

# Business Processes Modelling

## MPB (6 cfu, 295AA)

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\* - P and NP problems



# Problems and instances

A **problem** defines a family of related questions

For example, the factorization problem is:  
*“given a number  $n$ , return all its prime factors”*

A **problem instance** is one such question

An instance of the factorization problem is:  
*“return all prime factors of 18”*

# Decision problem

A **decision problem** requires just a **boolean answer**

For example: “*given a number  $n$ , is  $n$  prime?*”

And an instance: “*is 18 prime?*”

# Computational Complexity Theory

**Computational complexity theory** deals with the resources needed to solve a problem

For example, how many steps (time) or memory (space) it takes to solve a problem

# P

The complexity class **P** is the set of decision problems that can be solved by a deterministic algorithm in a **Polynomial** number of steps (time) w.r.t. input size

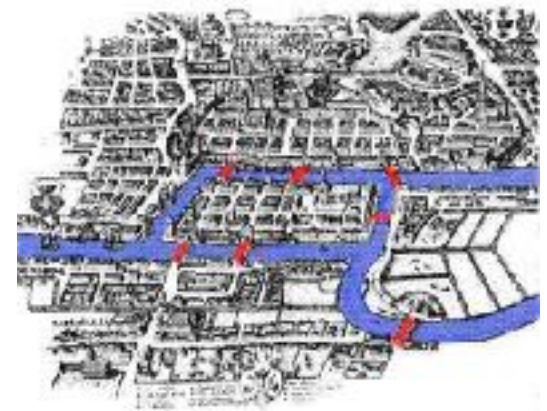
Problems in **P** can be (checked and) **solved effectively**

# Eulerian circuit (P)

*Given a graph  $G$ ,*  
*is it possible to draw an Eulerian circuit over it?*  
(i.e. a circuit that traverses each edge exactly once)

We have seen that it is the same problem as:

*Given a graph  $G$ ,*  
*is the degree of each vertex even?*



The problem can be (checked and) solved effectively!

# NP

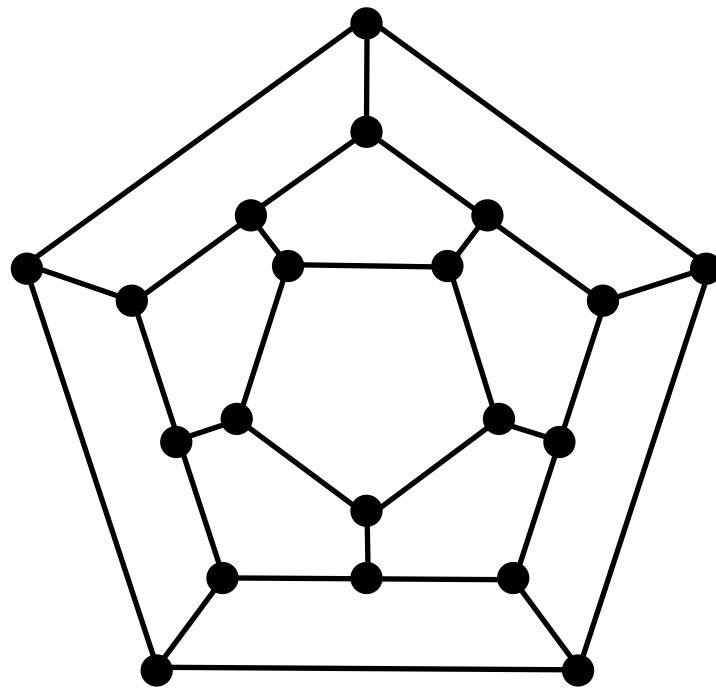
The complexity class **NP** is the set of decision problems that can be **solved** by a **Non-deterministic** algorithm in a **Polynomial** number of steps (time)

Equivalently **NP** is the set of decision problems whose solutions can be **checked** by a deterministic algorithm in a polynomial number of steps (time)

