# Ex 1 (A - B - C)

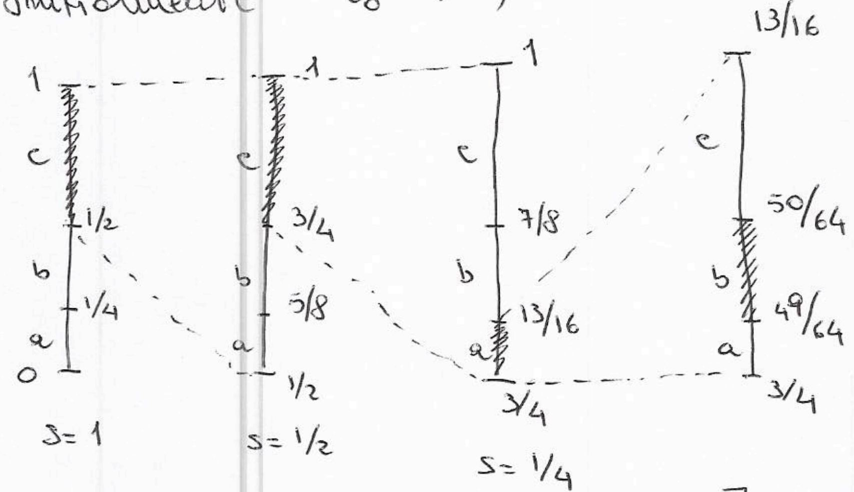
you can find the answers in the class notes.

# Ex 2

$p(a) = \frac{1}{4}$, $p(b) = \frac{1}{4}$, $p(c) = 1/2$

$F(a) = 0$, $F(b) = \frac{1}{4}$, $F(c) = 1/2$

Inizialmente $l_0 = \emptyset$, $s_0 = 1$,




$\longrightarrow \left[\frac{49}{64}, \frac{50}{64}\right)$

size $= \frac{1}{64}$

$$\#\text{bits} = \left\lceil \log_2 \frac{2}{\text{size}} \right\rceil = \left\lceil \log_2 \frac{2}{1/64} \right\rceil = \log_2 128 = 7$$

$$\text{numero da codificare} = \underbrace{\frac{49}{64}}_{l_4} + \frac{\text{size}}{2} = \frac{49}{64} + \frac{1/64}{2} = \frac{98}{128} + \frac{1}{128} = \frac{99}{128}$$

$2 \cdot \frac{99}{128} = \frac{99}{64} \geq 1 \longrightarrow$ output 1, $\frac{99 - 64}{64} = \frac{35}{64}$

$2 \cdot \frac{35}{64} = \frac{35}{32} \geq 1 \longrightarrow$ output 1, $\frac{35 - 32}{32} = \frac{3}{32}$

$2 \cdot \frac{3}{32} = \frac{6}{32} < 1 \longrightarrow$ output 0

$2 \cdot \frac{6}{32} = \frac{12}{32} < 1 \longrightarrow$ output 0

$2 \cdot \frac{12}{32} = \frac{24}{32} < 1 \longrightarrow$ output $\emptyset$

$2 \cdot \frac{24}{32} = \frac{48}{32} \geqslant 1 \longrightarrow$ output 1, $\frac{48-32}{32} = \frac{16}{32}$

$2 \cdot \frac{16}{32} = \frac{32}{32} \geqslant 1 \longrightarrow$ output 1

$C_{Arith}(CCAB) = \langle 4, 1100011 \rangle$

Ex 3

$S = \{6, 4, 5, 8, 7, 14, 11, 16, 18\}$  $n = 5$

Since the prima table has size 5, we remap the keys $\underline{\text{mod } 5}$

$6 \longrightarrow 1$
$4 \longrightarrow 4$
$5 \longrightarrow 0$
$8 \longrightarrow 3$
$7 \longrightarrow 2$
$14 \longrightarrow 4$
$11 \longrightarrow 1$
$16 \longrightarrow 1$
$18 \longrightarrow 3$

So, whichever is the function we derive we will have

$\sum (n_i)^2 = 3^2 + 1^2 + 1^2 + 2^2 + 2^2 = 19 > 2n = 18$ [Thm]

In any case it is $< 3n$ and thus, since the constant is arbitrary because we care about $O(n)$ space, we accept this distribution given by $h(x) = x \bmod 5$ and

