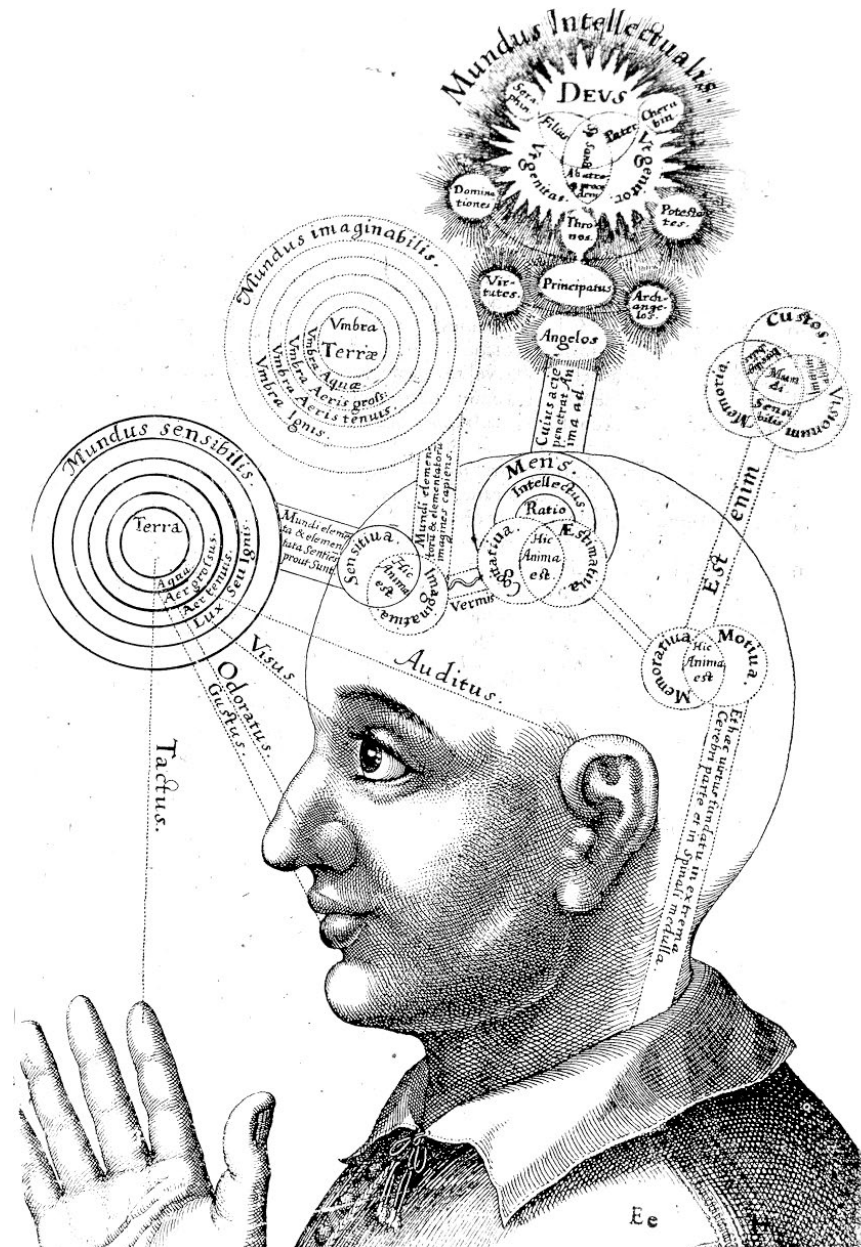


# 3B • VISUAL VARIABLES

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Course: Visual Analytics (602AA)  
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# Data Types

- **Nominal (N)**
  - Equality relation
  - Apples, bananas, pears...
- **Ordinal (O)**
  - Ordering relation
  - Small, medium, large, darker, light...
- **Quantitative (Q)**
  - Arithmetic relations
  - 10m, 32 degrees, 2 bars...

# Data Types - Operators

- **Nominal**
  - $\neq, =$
- **Ordinal**
  - $\neq, =, >, <$
- **Quantitative Interval** (no reference point)
  - $\neq, =, >, <, +, -$
  - Dates, Location
  - Distances: A is 3 degrees hotter than B
- **Quantitative Ratio** (reference point)
  - $\neq, =, >, <, +, -, \times, \div$
  - Length, mass
  - Proportions: A is twice as large as B

# From Data to Conceptual Model

- **Data Model:** low-level representation of data and operations
- **Conceptual Model:** mental and semantic construction

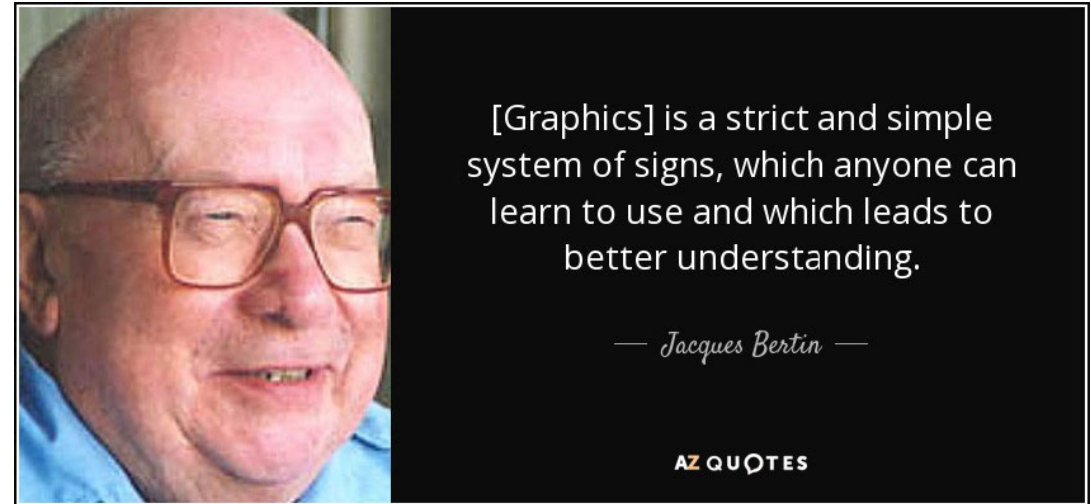
Data	Concept
1D number	Temperature
2D numbers	Geographic Coordinate
3D numbers	Spatio-temporal position

# From Data to Conceptual Model

- From **data model**...
  - 70.8, 27.2, -10.2...
- ...using **conceptual model**...
  - Temperature
- ...to **data type**
  - Continuous variation
  - Warm, hot, cold
  - Burned vs not burned

