

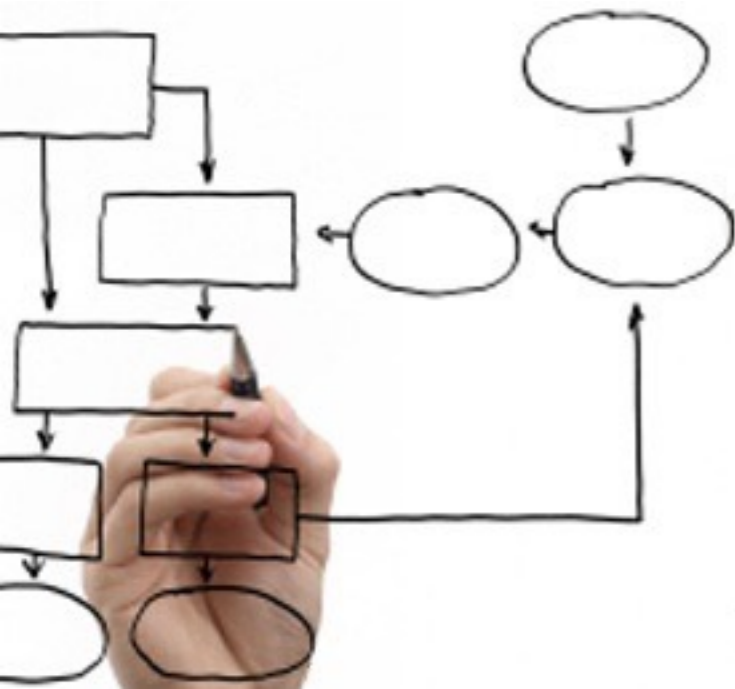
Business Processes Modelling

MPB (6 cfu, 295AA)

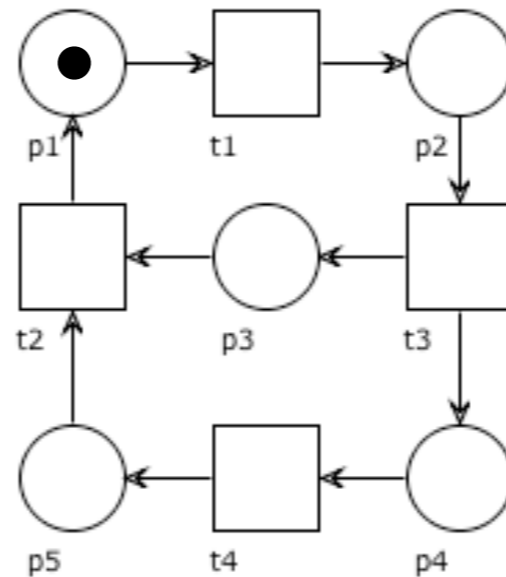
Roberto Bruni

<http://www.di.unipi.it/~bruni>

17 - T-systems



Object



We study some “good” properties of T-systems

Free Choice Nets (book, optional reading)

<https://www7.in.tum.de/~esparza/bookfc.html>

T-systems

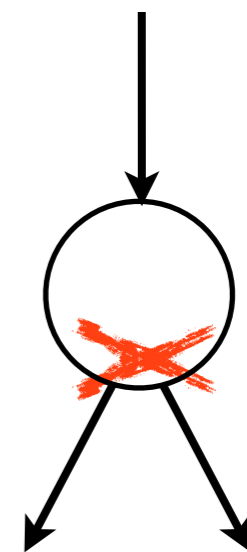
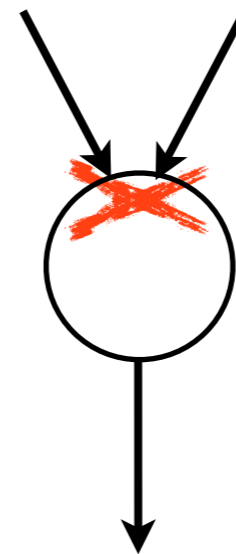
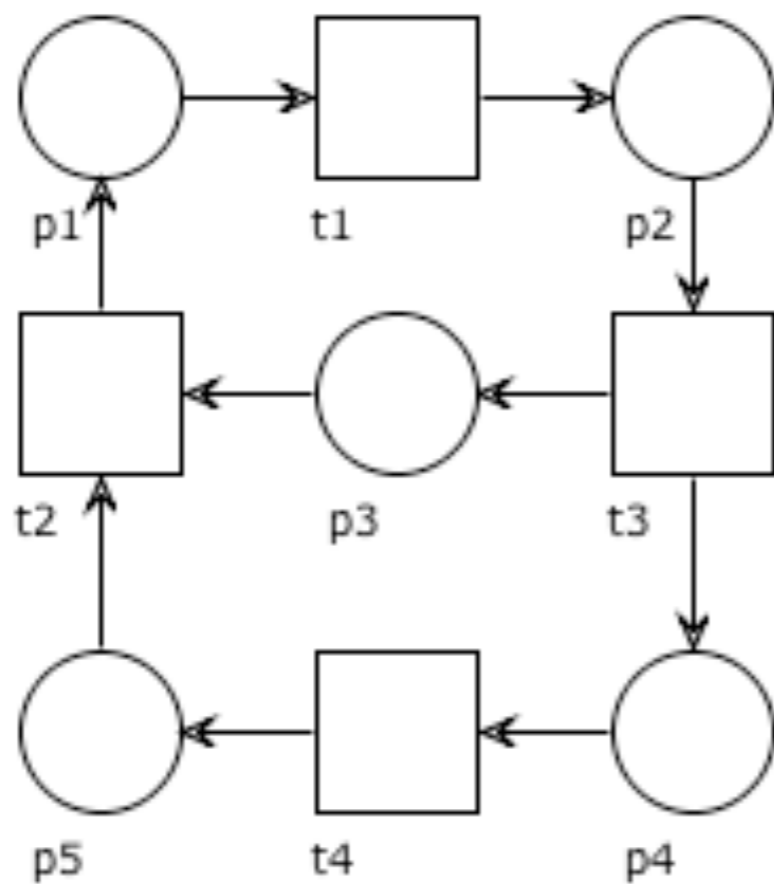
T-system

Definition: We recall that a net N is a **T-net** if each place has exactly one input transition and exactly one output transition

$$\forall p \in P, \quad |\bullet p| = 1 = |p \bullet|$$

A system (N, M_0) is a **T-system** if N is a T-net

T-net: example



T-systems: an observation

Notably, computation in T-systems is concurrent,
but essentially deterministic:

the firing of a transition t in M cannot disable
another transition t' enabled at M

T-net N^*

Is the following conjecture true?
A workflow net N is a T-net
iff N^* is a T-net

T-net N^*

Is the following conjecture true?

A workflow net N is a T-net
iff N^* is a T-net

No, a workflow net cannot be a T-net because
the place i has no incoming arc
and the place o has no outgoing arc

(N^* can be a T-net)

T-systems: another observation

Determination of control:

