $M$ be a marking reachable in $N^*$
By the technical lemma, $M$ is reachable in $N$

Since $N$ is sound: $\exists \sigma \in T^*$ with $M \xrightarrow{\sigma} o$
Since $N$ is sound: $\exists \sigma' \in T^*$ with $i \xrightarrow{\sigma'} M'$ and $M' \xrightarrow{t}$

Let $\sigma'' = \sigma \, reset \, \sigma'$, then:
$M \xrightarrow{\sigma''} M'$ in $N^*$ and $M' \xrightarrow{t}$

# Exercise

Use some tools to check if the net below is a sound workflow net or not



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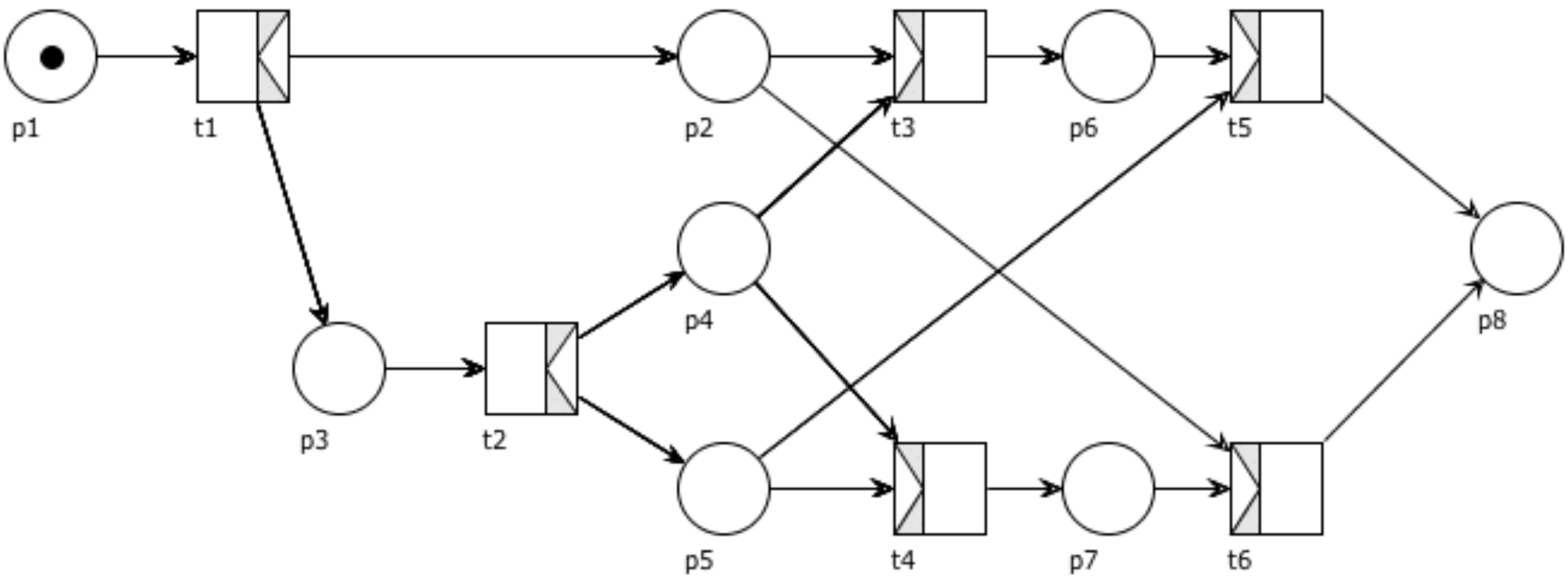

# Exercise

Analyse the following net