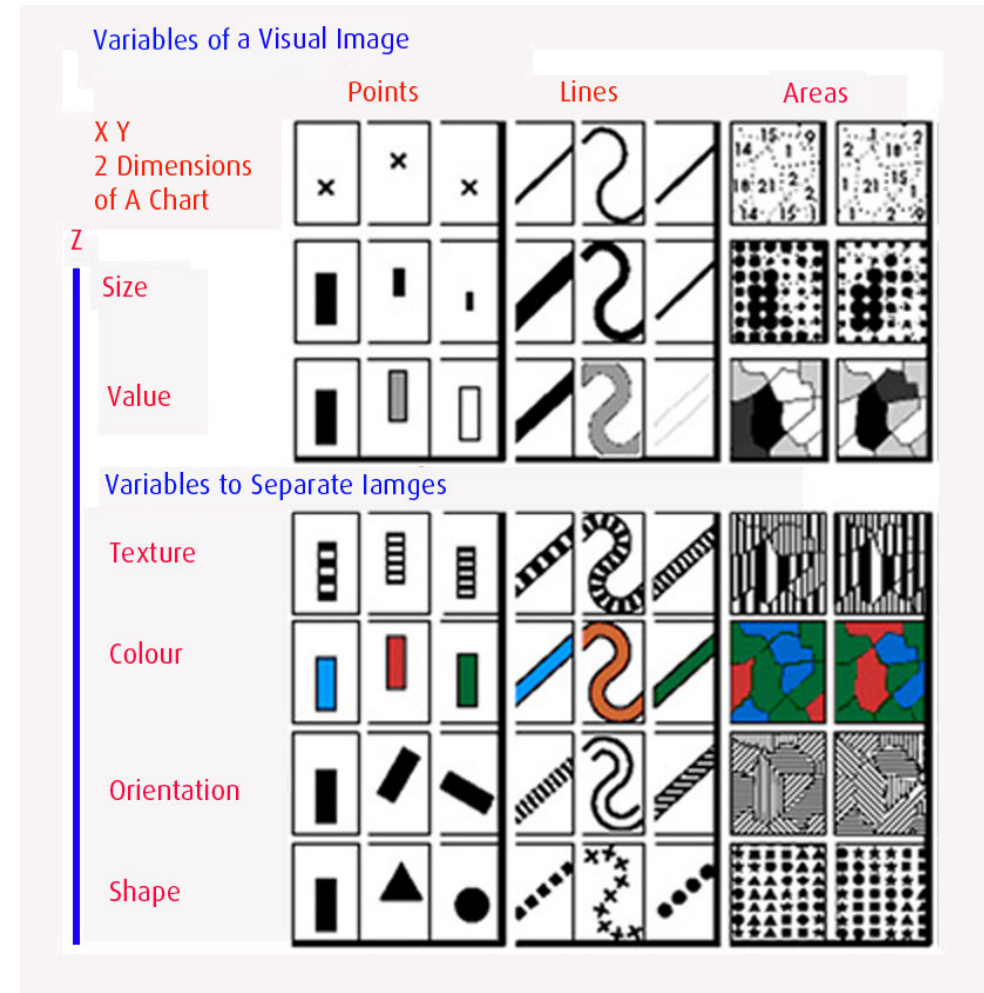
# Visual Variables

- Jacques Bertin (1918-2010), cartographer
- Theoretical principles of visual encodings
- Semiology of Graphics (1967)



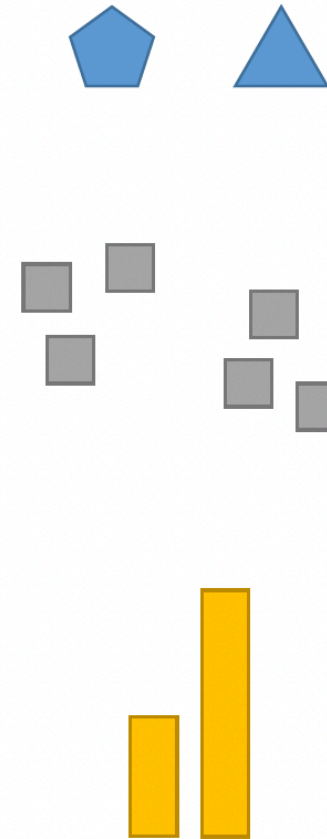
# Bertin's Visual Variables

This is a visual summary of Bertin's classification of visual variables



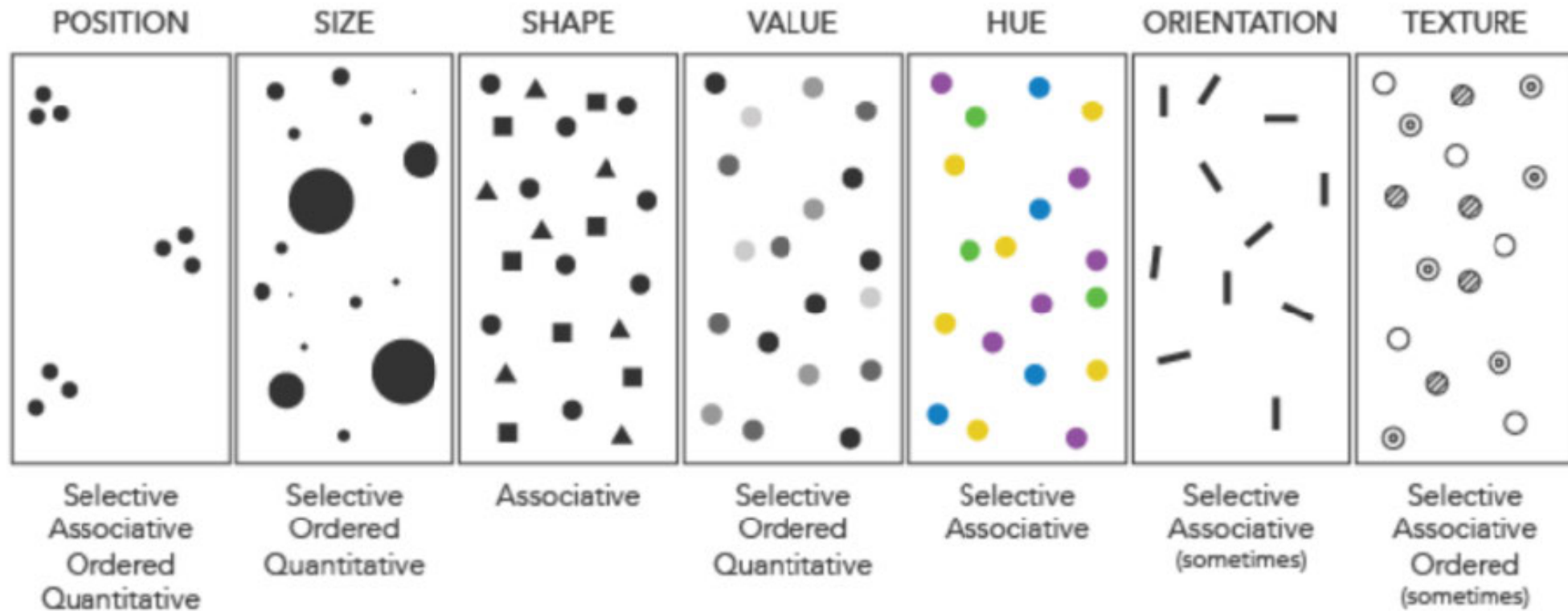
# Characteristics of Visual Variables

- **Selective**
  - May I distinguish a symbol from the others?
- **Associative**
  - May I identify groups?
- **Quantitative**
  - May I quantify the difference of two values?
- **Order**
  - May I identify an ordering?