the transitions responsible for enabling t are
one for each input place of t

Notation: token count of a circuit

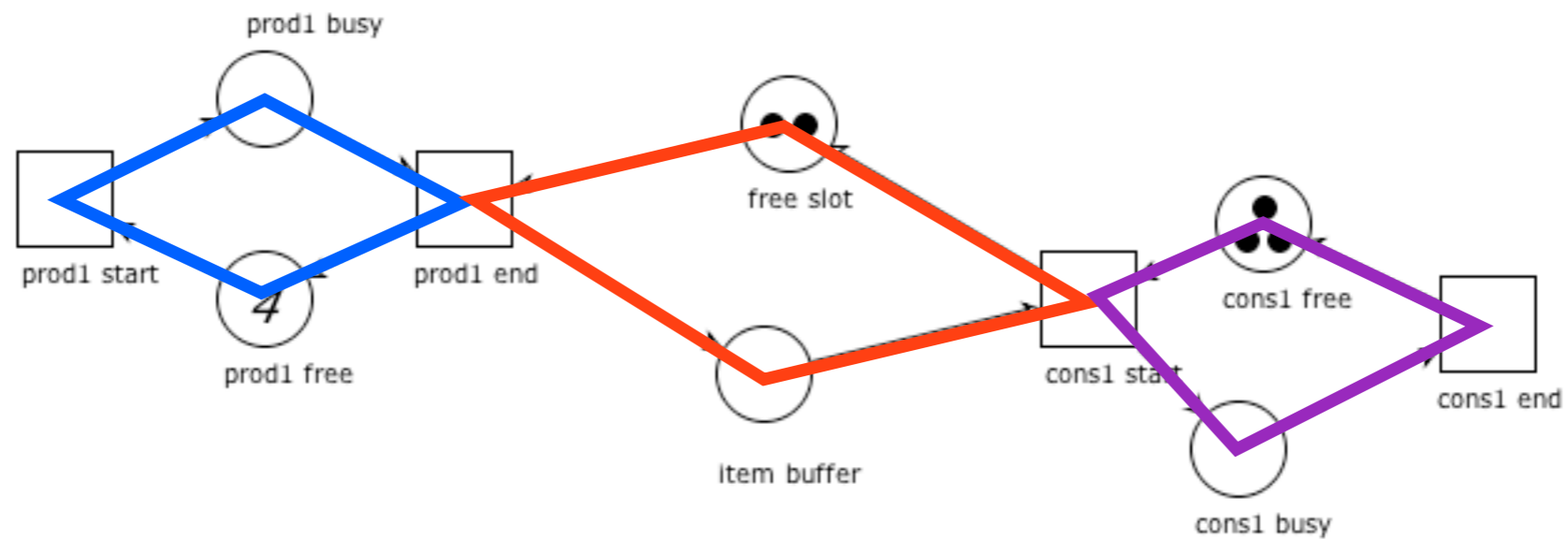
Let $\gamma = (x_1, y_1)(y_1, x_2)(x_2, y_2)\dots(x_n, y_n)$ be a circuit.

Let $P|_{\gamma} \subseteq P$ be the set of places in γ .

$$M(\gamma) = M(P|_{\gamma}) = \sum_{p \in P|_{\gamma}} M(p)$$

We say that γ is **marked at** M if $M(\gamma) > 0$

Example



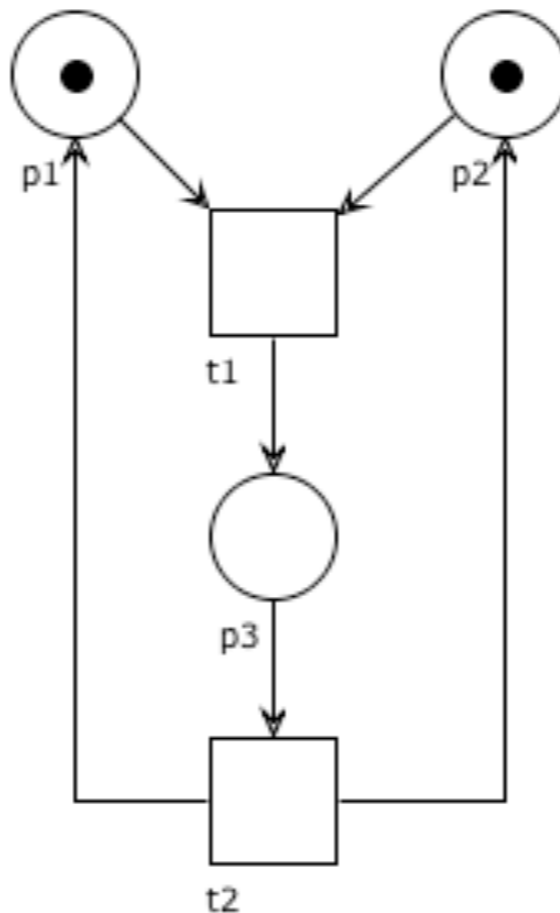
$$M(\gamma_1) = 4$$

$$M(\gamma_2) = 2$$

$$M(\gamma_3) = 3$$

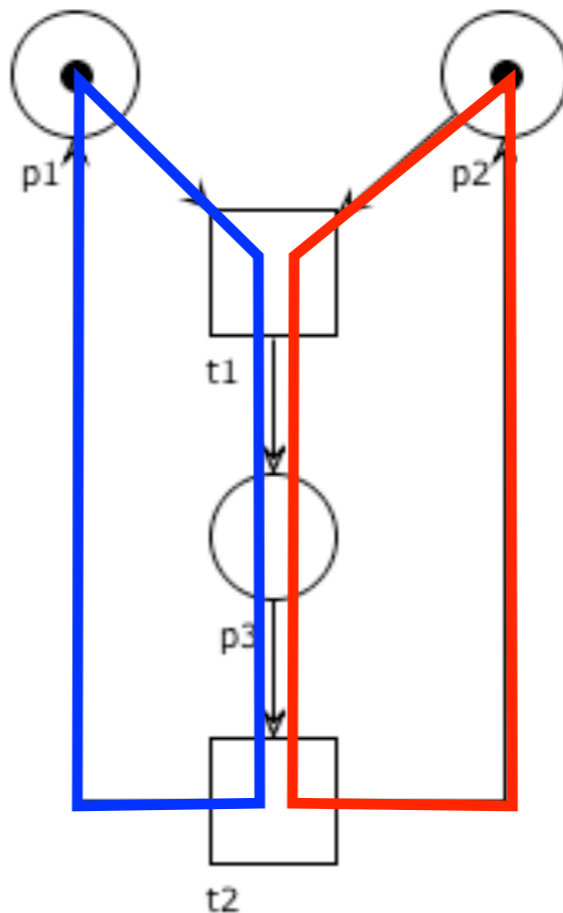
Question time

Trace two circuits over the T-system below



Question time

Trace two circuits over the T-system below



Fundamental property of T-systems

The token count of a circuit is invariant under any firing.

Fundamental property of T-systems

Proposition: Let γ be a circuit of a T-system (P, T, F, M_0) .
If M is a reachable marking, then $M(\gamma) = M_0(\gamma)$

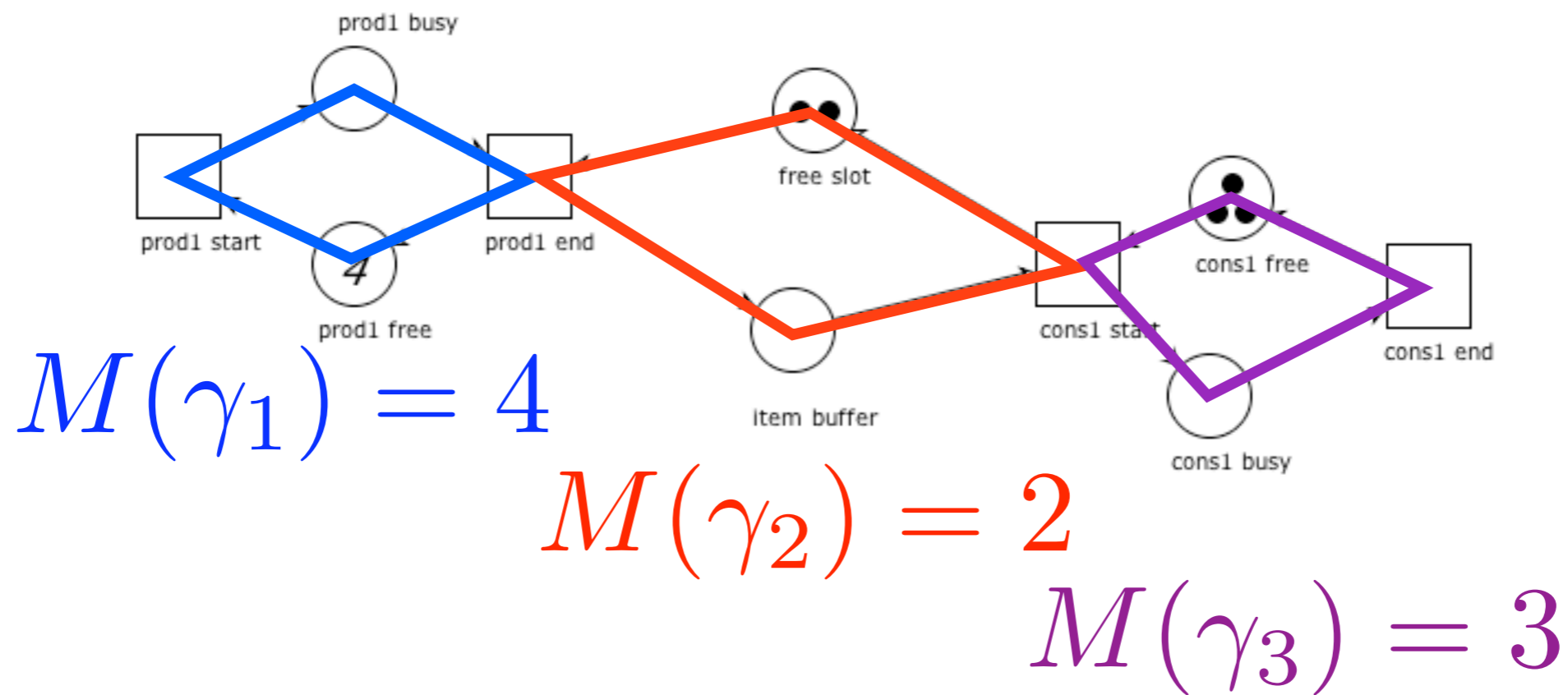
Take any $t \in T$: either $t \notin \gamma$ or $t \in \gamma$.