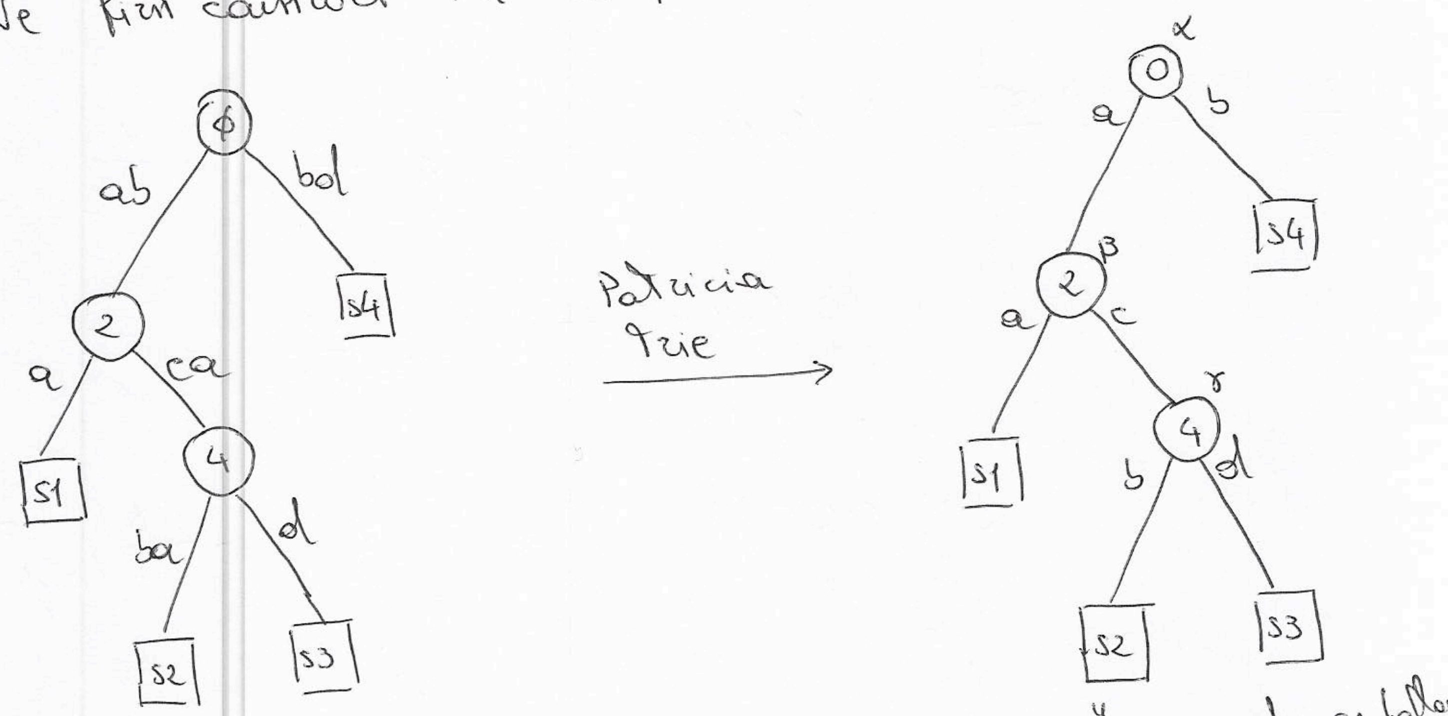
then define:

$h_0(x) = x \bmod 1$
$h_1(x) = x \bmod 9$
$h_2(x) = x \bmod 1$
$h_3(x) = x \bmod 4$
$h_4(x) = x \bmod 4$

all of them are perfect for the previous distribution

## EX 4

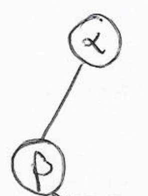
We first construct the compacted trie for the string in $S$




The lexicographic search for $P1 =$ "a c d a c" proceeds as follows:

1) checks chars $\frac{a}{0}$ c $\frac{d}{2}$ a c ($\alpha$, $\beta$) → there is no outgoing edge labeled with "d" so we pick any leaf descending from $\beta$. Say $\boxed{S1}$

2) $lcp(P1, \boxed{S1}) = lcp(acdac, aba)$ "c" vs "b"
$= 1$ → mismatch

3) Stop upward on the edge  and since $c > b$ then P1 is to the right of the subtree descending from $\beta$. This means that $S3 < P1 < S4$

The lexicographic search for P2 proceeds equally as before, reaching leaf S1, finding an $lcp = 1$, but now the mismatch character on P2 is "a" < character on the edge  which is "b". So P2 occurs to the left of the subtree descending from $\beta$, hence $P2 < S1$.