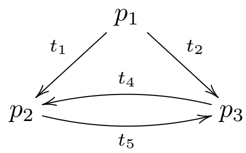
MPB – Selected solutions of mid-term exams

October 31, 2018

Ex.3 $\exists M \in [M_0\rangle.\ M(p_1) > 1$

Ex.4

1. 

2. no, because t_3 is dead (G has no arc labelled by t_3)
3. yes, all states have outgoing arcs
4. yes, because G is finite
5. yes, in each reachable marking there is at most one token in each place
6. no, the initial marking p_1 has no incoming arc

Ex.5

1. the Parikh vector is $\vec{\sigma} = [\ 3\ 2\ 3\ 2\ ]$, thus $M = M_0 + \mathbf{N} \cdot \vec{\sigma} = 2p_2 + 2p_4 + 2p_5$
2. the Parikh vector is $\vec{\sigma'} = [\ 3\ 3\ 2\ 1\ ]$, but $M' = M_0 + \mathbf{N} \cdot \vec{\sigma} = [\ -1\ 1\ 1\ 0\ 2\ 2\ ]$ is not a marking

Ex.6

1. yes, every place has exactly one incoming arc and one outgoing arc
2. $\mathbf{I} = [\ 3\ 1\ 2\ 1\ 1\ 2\ ]$
3. $\mathbf{I} \cdot M_0 = 6$ and $\mathbf{I}(p_1) = 3$ thus for any $M \in [M_0\rangle$ we have $M(p_1) \leq 6/3 = 2$; note that by firing $t_2\ t_4$ we get two tokens in p_1

November 2, 2017

Ex.4

1. no because p_2 has two outgoing arcs
2. the firing sequence $t_5\ t_1\ t_1\ t_3\ t_3\ t_4\ t_5\ t_1\ t_3$ leads to $2p_3$ which is a deadlock
3. no, because it is not deadlock free
4. places p_1, p_2, p_3, p_5 must be assigned the same weight x because of t_1, t_3, t_5; then because of t_2 we have $x = x + y$, for y the weight of p_4, hence $y = 0$
5. no

Ex.5

1. the Parikh vector is $\vec{\sigma} = [\ 3\ 1\ 2\ 2\ 2\ ]$, thus $M = M_0 + \mathbf{N} \cdot \vec{\sigma} = p_3 + p_5$

2. the Parikh vector is $\vec{\sigma'} = [\ 2\ 1\ 2\ 1\ 1\ ]$, but $M' = M_0 + \mathbf{N} \cdot \vec{\sigma} = [\ 0\ \ -1\ 2\ 1\ 1\ ]$ is not a marking

Ex.6
1. $\mathbf{I} = [\ 2\ 1\ 1\ 1\ 1\ ]$
2. $\mathbf{I} \cdot M_0 = 2$ and $\mathbf{I}(p_1) = 2$ thus for any $M \in [M_0\rangle$ we have $M(p_1) \leq 2/2 = 1$
3. the only possible T-invariants are of the form $[\ x\ x\ x\ x\ 0\ 0\ ]$
4. from (i) the system is bounded; from a theorem: "if a bounded system is live then it has a positive T-invariant"; since the system has no positive T-invariant then it is not live

November 3, 2016

Ex.4
1. no: t_2 and t_5 have different pre-sets with p_2 in common
2. a positive S-invariant is $\mathbf{I} = [\ 2\ 1\ 1\ 1\ 1\ 2\ 2\ ]$
3. $\mathbf{I} \cdot M = 4 \neq 3 = \mathbf{I} \cdot M_0$
4. $t_1\ t_2\ t_3$ leads to the marking $2p_4 + p_5$ that is deadlock
5. no, because it is not deadlock free

Ex.5
1. the sequence $t_1\ t_2$ leads to the marking $M_0 + p_3$
2. $\mathbf{I} = [\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ ]$ is semi-positive and $\mathbf{I} \cdot M_0 = 0$
3. the Parikh vector is $\vec{\sigma} = [\ 2\ 3\ 3\ 5\ 0\ 0\ 0\ ]$, then $M = M_0 + \mathbf{N} \cdot \vec{\sigma} = [\ 1\ 3\ 0\ \ -1\ 0\ 0\ 0\ ]$ is not a marking