If $t \notin \gamma$, then no place in $\bullet t \cup t\bullet$ is in γ
(otherwise, by definition of T-nets, t would be in γ).

Then, an occurrence of t does not change the token count of γ .

If $t \in \gamma$, then exactly one place in $\bullet t$ and one place in $t\bullet$ are in γ .
Then, an occurrence of t does not change the token count of γ .

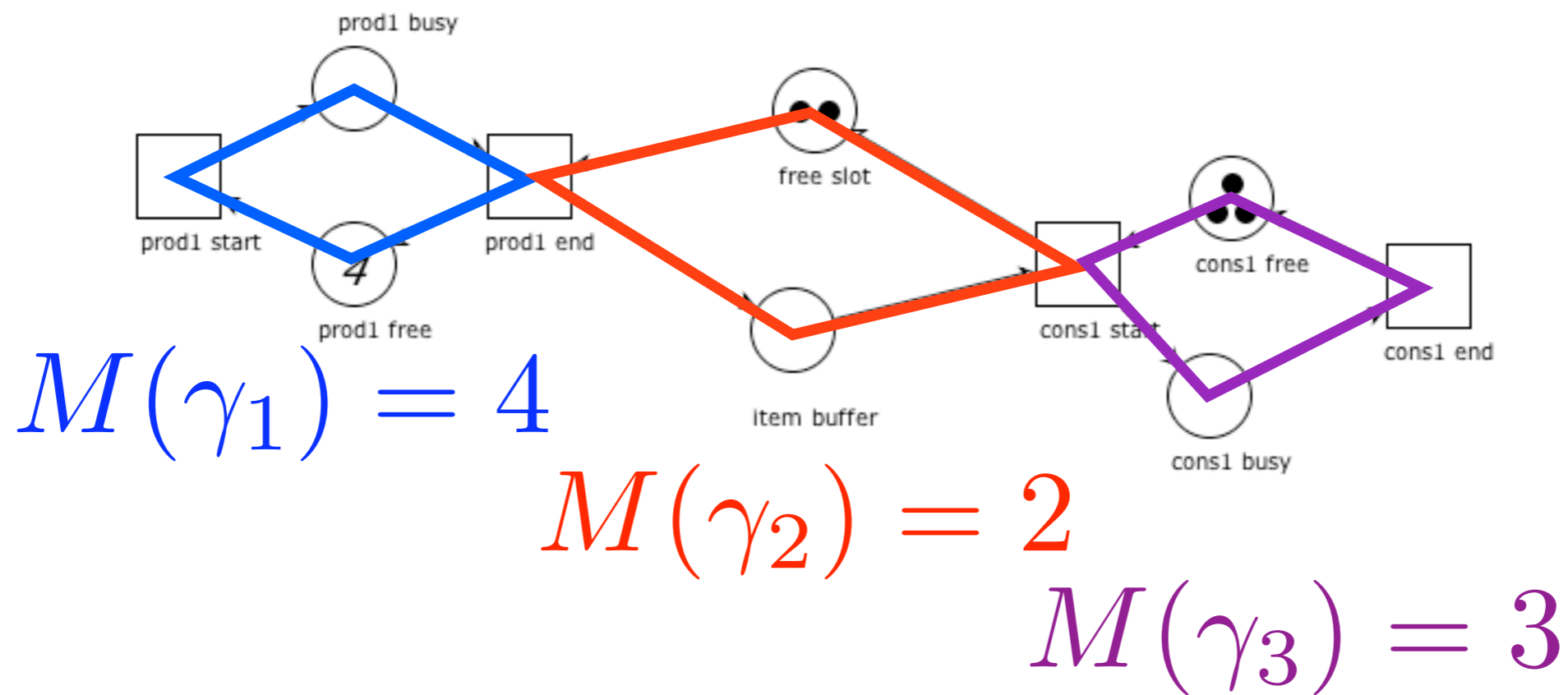
Example



$$M_0 = [0 \ 4 \ 2 \ 0 \ 3 \ 0]$$

$$M = [2 \ 2 \ 1 \ 2 \ 2 \ 1]$$

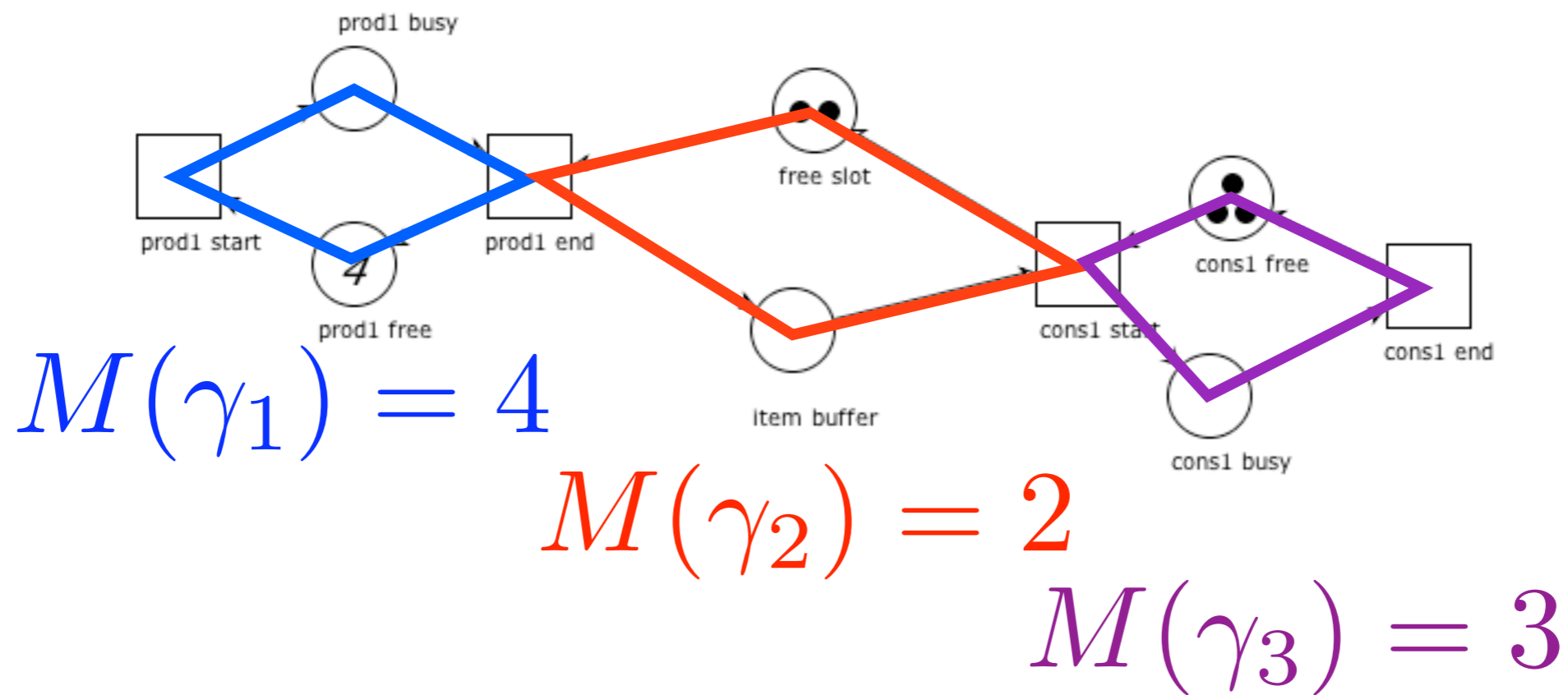
Example



$$M_0 = [0 \ 4 \ 2 \ 0 \ 3 \ 0]$$