Solutions of problems in **NP** can be **checked effectively**

# Hamiltonian circuit

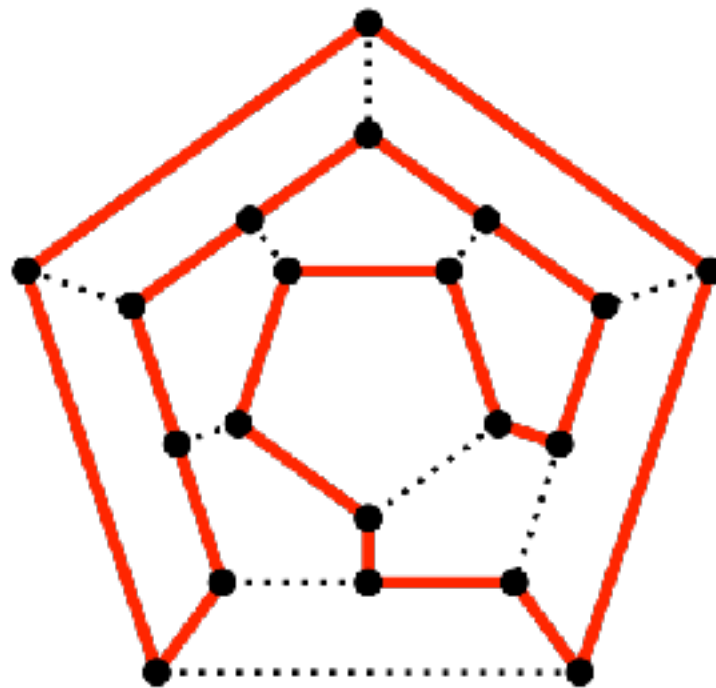
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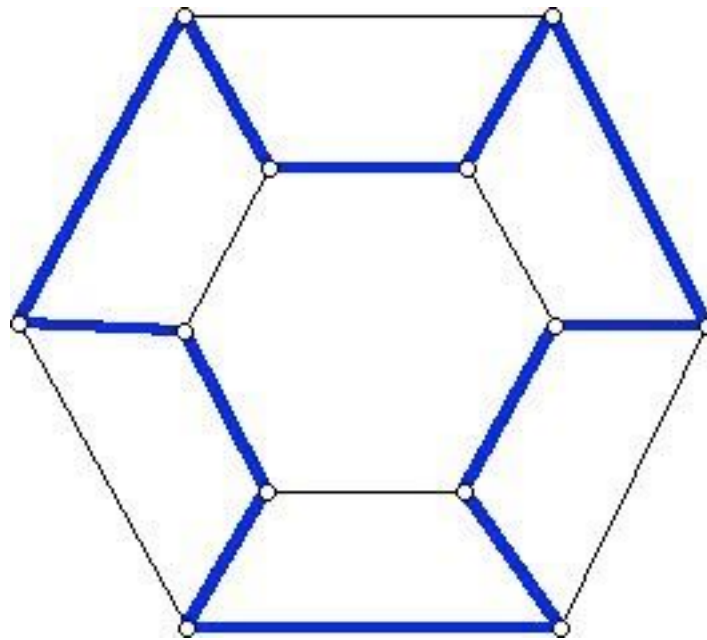
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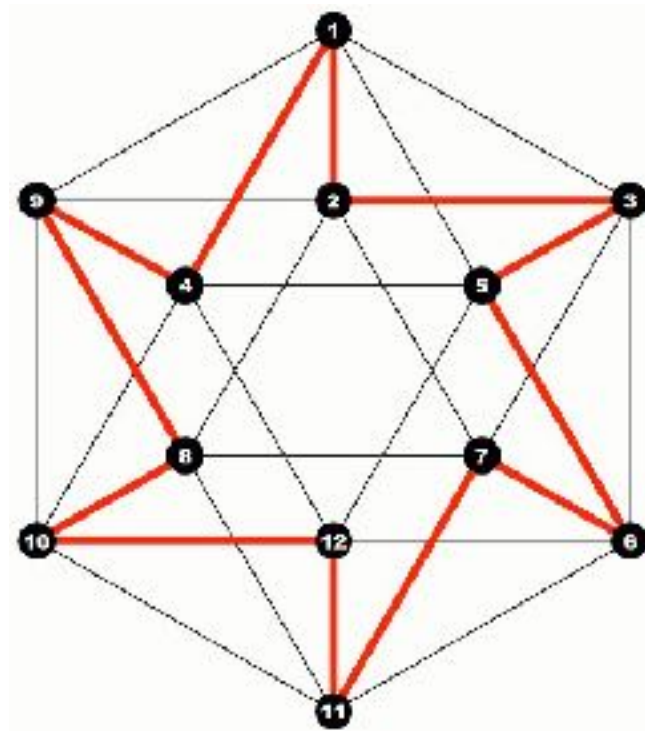
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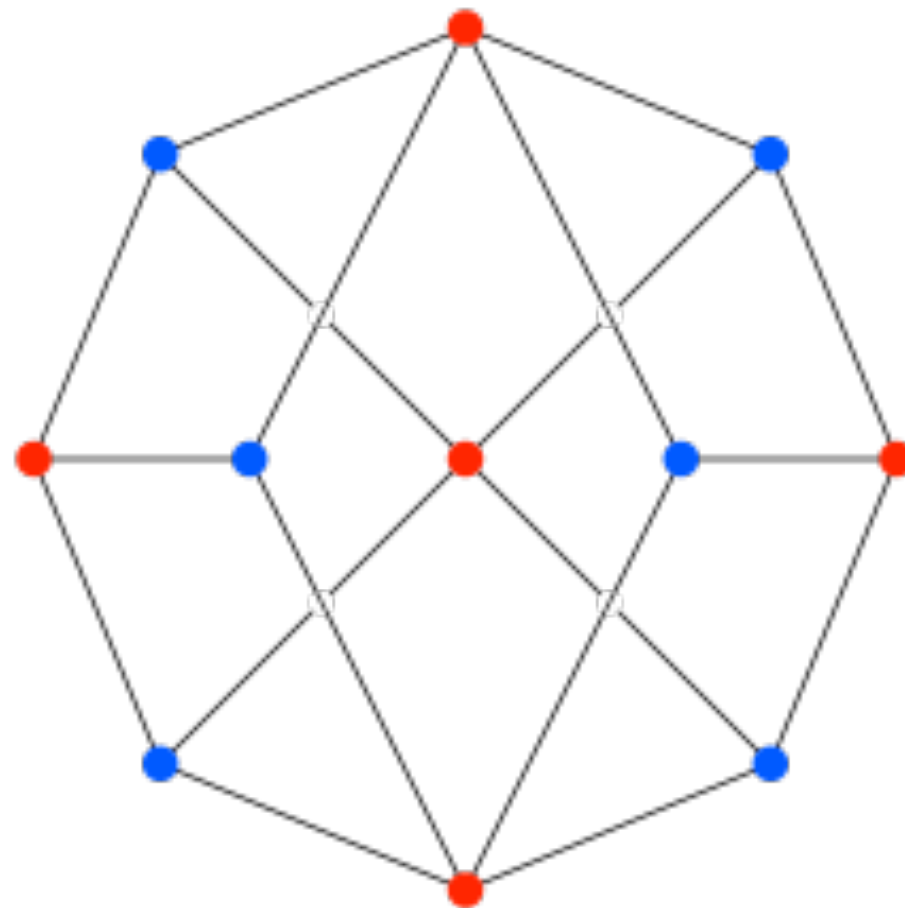