November 5, 2015

Ex.3
1. no, p_1 has two incoming arcs
2. yes, $\mathbf{I} = [\ 1\ 1\ 1\ 1\ ]$ is a positive S-invariant
3. no, transitions t_1, t_3, t_4 must be assigned the same weight x; then we have $x = x + y$, for y the weight of t_2, hence $y = 0$
4. no positive T-invariant + boundedness implies the system is not live

Ex.4
1. the Parikh vector is $\vec{\sigma} = [\,4\,2\,3\,4\,]$, then $M = M_0 + \mathbf{N} \cdot \vec{\sigma} = [\,3\,0\,0\,1\,] = 3p_1 + p_4$
2. $\mathbf{I} = [\ 1\ 1\ 1\ 1\ ]$ is an S-invariant with $\mathbf{I} \cdot M = 5 \neq 4 = \mathbf{I} \cdot M_0$

Ex.5
1. yes
2. it has no initial place
3. not strongly connencted: p_3 has no outgoing arc
4. $\mathbf{I} = [\ 2\ 1\ 1\ 2\ 1\ ]$ is a positive S-invariant
5. from (iv) it is bounded; bounded + not strongly connected implies non live

November 7, 2014

Ex.3
1. the firing of t_2 leads to the marking p_3 that is deadlock
2. not live because not deadlock free
3. no, t_3 and t_4 have different pre-sets with p_2 in common
4. the firing sequence t_1 t_4 t_6 t_5 leads to $M_0 + p_3$

Ex.4
1. the Parikh vector is $\vec{\sigma} = [\ 3\ 1\ 0\ 3\ 2\ 3\ ]$, then $M = M_0 + \mathbf{N} \cdot \vec{\sigma} = [\ 0\ 0\ 3\ 1\ 0\ ] = 3p_3 + p_4$
2. the Parikh vector is $\vec{\sigma'} = [\ 3\ 0\ 2\ 3\ 2\ 2\ ]$, then $M' = M_0 + \mathbf{N} \cdot \vec{\sigma} = [\ 0\ -2\ 0\ 3\ 1\ ]$ is not a marking

Ex.5
1. $\mathbf{I} = [\ 3\ 1\ 2\ 1\ 1\ 1\ 2\ 3\ ]$ is a positive S-invariant
2. $\mathbf{I} \cdot M = 6 \neq 5 = \mathbf{I} \cdot M_0$

November 6, 2013

Ex.3
1. not live
2. not place-live
3. not deadlock free
4. bounded
5. safe
6. not cyclic

Ex.4
1. the Parikh vector is $\vec{\sigma} = [\ 2\ 0\ 2\ 1\ 0\ 1\ 1\ 2\ ]$, then $M = M_0 + \mathbf{N} \cdot \vec{\sigma} = [\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ ] = p_2 + p_5$

Ex.5
1. $\mathbf{I} = [\ 2\ 2\ 1\ 1\ 1\ ]$ is a positive S-invariant from which we have $\mathbf{I} \cdot M = 6 \neq 5 = \mathbf{I} \cdot M_0$

November 7, 2012

Ex.3 The net is alike the producer-consumer example with bounded buffer (2 producers, 2 consumers, 2 slots):

1. live
2. deadlock free
3. bounded
4. not safe
5. cyclic

Ex.5
1. the Parikh vector is $\vec{\sigma} = [\ 3\ 2\ 1\ 1\ ]$, then $M = M_0 + \mathbf{N} \cdot \vec{\sigma} = [\ 0\ 2\ 1\ 0\ ] = 2p_2 + p_3$
2. the Parikh vector is $\vec{\sigma'} = [\ 4\ 3\ 2\ 2\ ]$, then $M = M_0 + \mathbf{N} \cdot \vec{\sigma} = [\ 0\ 2\ 1\ 0\ ] = 2p_2 + p_3$

November 21, 2011

Ex.3 The analysis of the T-system relies on the identification of its circuits. Let:

$$\begin{aligned}\gamma_0 &= (p_5, t_4)(t_4, p_6)(p_6, t_3)(t_3, p_4)(p_4, t_2)\\ \gamma_1 &= (p_3, t_3)(t_3, p_4)(p_4, t_2)\\ \gamma_2 &= (p_1, t_1)(t_1, p_2)(p_2, t_2)\end{aligned}$$

Note that $M_0(\gamma_0) = 3$, $M_0(\gamma_1) = 2$, and $M_0(\gamma_2) = 1$.