$$M = [2 \ 2 \ 1 \ 2 \ 2 \ 1] \quad \text{Not reachable!}$$

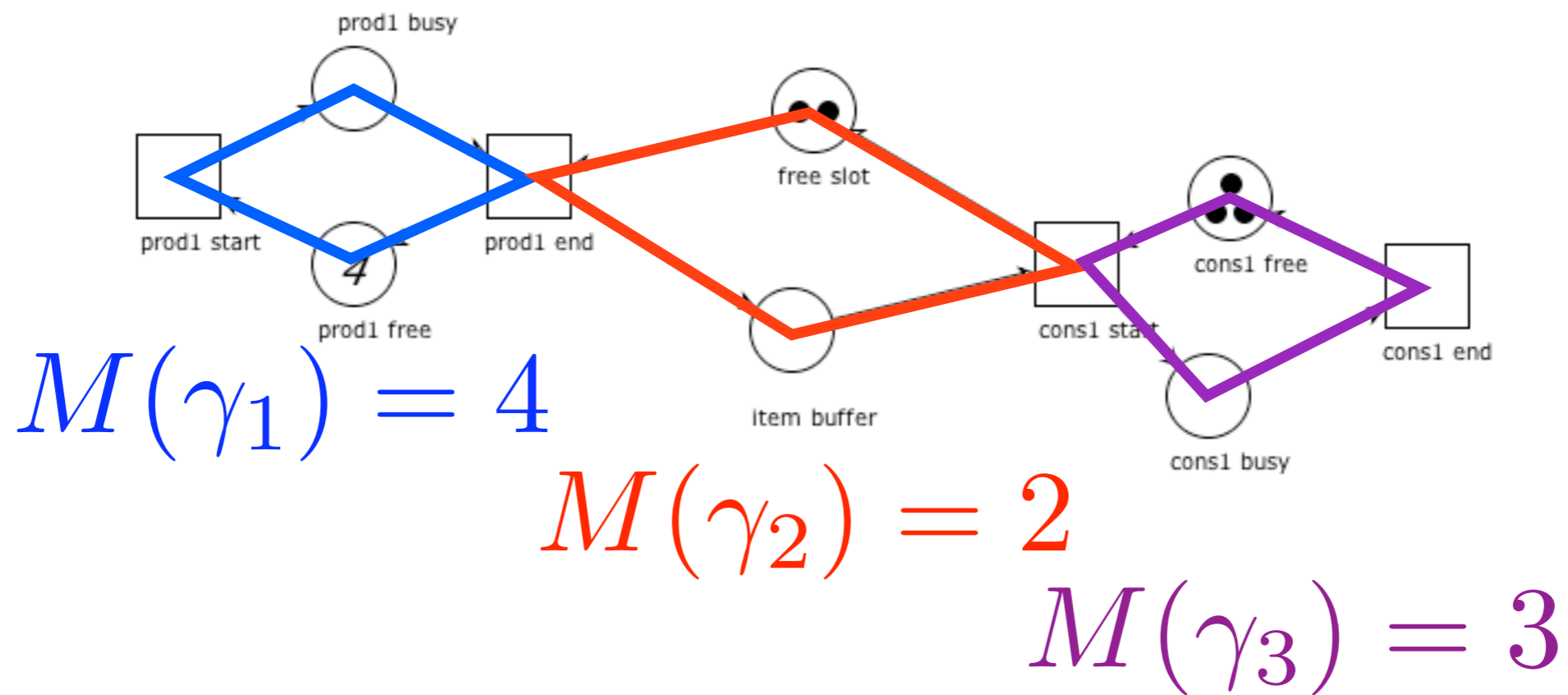
Example



$$M_0 = \begin{bmatrix} 0 & 4 & 2 & 0 & 3 & 0 \end{bmatrix}$$

$$M' = \begin{bmatrix} 2 & 1 & 1 & 1 & 2 & 2 \end{bmatrix}$$

Example

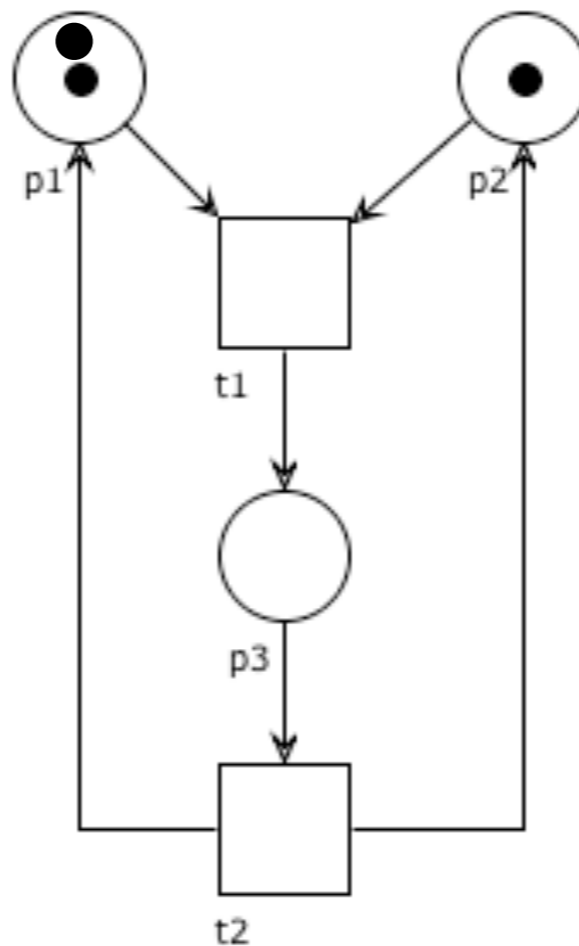


$$M_0 = [0 \ 4 \ 2 \ 0 \ 3 \ 0]$$

$$M' = [2 \ 1 \ 1 \ 1 \ 2 \ 2] \quad \text{Not reachable!}$$

Question time

Is the marking $p_1 + 2p_2$ reachable? (why?)



Question time

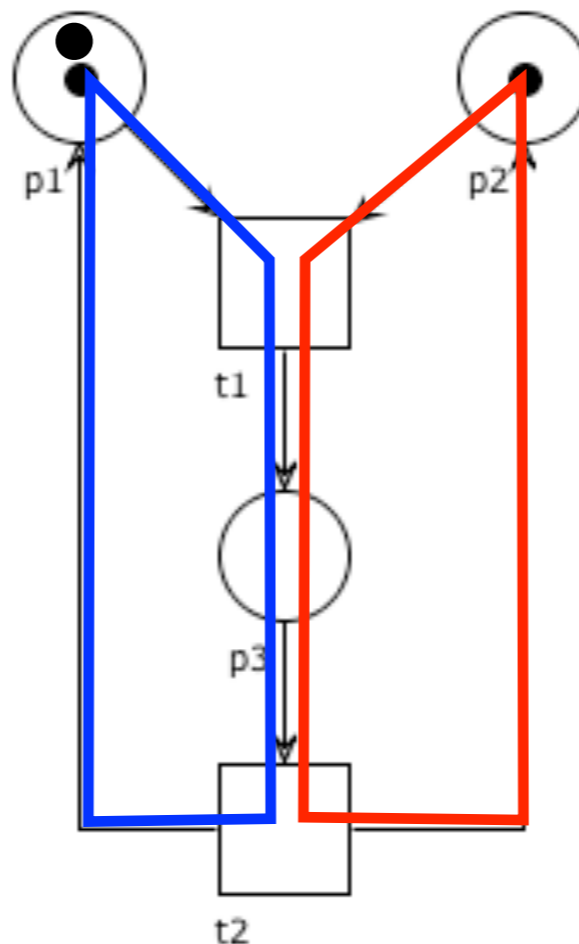
Is the marking $p_1 + 2p_2$ reachable? (why?)