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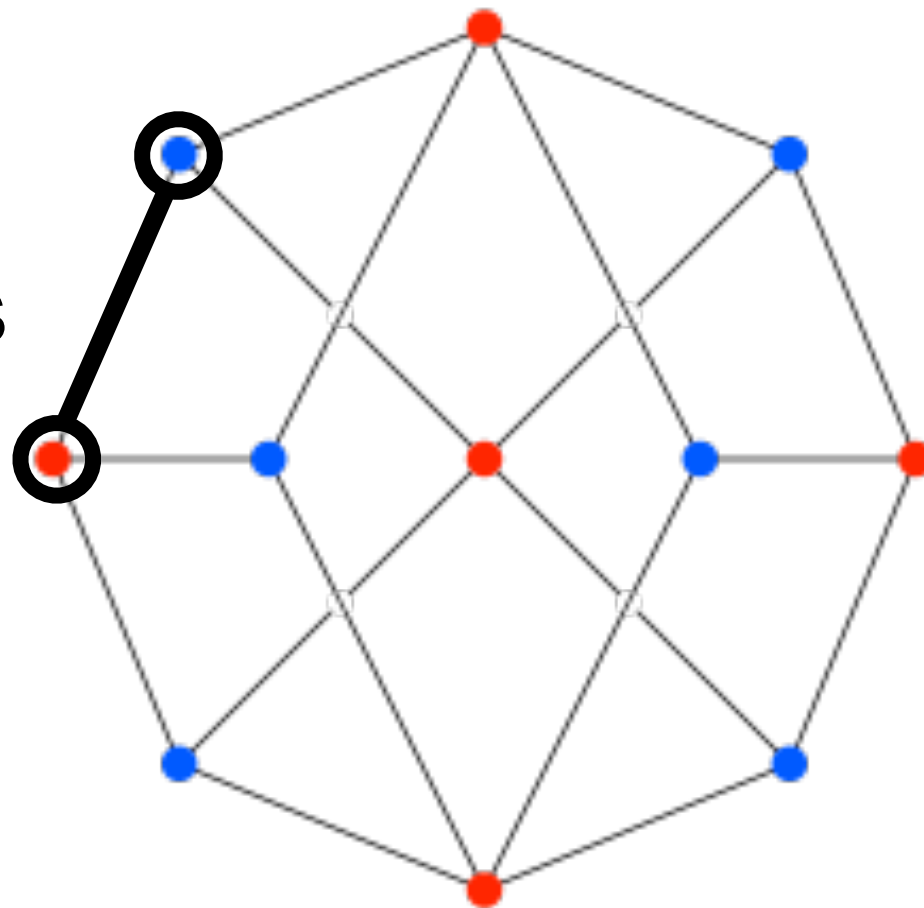
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