1. It is immediate to check that the T-system is strongly connected. By a known Lemma, since it is strongly connected, then it is bounded. Place p_5 is not 1-bounded, hence the T-system is not safe (e.g., take the firing $M_0 \xrightarrow{t_2} M'$).
2. It is immediate to check that all places belong to γ_0 or γ_1 or γ_2. By a known Theorem, a T-system is live iff all of its circuits are marked at M_0. Since γ_0, γ_1 and γ_2 are all marked, we can conclude that the T-system is live. Note that, by a known Theorem, a live T-system is k-bounded iff every place p belongs to some circuit γ_p such that $M_0(\gamma_p) \leq k$. We can exploit this property to confirm that the T-system is not safe by noting that no such circuit γ with $M_0(\gamma) \leq 1$ can be found for p_5.
3. The fundamental property of T-systems guarantees that the token count of a circuit is invariant under any firing. By noting that $M(\gamma_0) = M(p_5)+M(p_6)+M(p_4) = 2 \neq 3 = M_0(\gamma_0)$ we can conclude that M is not reachable from M_0.

Ex.4 The set $\mathbf{M}$ is stable. In fact, the fundamental property of S-systems guarantees that the token count is invariant under any firing. Therefore, taken any $M \in \mathbf{M}$, and any firing $M \xrightarrow{t} M'$, we know that $M'(P) = M(P) = M_0(P)$ and thus $M' \in \mathbf{M}$.

By the Reachability Lemma for S-system, $\mathbf{M} = [\, M_0 \,\rangle$ iff the S-system is strongly connected.

Ex.5 For each vector $\mathbf{I}_i$ we need to check that

$$\forall t \in T. \sum_{p \in \bullet t} \mathbf{I}_i(p) = \sum_{p \in t \bullet} \mathbf{I}_i(p)$$

- $\mathbf{I}_1 = [\,1\ 1\ 0\ 0\ 0\,]$ is an S-invariant.
- $\mathbf{I}_2 = [\,0\ 0\ 1\ 1\ 1\,]$ is not an S-invariant, because the above equality does not hold, e.g., for t_1.
- $\mathbf{I}_3 = [\,2\ 2\ 1\ 2\ 1\,]$ is an S-invariant.

For each vector $\mathbf{J}_i$ we need to check that

$$\forall p \in P. \sum_{t \in \bullet p} \mathbf{J}_i(t) = \sum_{t \in p\bullet} \mathbf{J}_i(t)$$

- $\mathbf{J}_1 = [\,1\ 2\ 2\ 1\,]$ is not a T-invariant, because the above equality does not hold, e.g., for p_4.
- $\mathbf{J}_2 = [\,1\ 1\ 1\ 0\,]$ is a T-invariant.
- $\mathbf{J}_3 = [\,0\ 1\ 0\ 1\,]$ is not a T-invariant, because the above equality does not hold, e.g., for p_3.

May 2, 2011

Ex.3
1. S-net
2. not a T-net
3. free-choice (because S-net)

Ex.4
1. there can exist nets with a semi-positive S-invariant but with some unbounded place (e.g., producer-consumer example with unbounded buffer)
2. the existence of a positive S-invariant implies the boundedness but not the safeness (e.g., put two tokens in the same place in the initial marking)
3. the existence of a positive S-invariant implies the boundedness (known theorem)

Ex.5 For each vector $\mathbf{I}_i$ we need to check that

$$\forall t \in T. \sum_{p \in \bullet t} \mathbf{I}_i(p) = \sum_{p \in t\bullet} \mathbf{I}_i(p)$$

1. $\mathbf{I}_1 = [\,1\ 1\ 0\ 0\ 0\,]$ is not an S-invariant, because the above equality does not hold, e.g., for t_3.
2. $\mathbf{I}_2 = [\,0\ 0\ 1\ 1\ 1\,]$ is an S-invariant.
3. $\mathbf{I}_3 = [\,1\ 1\ 2\ 2\ 1\,]$ is an S-invariant.
4. $\mathbf{I}_4 = [\,2\ 2\ 1\ 1\ 1\,]$ is not an S-invariant, because the above equality does not hold, e.g., for t_3.
5. $\mathbf{I}_5 = [\,1\ 1\ 1\ 1\ 0\,]$ is an S-invariant ($\mathbf{I}_5 = \mathbf{I}_3 - \mathbf{I}_2$).
6. $\mathbf{I}_6 = [\,0\ 1\ 0\ 1\ 1\,]$ is not an S-invariant, because the above equality does not hold, e.g., for t_1.