No

the token count in
the left (blue) circuit
must remain 2

and

the token count in
the right (red) circuit
must remain 1



T-invariants of T-nets

Proposition: Let $N=(P,T,F)$ be a (connected) T-net. J is a T-invariant of N iff $J=[k \dots k]$ for some value k

(the proof is dual to the analogous proposition for S-invariants of S-nets)

Boundedness in strongly connected T-systems

Lemma: If a T-system (N, M_0) is strongly connected, then it is bounded

Let Γ be the set of the circuits of N and let $k = \max_{\gamma \in \Gamma} M_0(\gamma)$.

Since N is strongly connected, every place p belongs to some circuit γ_p .

By the fundamental property of T-systems: token count of γ_p is invariant.

Thus, for any reachable marking M , we have $M(p) \leq M(\gamma_p) = M_0(\gamma_p) \leq k$.
Hence the net is k -bounded.

Safeness in strongly connected T-systems

Corollary: If a T-system (N, M_0) is strongly connected and $M_0(P)=1$, then it is safe

Let Γ be the set of the circuits of N and let $k = \max_{\gamma \in \Gamma} M_0(\gamma) = 1$

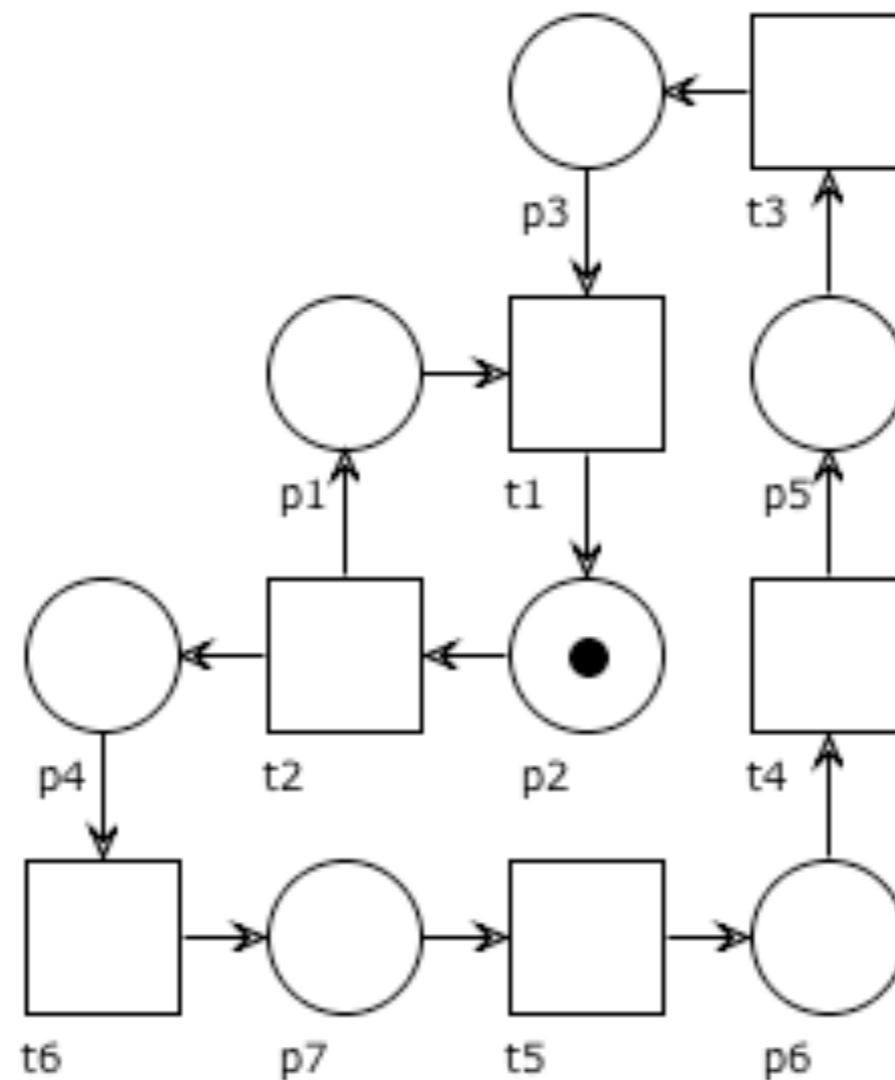
Since N is strongly connected, every place p belongs to some circuit γ_p .

By the fundamental property of T-systems: token count of γ_p is invariant.

Thus, for any reachable marking M , we have $M(p) \leq M(\gamma_p) = M_0(\gamma_p) \leq k$.
Hence the net is k -bounded. =1

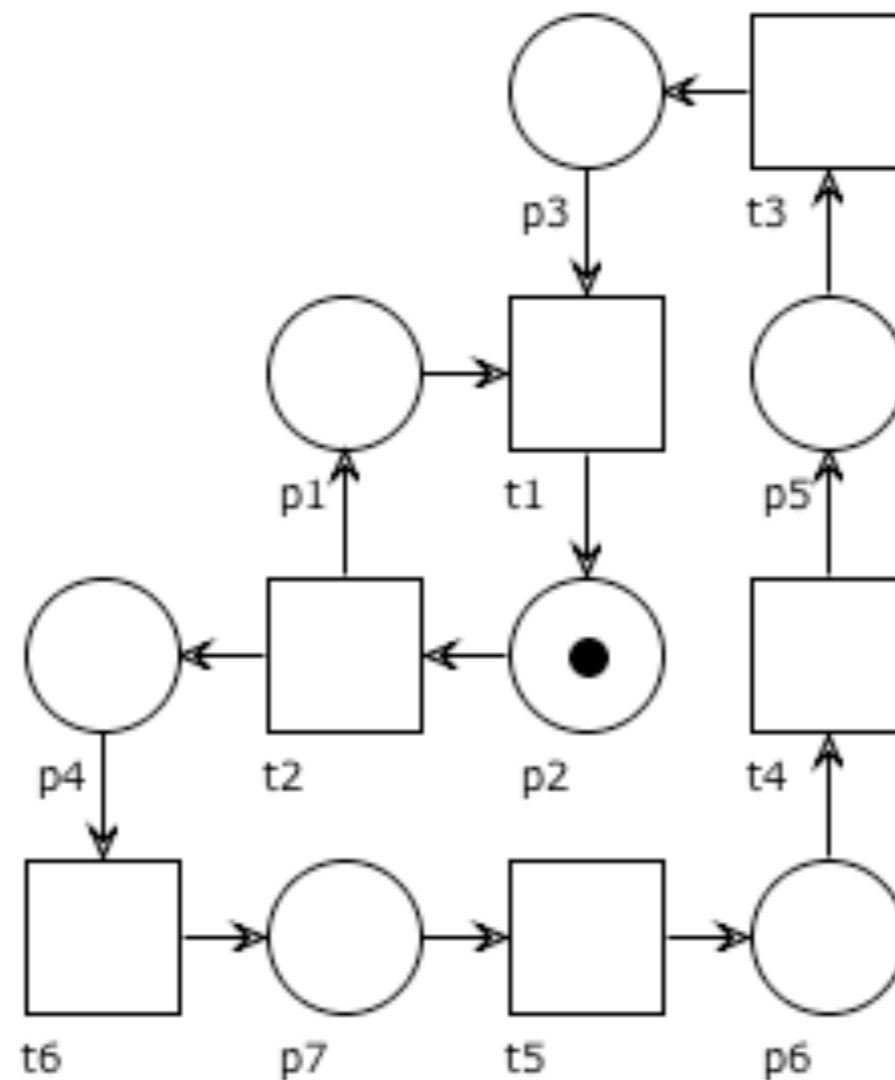
Question time

Is the T-systems below bounded? (why?)