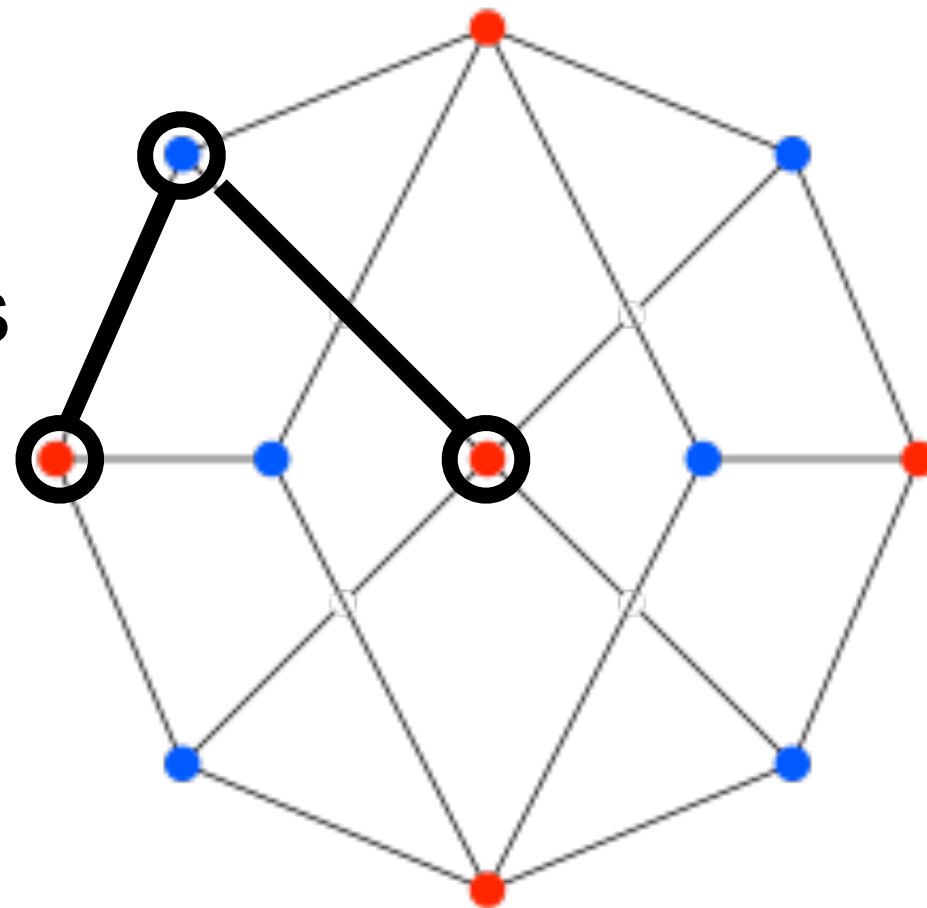
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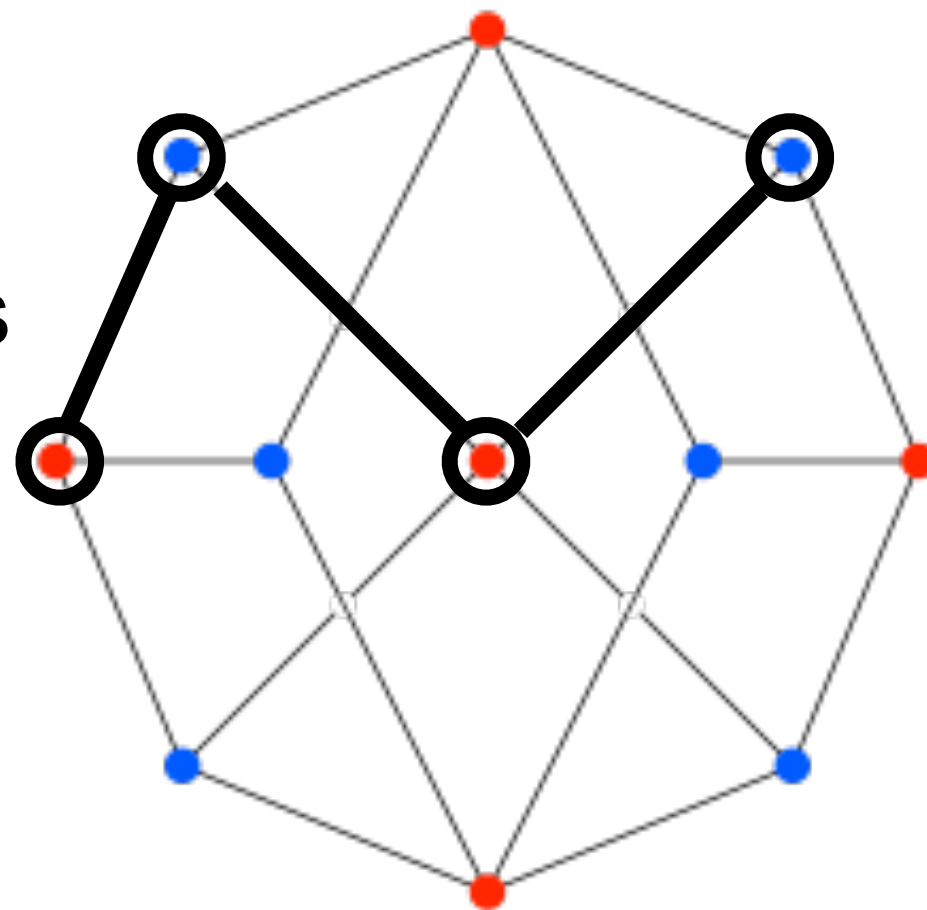
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