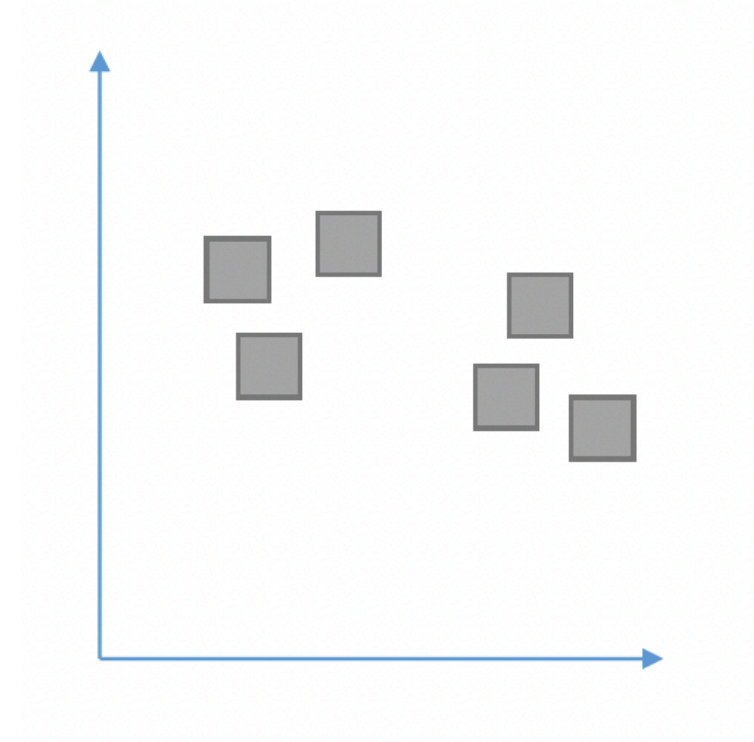


# Characteristics of Visual Variables



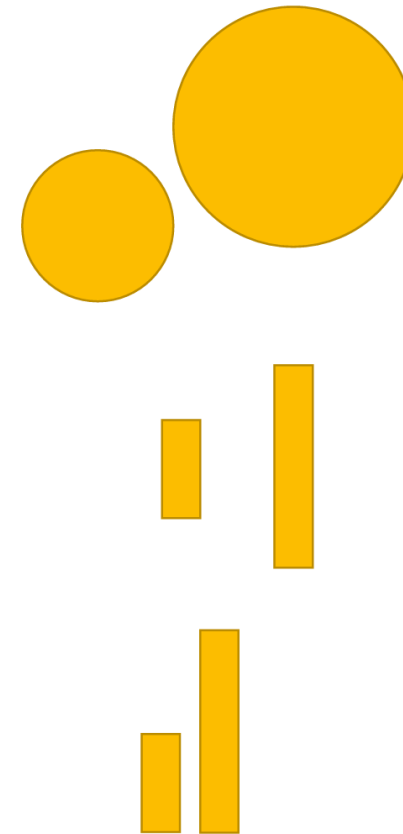
## VV: Position

- Strongest visual variable
- Compatible for all data types
- **Cons:**
  - Not always applicable (e.g. nD data)
  - Cluttering



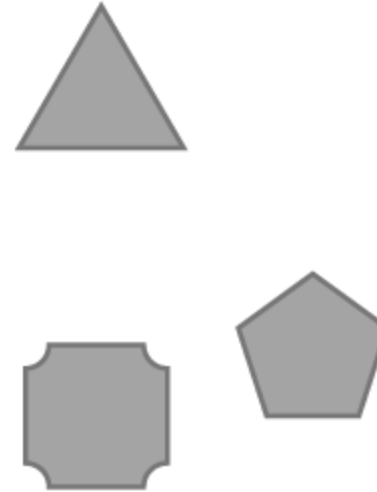
## VV: Size and Length

- Easy to compare dimensions
- Grouping
- Estimate differences
  - Quantitative encoding
  - Changes in lengths
  - Worse for change in area



## VV: Shapes

- Strong for nominal encoding
- No ordering
- No grouping



## VV: Value (Intensity)

- Quantitative representation (when size and length are used)
- Limited number of shades
- Support grouping



## VV: Color (Tint)

- Good for qualitative data
- Limited number of classes (!!!)
- Not good for quantitative data
- Be careful!!

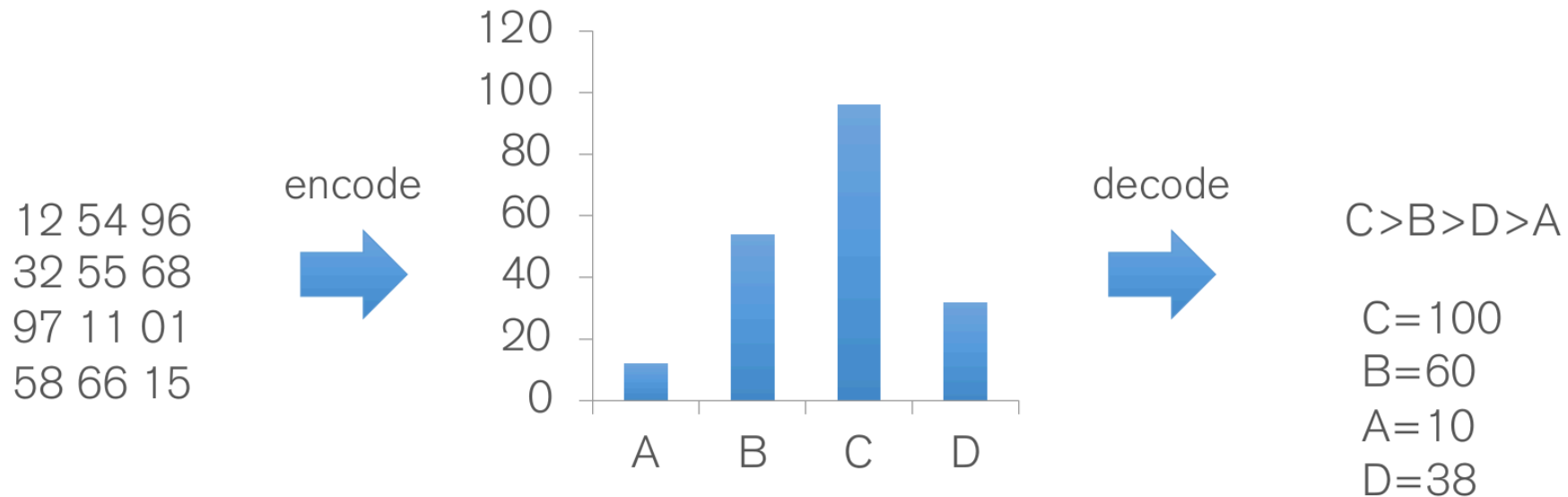


# Bertin Visual Variables

	Nominal	Ordinal	Quantitative
Position	✓	✓	✓
Size	✓	✓	~
Value (intensity)	✓	✓	~
Texture	✓	~	✗
Color	✓	✗	✗
Orientation	✓	✗	✗
Shape	✓	✗	✗

# Visual Encoding/Decoding

- A graph encodes a set of information as a set of graphical attributes
- The observer has to decode the graphical attributes to extract the original information





# TAXONOMY OF VISUAL VARIABLES

# Cleveland McGill [1984]

William S. Cleveland; Robert McGill, "Graphical Perception: Theory, Experimentation, and Application to the Development of Graphical Methods." 1984