Question time

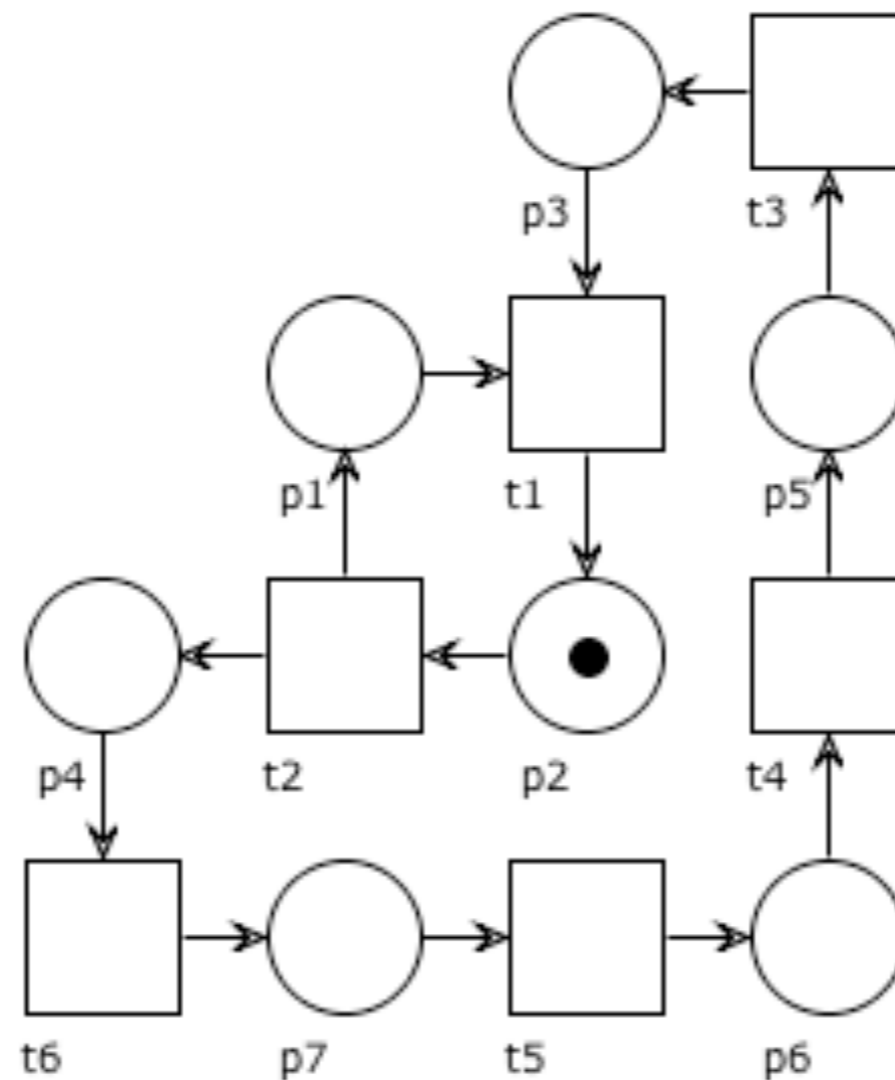
Is the T-systems below bounded? (why?)



Strongly connected
=> bounded

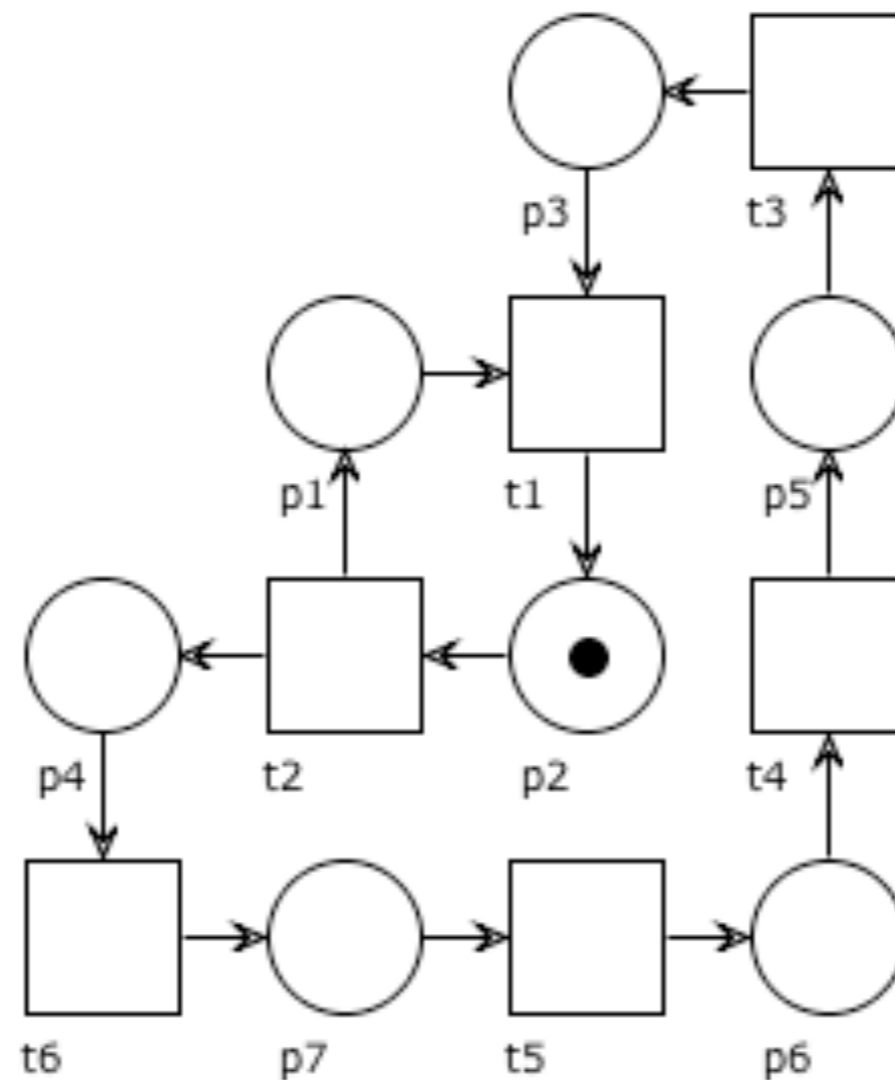
Question time

Is the T-systems below safe? (why?)



Question time

Is the T-systems below safe? (why?)



Strongly connected
+ $M_0(P)=1$
=> safe

Liveness theorem for T-systems

Theorem: A T-system (N, M_0) is live
iff every circuit of N is marked at M_0

\Rightarrow) (quite obvious)

By contradiction, let γ be a circuit with $M_0(\gamma) = 0$.

By the fundamental property of T-systems: $\forall M \in [M_0 \rangle, M(\gamma) = 0$.

Take any $t \in T|_\gamma$ and $p \in P|_\gamma \cap \bullet t$.

For any $M \in [M_0 \rangle$, we have $M(p) = 0$.

Hence t is never enabled and the T-system is not live.

Liveness theorem for T-systems

Theorem: A T-system (N, M_0) is live
iff every circuit of N is marked at M_0

\Leftarrow) (more involved)

Take any $t \in T$ and $M \in [M_0 \rangle$.

We need to show that some marking M' reachable from M enables t .