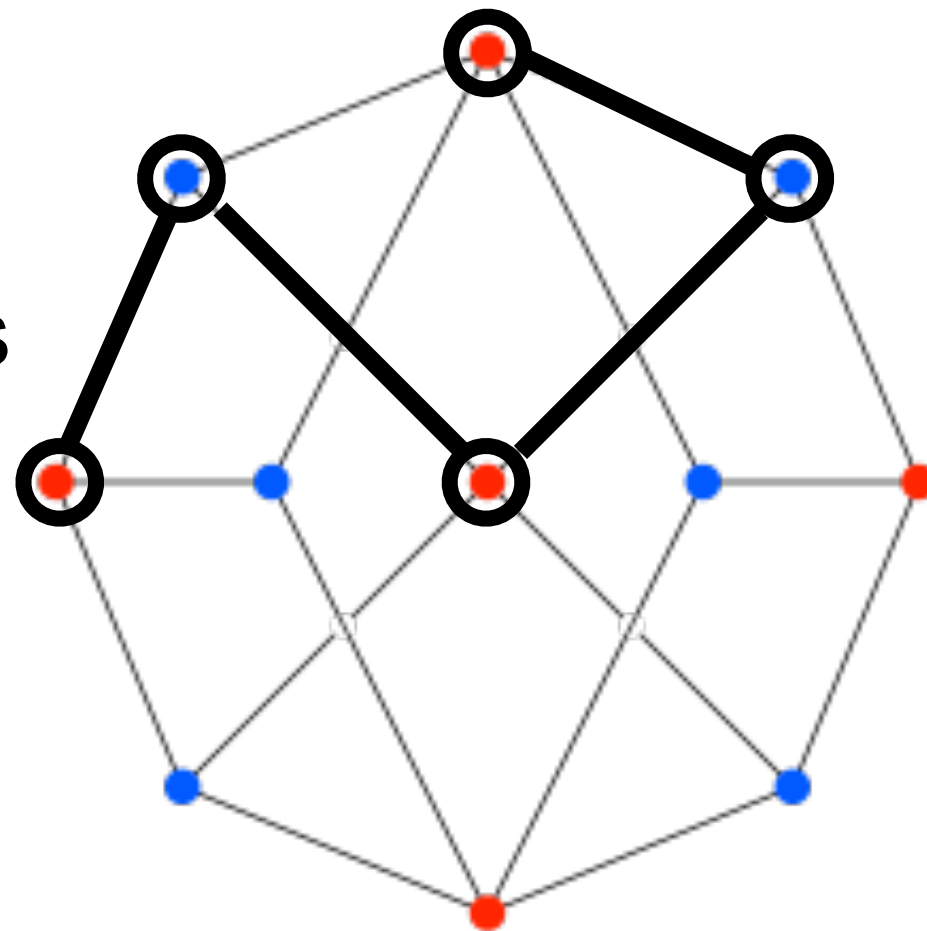
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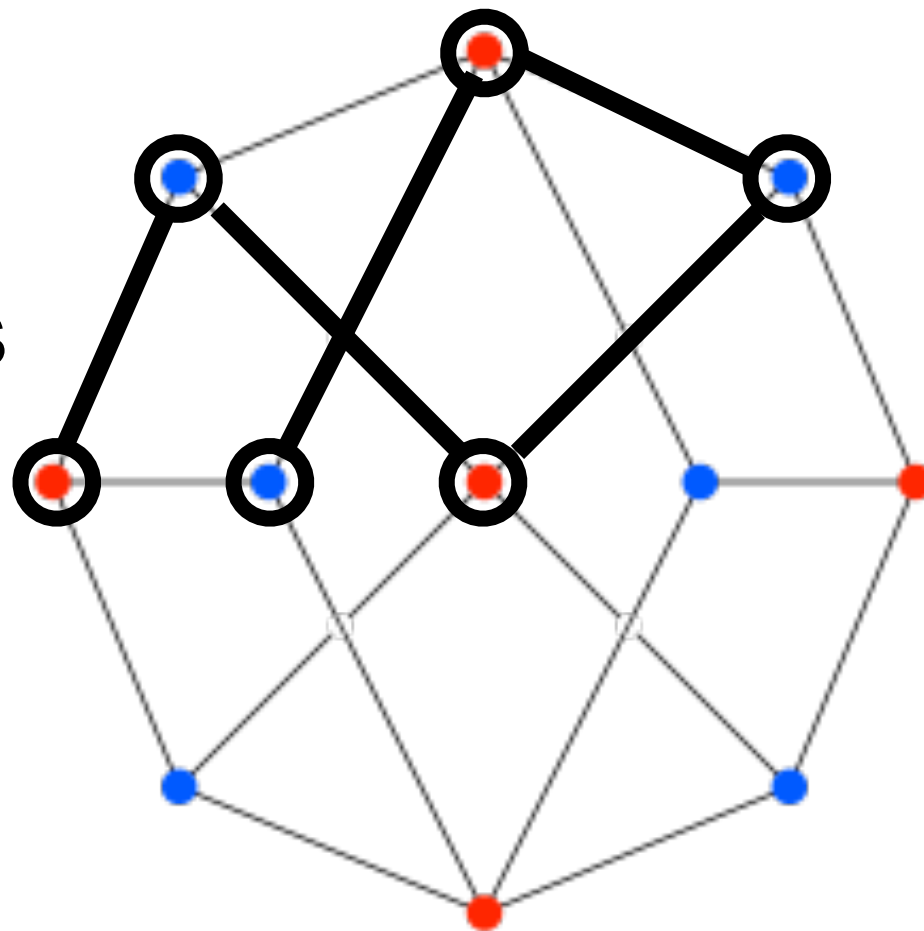