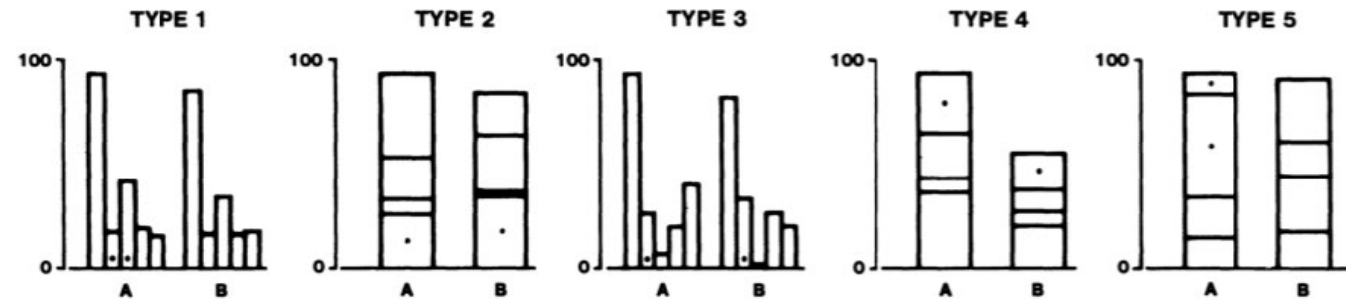


Figure 4. Graphs from position-length experiment.

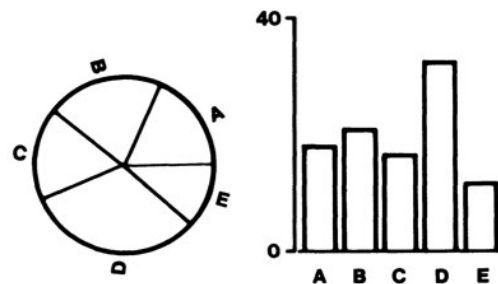


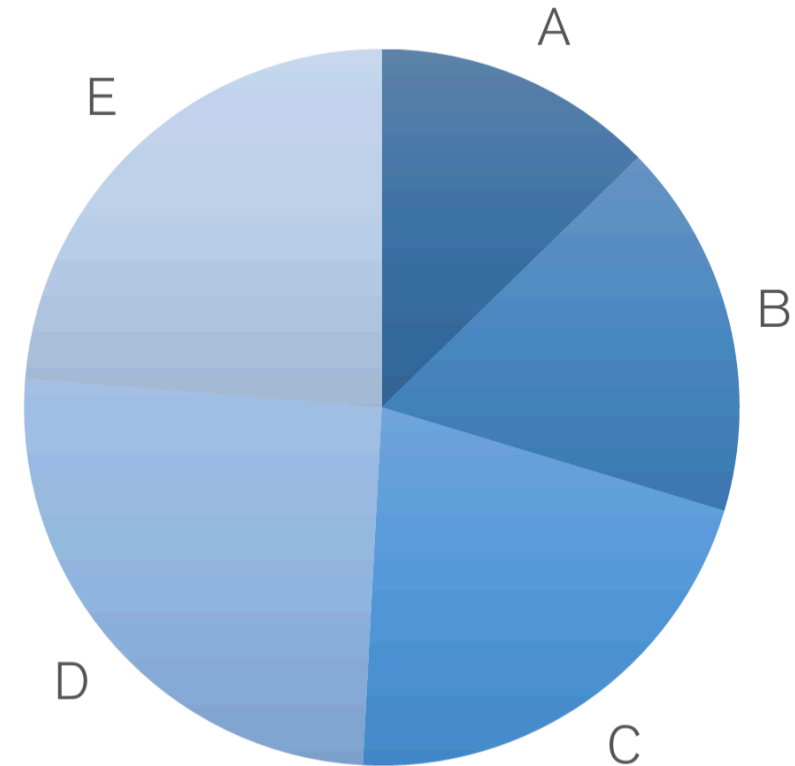
Figure 3. Graphs from position-angle experiment.

# Cleveland & McGill: Graphical Encodings

- Angle
- Area
- Color Hue
- Color Saturation
- Density
- Length
- Position on a common scale
- Position on non-aligned scale
- Slope
- Volume

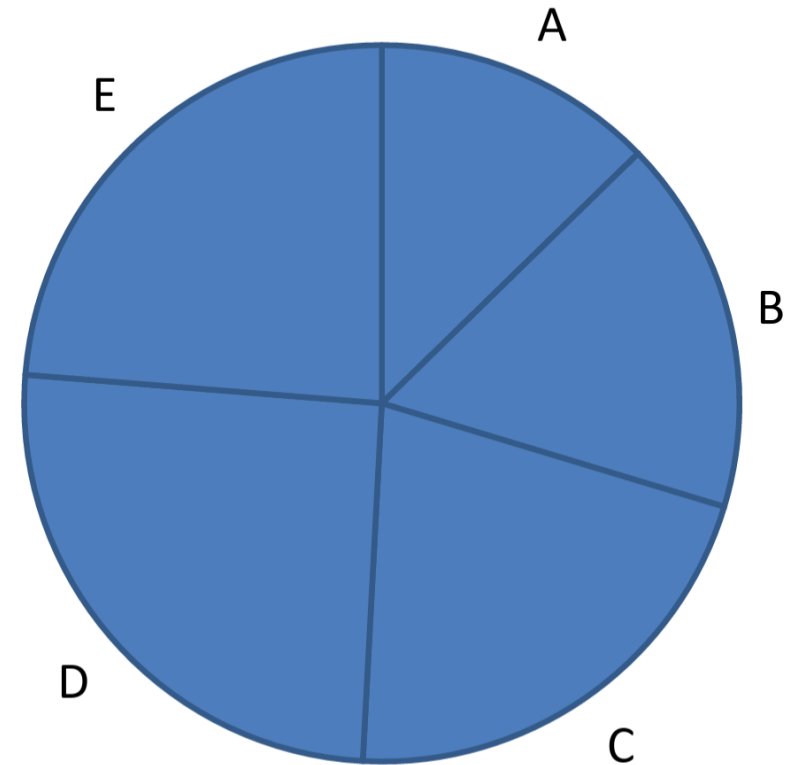
# Angle Decoding

- It is difficult to compare angles
  - Underestimation of acute angles
  - Overestimation of obtuse angles
  - Easier if bisectors are aligned
- Area estimation helps