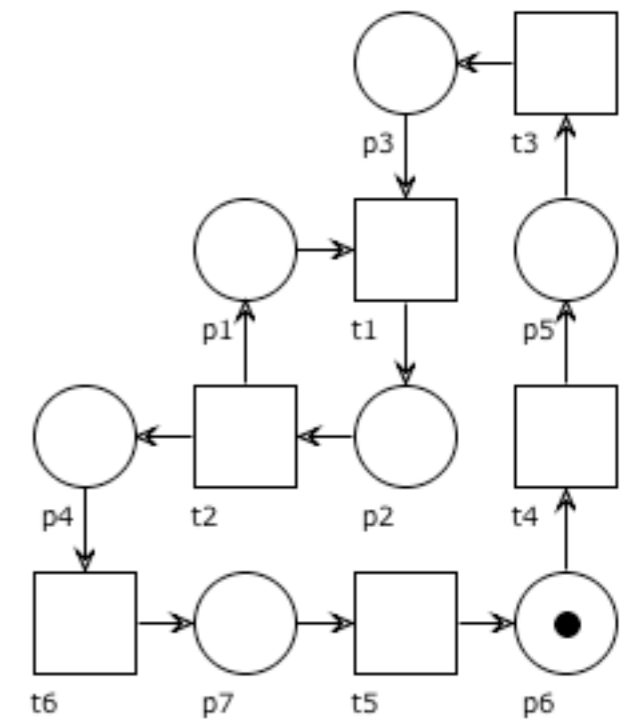
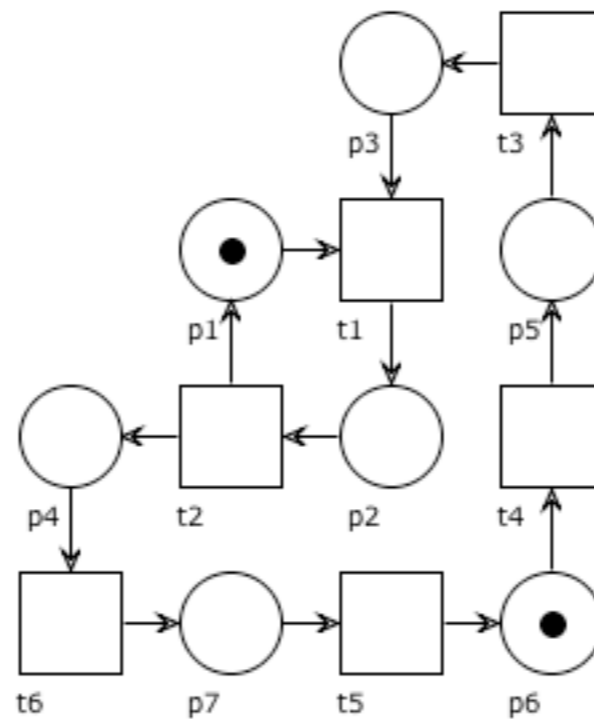
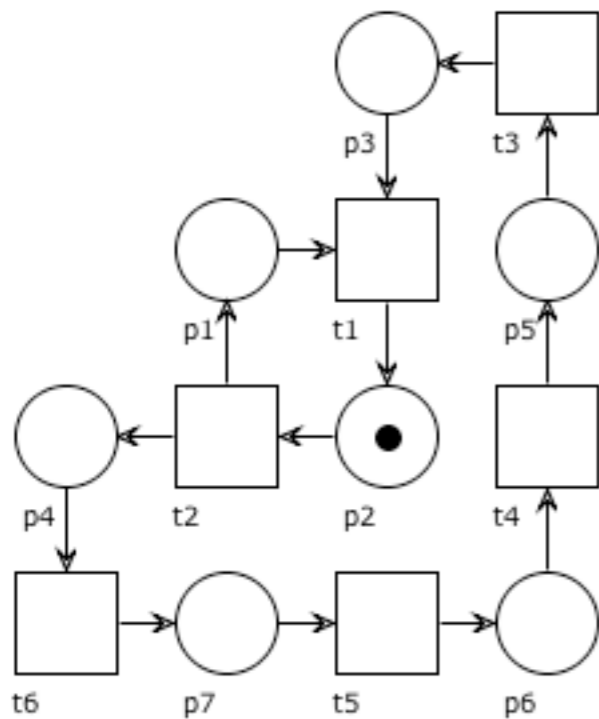
The key idea is to collect the places that control the firing of t :

$p \in P_{M,t}$ if there is a path from p to t through places unmarked at M .

We then proceed by induction on the size of $P_{M,t}$.

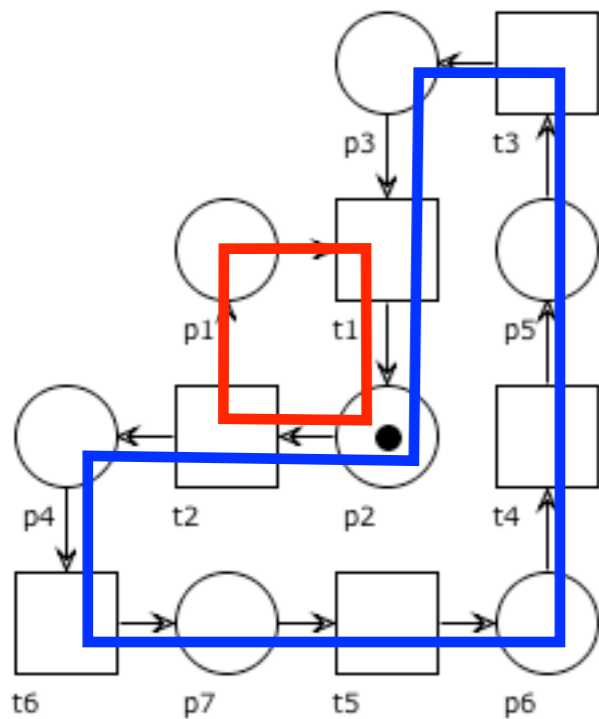
Question time

Which of the T-systems below is live? (why?)

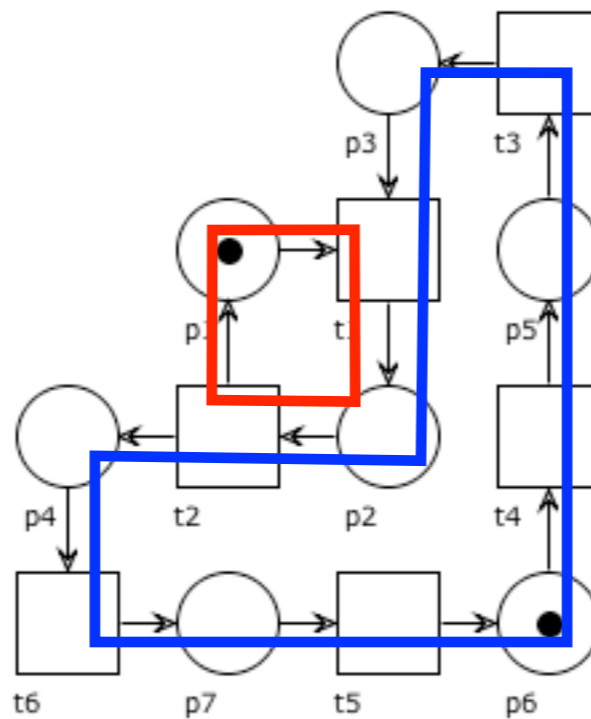


Question time

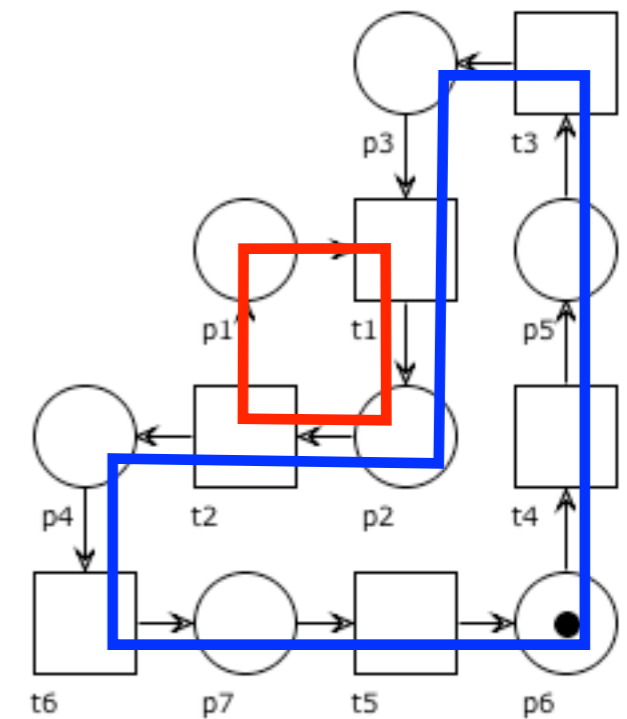
Which of the T-systems below is live? (why?)