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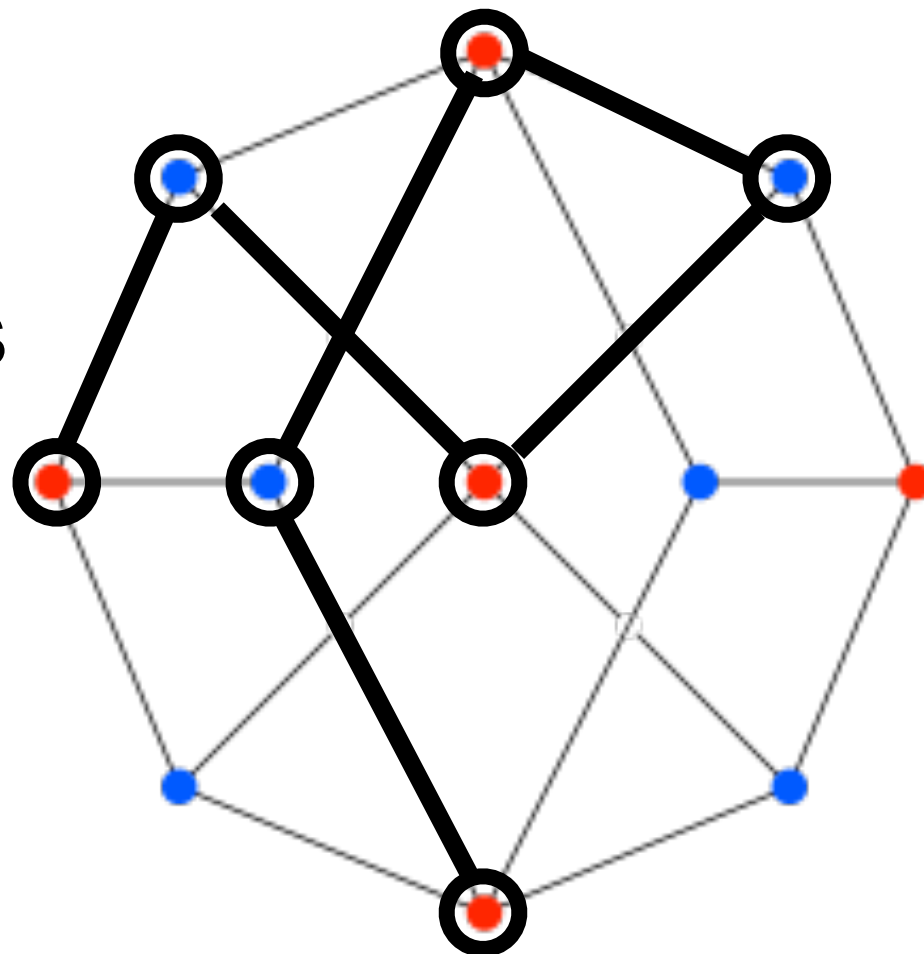
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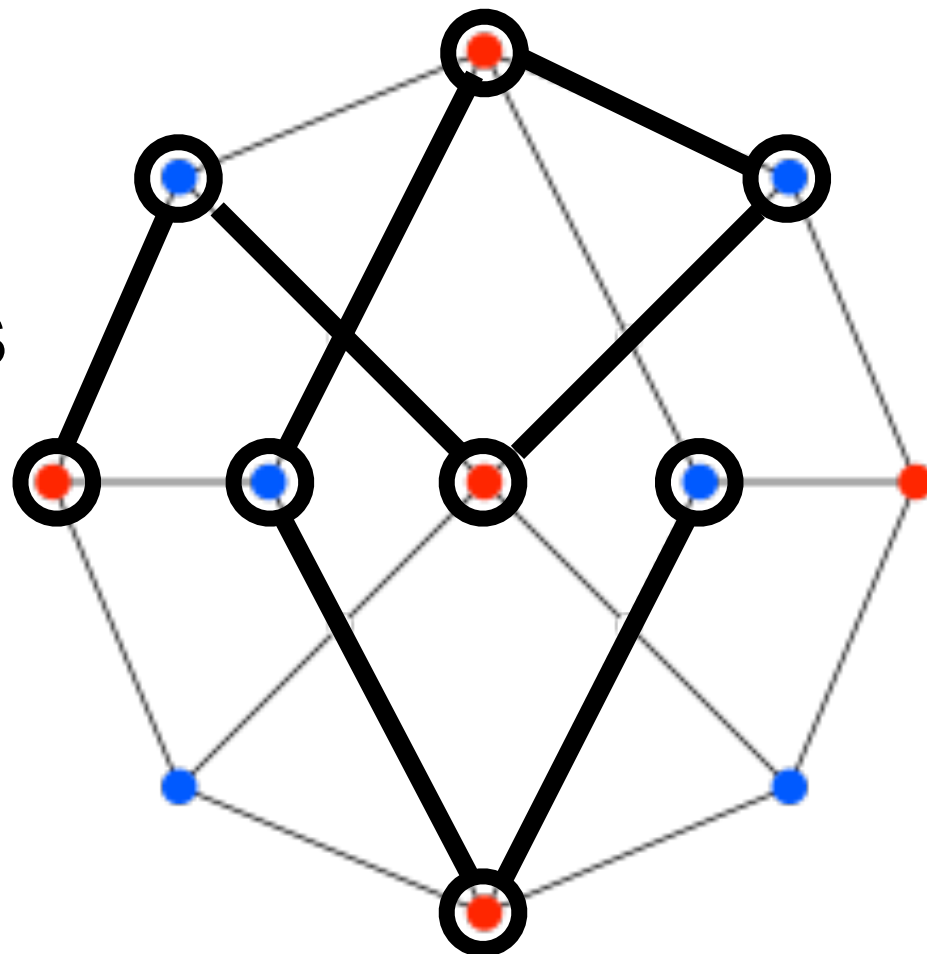
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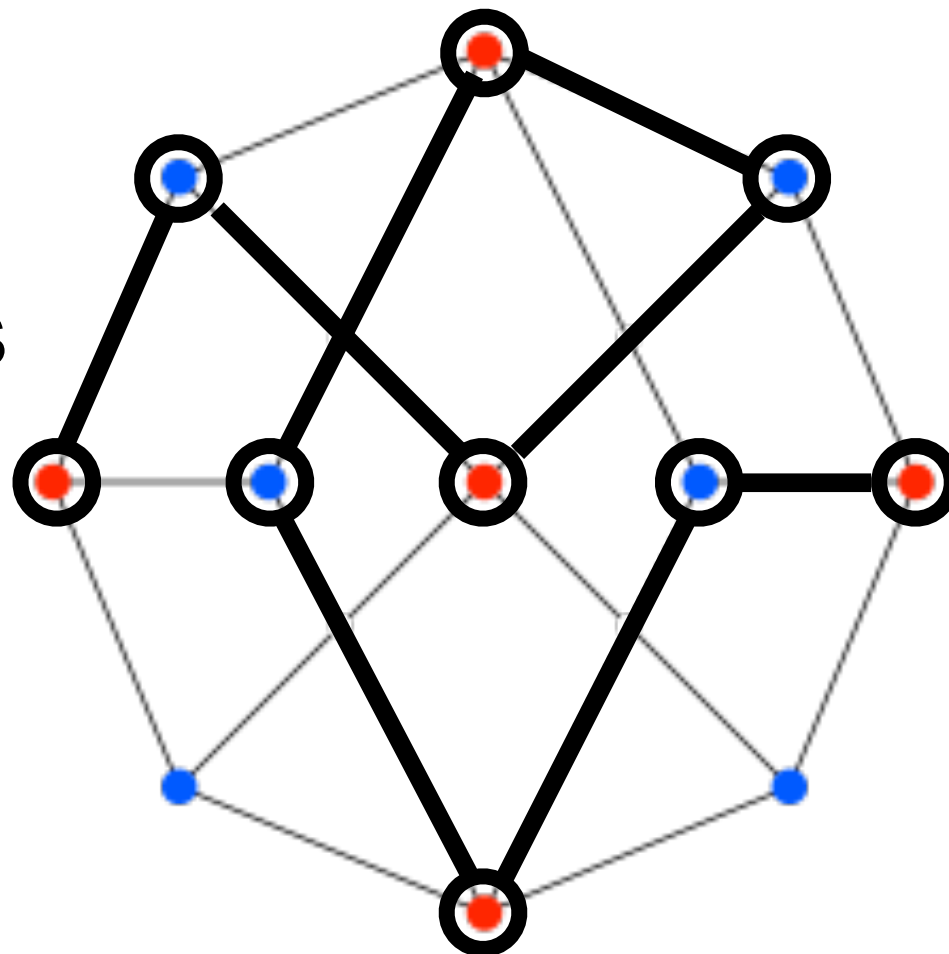
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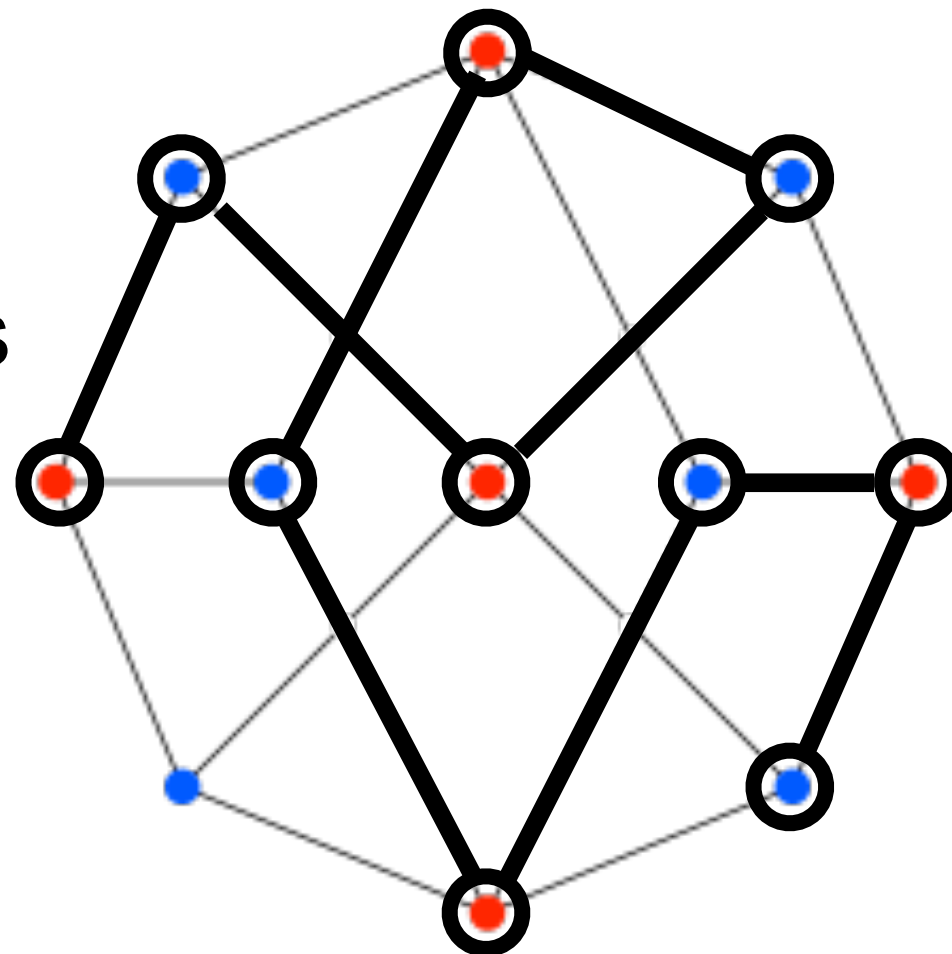
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# P vs NP

The question of whether **P** is the same set as **NP** is the most important open question in computer science

Intuitively, it is much harder to solve a problem than to check the correctness of a solution

A fact supported by our daily experience,  
which leads us to conjecture **P**  $\neq$  **NP**

What if “solving” is not really harder than “checking”?  
what if **P** = **NP**?

# NP-completeness

A problem  $Q$  in **NP** is **NP-complete** if every other problem in **NP** can be reduced to  $Q$  (in polynomial time)

(finding an effective way to solve such a problem  $Q$  would allow to solve effectively any other problem in **NP**)