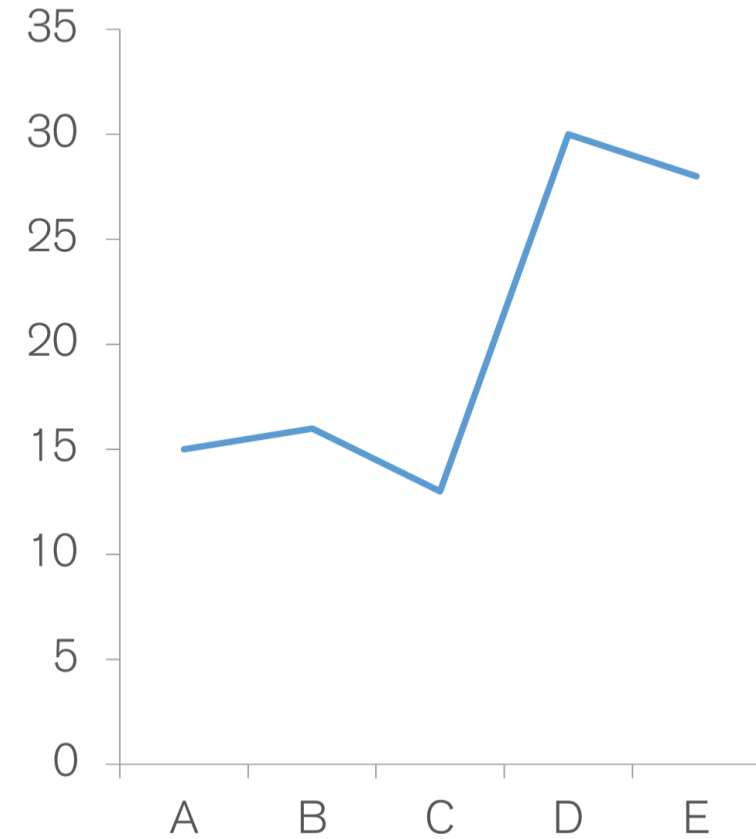
# Angle Decoding

- It is difficult to compare angles
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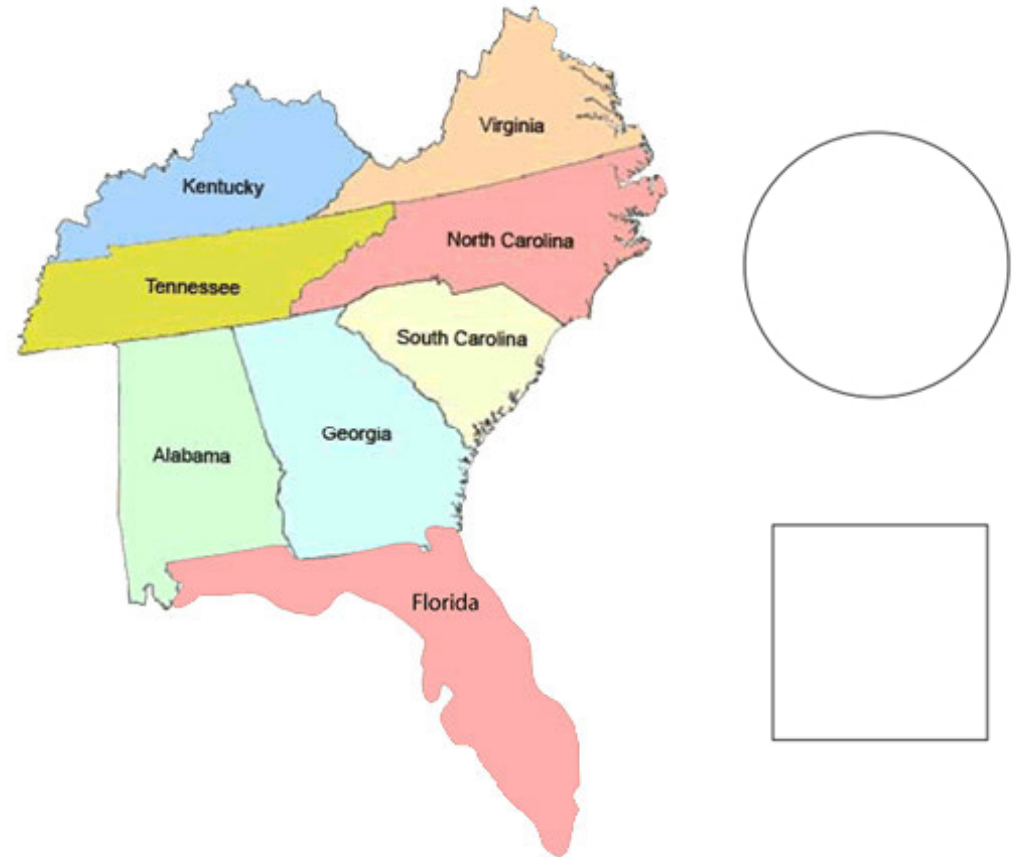
# Slopes Decoding

- Same difficulties as angles
- Easier task since one branch is aligned with x-axis



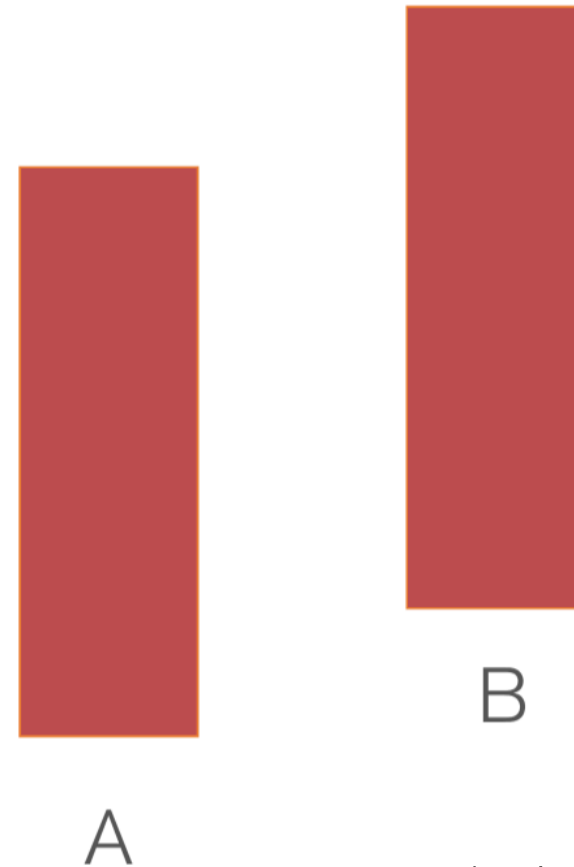
# Area Decoding

- Area is not well decoded
- Different regular shapes
- Irregular shapes
- Context influences (thin area within compact thick area)



# Length Decoding

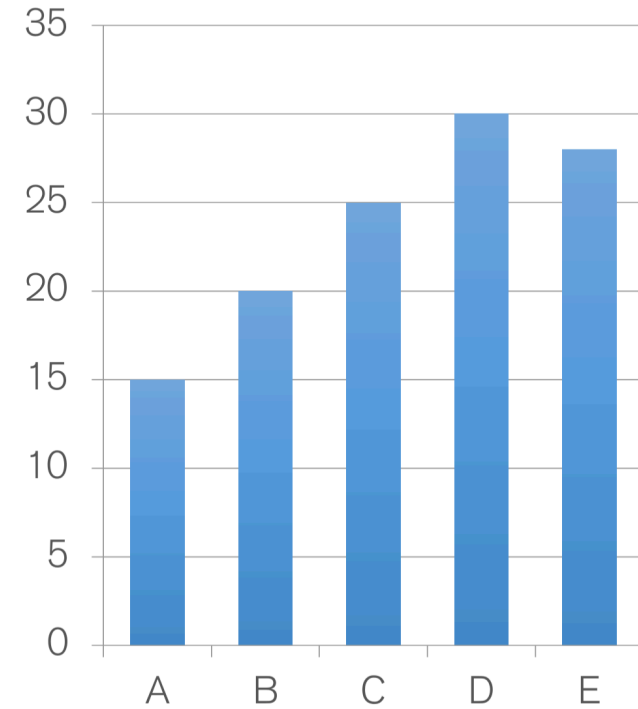
- Straight forward to encode numerical values
- Difficulties with relative lengths