Both circuits are marked
=> live



Both circuits are marked
=> live



red circuit is not marked
=> not live

T-systems: recap

T-system + γ circuit + M reachable $\Rightarrow M(\gamma) = M_0(\gamma)$

T-system + γ circuit + $M(\gamma) \neq M_0(\gamma) \Rightarrow M$ not reachable

T-system + $\gamma_1 \dots \gamma_n$ circuits: $\exists i. p \in \gamma_i \Leftrightarrow p$ bounded

T-system: $M_0(\gamma) > 0$ for all circuits $\gamma \Leftrightarrow$ live

T-system: strongly connected \Rightarrow bounded

T-system + live: strongly connected \Leftrightarrow bounded

T-system: T-invariant $\mathbf{J} \Leftrightarrow \mathbf{J} = [k \ k \ \dots \ k]$

Consequences on workflow nets

Theorem: If N is a workflow net s.t. N^* is a T-system then
 N is safe and sound **iff**
every circuit of N^* is marked

N workflow net $\Rightarrow N^*$ strongly connected

N^* strongly connected + T-system $\Rightarrow N^*$ bounded

N^* strongly connected + T-system + $M_0(P)=1 \Rightarrow N^*$ safe

all circuits of N^* are marked $\Leftrightarrow N^*$ live

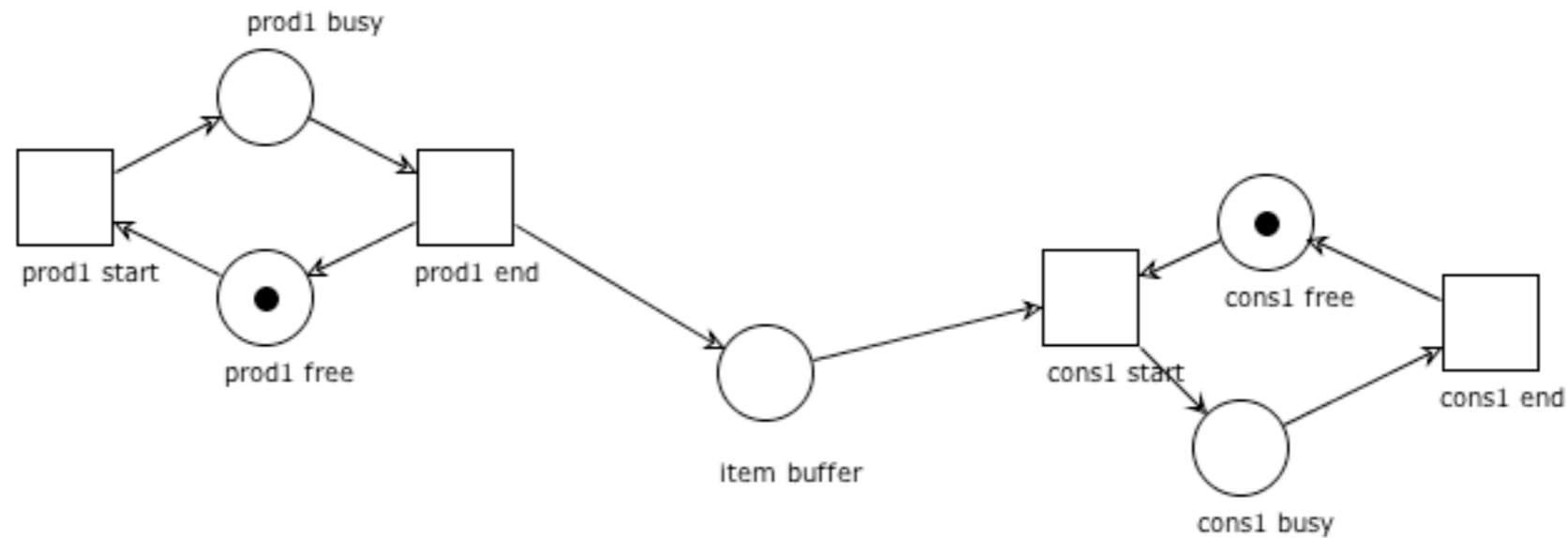
Exercises

Which are the circuits of the T-system below?

Is the T-system below live? (why?)

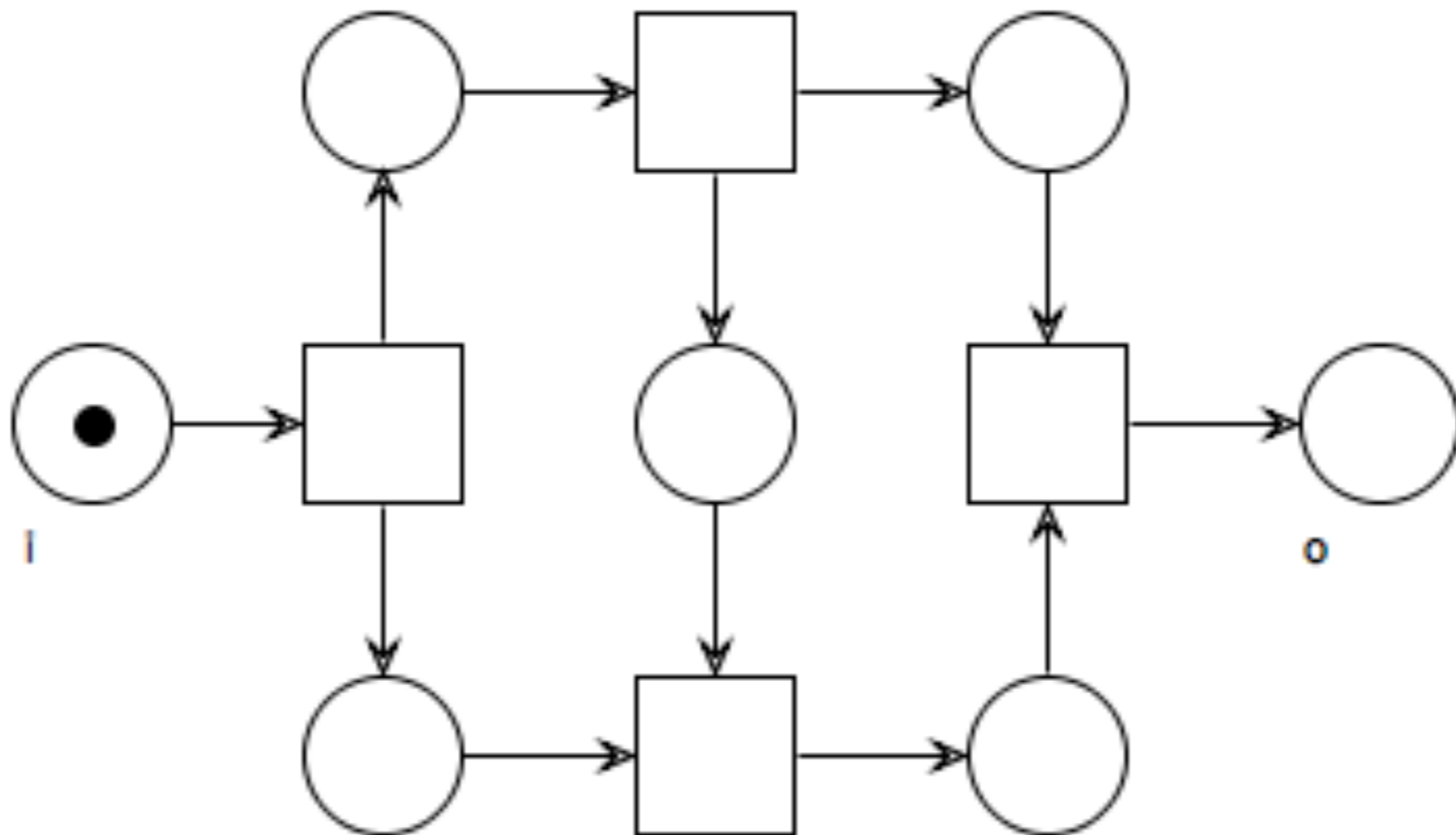
Which places are bounded? (why?)

Assign a bound to each bounded place.



Exercise

Is the net below a workflow net?
Is it sound?



Exercise

Is the net below a workflow net?
Is it sound?

