Algorithm Engineering 10/1/20

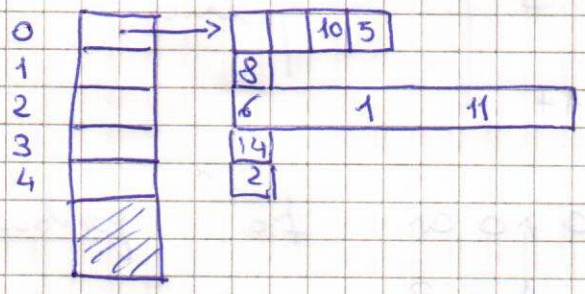
Exercise 1

We choose the hash function

$h_1(k) = 2x \bmod 5$ (first level)

$h_j(x) = 3x \bmod (n_j)^2$ (second level, slot j)

$\uparrow$ #keys colliding at slot j of the first level.



$$\sum_{j=0}^{4} (n_j)^2 = 16 = 2u \quad \checkmark$$

Exercise 2

For the Rice code we consider the gap-sequence

$$\Delta S = (1, 5, 9, 3, 1, 1, 1)$$

$$R_3(1) = \begin{cases} q = \lfloor \frac{x-1}{8} \rfloor = 0 \\ r = (x-1) \bmod 8 = 0 \end{cases} \qquad R_3(1) = 1\ 000$$

$$R_3(3) = \begin{cases} q = 0 \\ r = 2 \end{cases} \qquad R_3(3) = 1\ 010$$

$$R_3(5) = \begin{cases} q = 0 \\ r = 4 \end{cases} \qquad R_3(5) = 1\ 100$$

$$R_3(9) = \begin{cases} q = 1 \\ r = 0 \end{cases} \qquad R_3(9) = 01\ 000$$

Interpolative code manages the sequence S

- $l = 1,\ r = 7,\ low = 1,\ hi = 21$

$$m = \left\lfloor \frac{l + r}{2} \right\rfloor = 4$$

$$a = low + m - l = 1 + 4 - 1 = 4$$

$$b = hi + m - r = 21 + 4 - 7 = 18$$

BinEnc (18, 4, 18) → bin (18-4) in 4 bits → 1110

$\lceil \log_2 b - a + 1 \rceil$

- $l = 1,\ r = 3,\ low = 1,\ hi = 17$

$$m = \left\lfloor \frac{l + r}{2} \right\rfloor = 2$$

$$a = low + m - l = 1 + 2 - 1 = 2$$

$$b = hi + m - r = 17 + 2 - 3 = 16$$

BinEnc (6, 2, 16) → bin (6-2) in 4 bits =

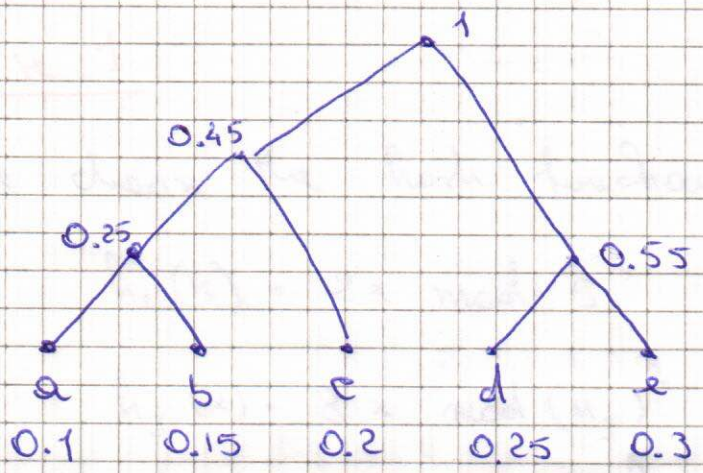
0100

- $l = 5,\ r = 7,\ low = 19,\ hi = 21$

$$a = low + m - l = 19 + 6 - 5 = 20$$
$$b = hi + m - r = 21 + 6 - 7 = 20$$

} no bit emitted

Exercise 3



level	nums	symbs
1	0	—
2	3	c, d, e
3	2	a, b

fc	2	1	0
level	1	2	3

decoding of 10 01 001

$l=1$, $v=1$: $v < fc[l] \rightarrow v = 2v + 0 = 2$

$l++ \rightarrow l = 2$

$l=2$, $v=2$: $v \geq fc[l] \rightarrow$ print symb[2, v - fc[2]] = d

$l=1$, $v=0$: $v < fc[l] \rightarrow v = 2 \cdot v + 1 = 1$

$l++ \Rightarrow l = 2$

$l=2$, $v=1$; $v \geq fc[l] \rightarrow$ print symb[2, v - fc[2]] = c

$l=1$, $v=0$: $v < fc[l] \rightarrow v = 2v + 0 = 0$

$l++ \rightarrow l = 2$

$l=2$, $v=0$: $v < fc[l] \rightarrow v = 2v + 1 = 1$

$l++ \rightarrow l = 3$

$l=3$, $v=1$: $v > fc[l] \Rightarrow$ print symb[3, v - fc[l]] = b