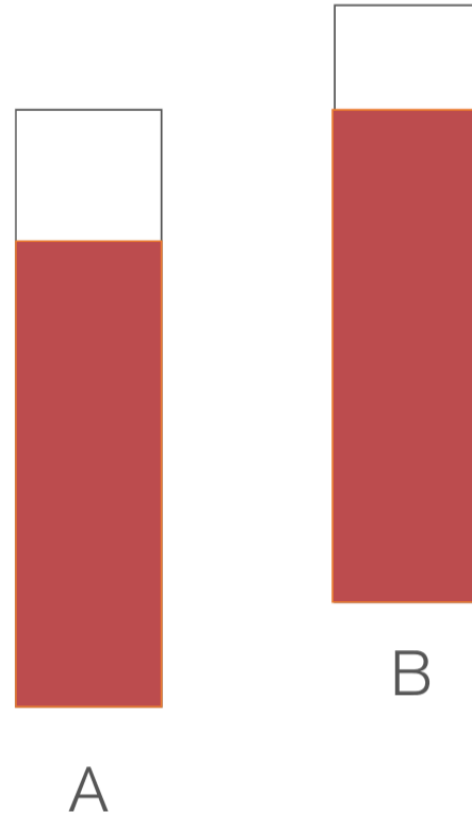
# Position on a Common Scale

- Widely used in statistical charts



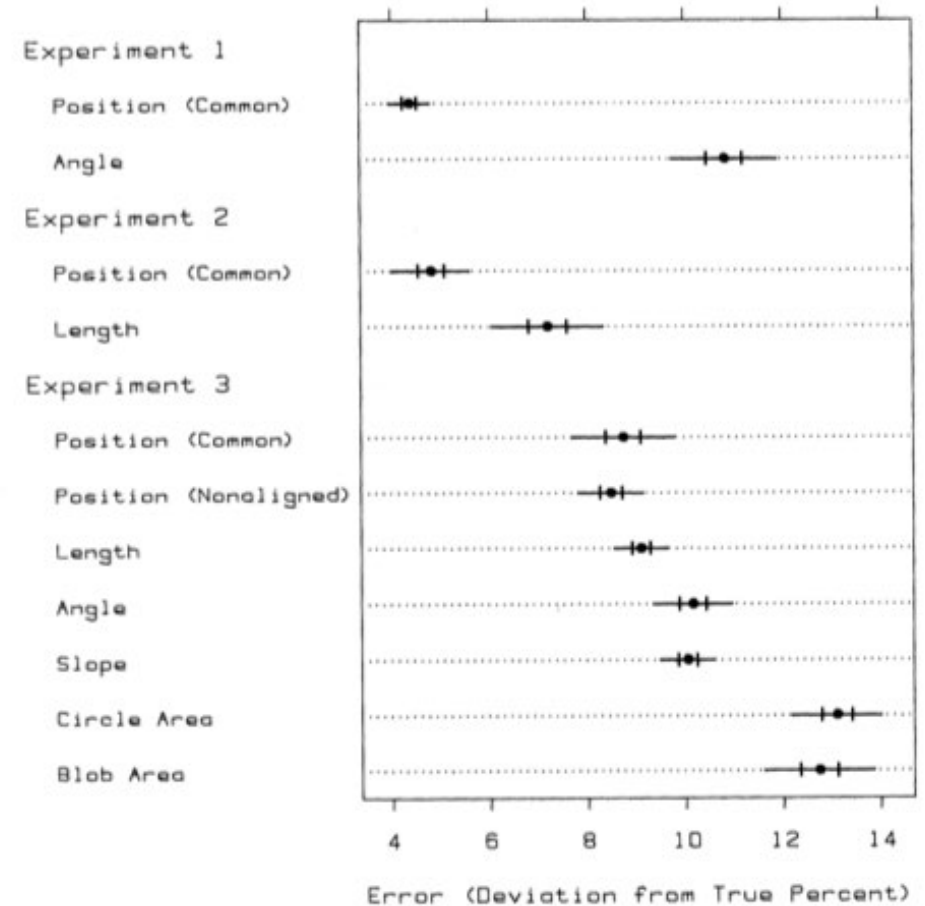
## Position on Non-Aligned Scale

- Not as good as common scale
- Still acceptable

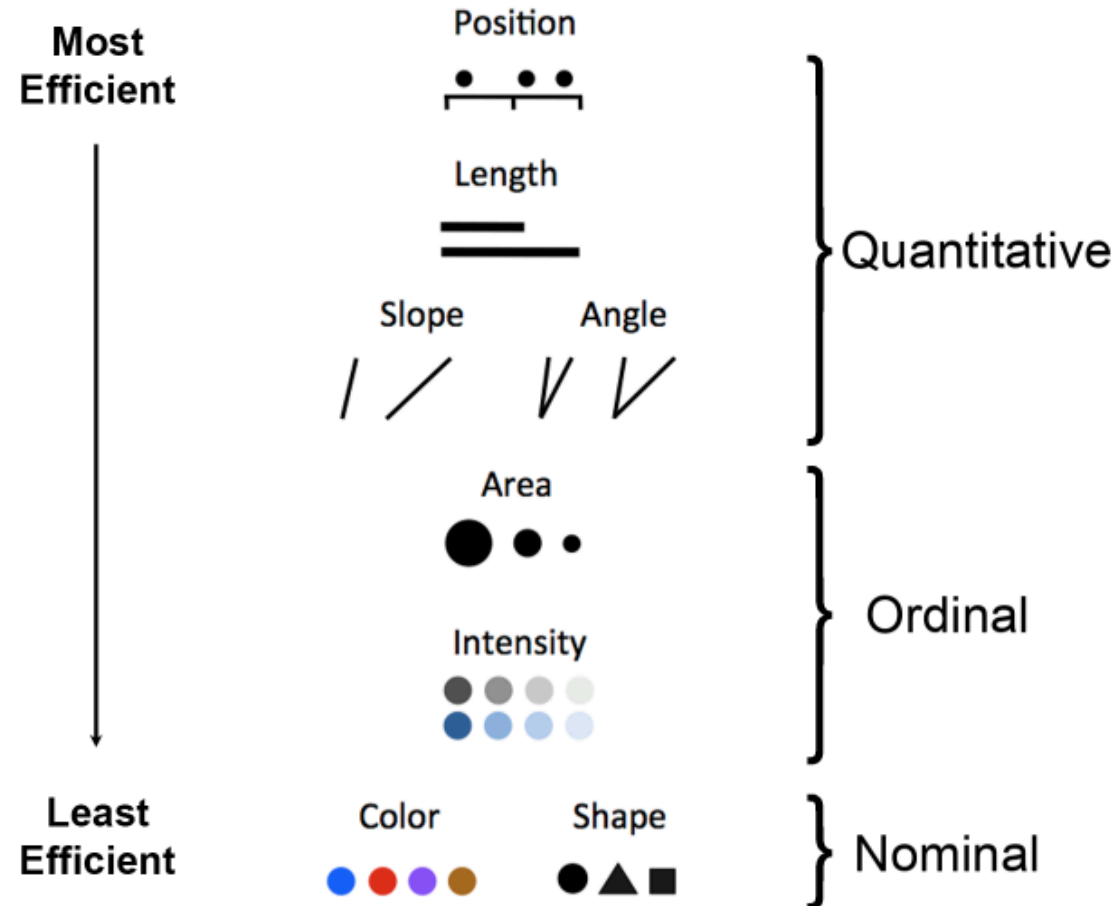


# Designing Effective Visualizations

- If possible, use graphical encoding that is easily decoded
- Graphical Attributes ordered (Cleveland & McGill):
  - i. Position along a common scale
  - ii. Position on non-aligned scales
  - iii. Length
  - iv. Angle and Slope
  - v. Area
  - vi. Volume, density, color saturation
  - vii. Color Hue



# Most Efficient to Least Efficient



# PERCEPTION LAWS

# Weber's Law

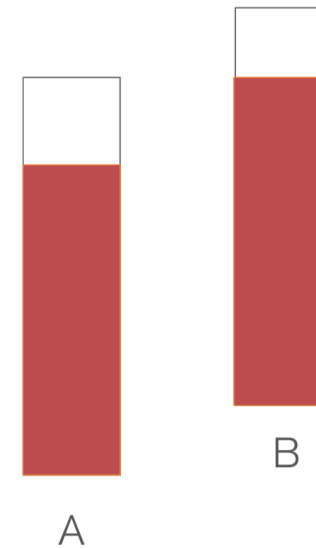
- **Just-noticeable difference** between two stimuli is proportional to their magnitudes
- Case study on length:
  - Given two lines with lengths  $x$  and  $x+w$
  - If  $w$  is small, it is difficult to notice difference between the two lines
  - If  $w$  is larger, it is easier to catch the difference
- How large should  $w$  be?
  - The probability of detecting the change is proportional to the relative value  $w/x$

# Weber's Law

- Given values (90, 92)
- Detect with probability of 2/90



- Given values (90, 92)
- Detect with probability of 2/10



# Stevens' Law

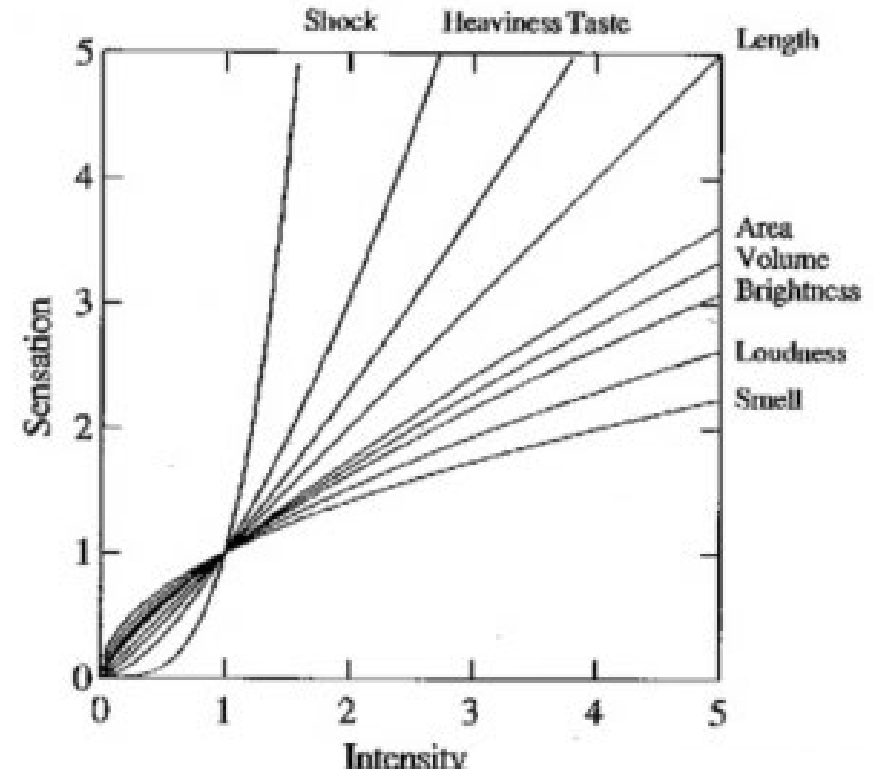
- Model the relation between a stimulus and its perceived intensity
- Given a stimulus  $x$  encoded with a visual attribute
- An observer decodes a perceived value  $p(x)$
- Stevens' law states that:
  - $p(x) = kx^\beta$
  - where  $k$  is constant and
  - $\beta$  is a constant that depends on the nature of stimulus



# Stevens' Law

- Better effectiveness when  $p(x) = kx^\beta$  is linear
- Linearity depends only on  $\beta$
- Different visual encodings yields typical ranges for  $\beta$ :
  - Lengths: 0.9 – 1.1
  - Area: 0.6 – 0.9
  - Volume: 0.5 – 0.8

*Stevens' Law graph showing over/underestimation of values*



## Weber and Stevens' Laws

- Given two values  $x_1$  and  $x_2$
- Let the perceived values be  $p(x_1)$  and  $p(x_2)$

$$\frac{p(x_1)}{p(x_2)} = \left( \frac{x_1}{x_2} \right)^\beta$$

## Weber and Stevens' Laws: Areas

- For areas  $\beta=0.7$
- Let  $x_1=2$  and  $x_2=1$
- The perceived difference will be:

$$\frac{p(2)}{p(1)} = \left(\frac{2}{1}\right)^{0.7} = 1.6245$$

- For areas  $\beta=0.7$
- Let  $x_1=0.5$  and  $x_2=1$
- The perceived difference will be:

$$\frac{p(\frac{1}{2})}{p(1)} = \left(\frac{1/2}{1}\right)^{0.7} = 0.6155$$

## Weber and Stevens' Laws: Areas vs Lengths

- For areas  $\beta=0.7$
- Let  $x_2=x_1+w$
- The perceived difference will be:

$$\left(\frac{x+w}{x}\right)^{0.7} \approx 1 + \frac{0.7w}{x}$$

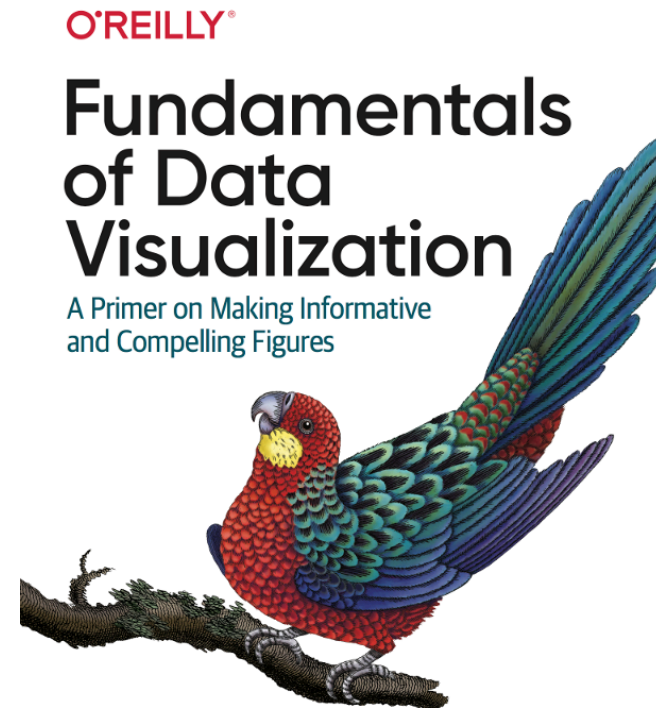
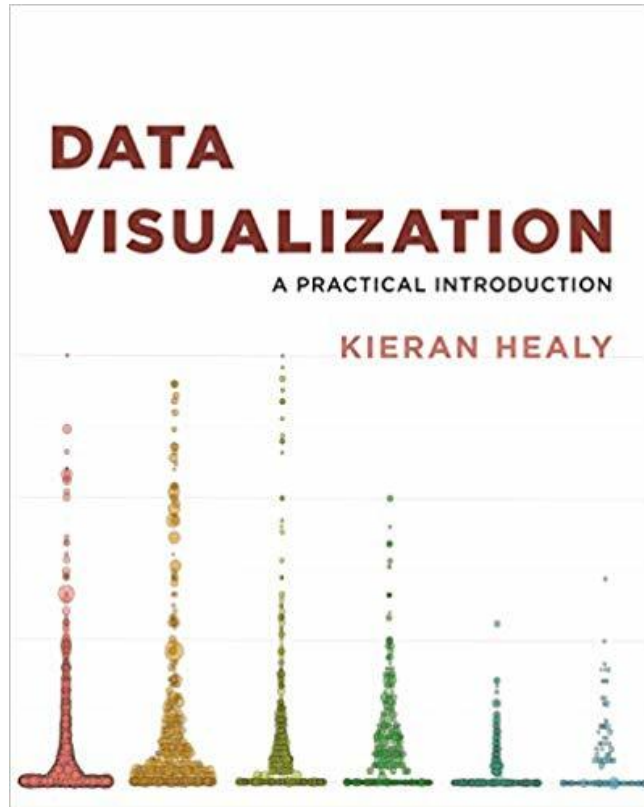
- For lengths  $\beta=1$
- Let  $x_2=x_1+w$
- The perceived difference will be:

$$\left(\frac{x+w}{x}\right)^1 = 1 + \frac{w}{x}$$

# Takeaway Messages

- Data type for entities and relationships
- Visual variables for representation
- Mapping of types to VVs
- Some VVs are more appropriate for specific data types

# Suggested Readings

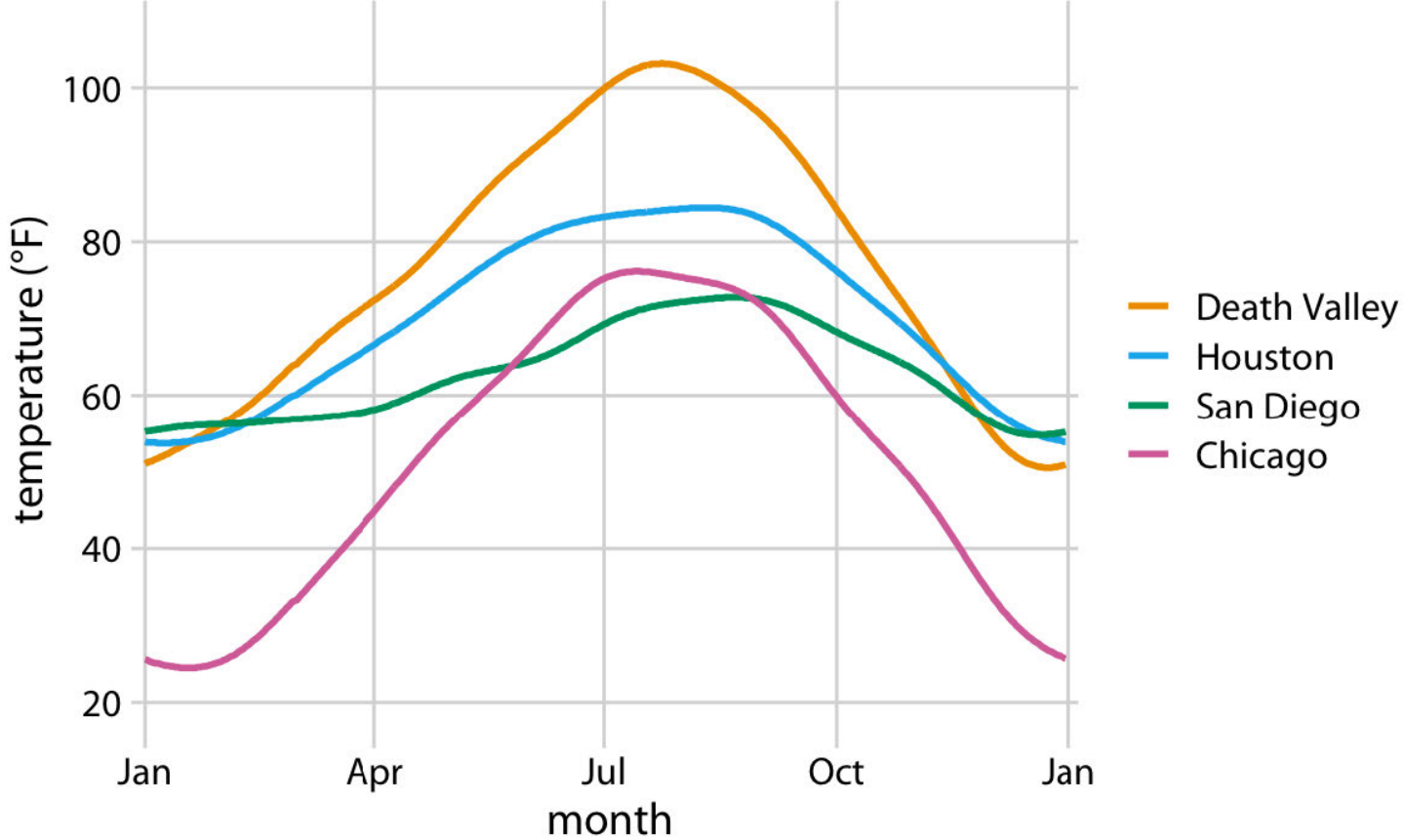


Claus O. Wilke

## Example

Month	Day	Location	Station ID	Temperature
Jan	1	Chicago	USW00014819	25.6
Jan	1	San Diego	USW00093107	55.2
Jan	1	Houston	USW00012918	53.9
Jan	1	Death Valley	USC00042319	51.0
Jan	2	Chicago	USW00014819	25.5
Jan	2	San Diego	USW00093107	55.3
Jan	2	Houston	USW00012918	53.8
Jan	2	Death Valley	USC00042319	51.2
Jan	3	Chicago	USW00014819	25.3
Jan	3	San Diego	USW00093107	55.3
Jan	3	Death Valley	USC00042319	51.3
Jan	3	Houston	USW00012918	53.8

# Visual Solution (1)





## Visual Solution (2)

