Algorithm Engineering [midterm]
11 November 2020 – time 45 minutes

Question #1 [ranks 4+3+1]. Consider the Snow Plow technique with memory M=2.

- Simulate Snow Plow over the sequence S = (1, 3, 9, 10, 7, 6, 5, 4, 3, 8).
- Provide an example of sequence of length 10 that generates exactly 5 runs, when the memory has size M=2.
- Show the average length of the run produced by Snow Plow, if we assume that the probability that an item goes to the Heap is ¾ rather than ½.

Question #2 [rank 3]. Given a Universal class of hash functions that map keys from a universe U to the range [0,22]. Compute the average number of collisions induced by a function h drawn randomly from that class and mapping a set of 10 keys.

Question #3 [rank 2]. Prove that the probability of having a 0 in a position of the binary array of the Bloom Filter is ½, when the number of hash functions is set to the optimal one.

Question #4 [rank 4]. Show the first 8 codewords of the (s,c)-code with s=3 and c=1, hence s+c = 4 (briefly explain your calculations).

Question #5 [rank 4]. Decompress the 8th integer encoded via Elias-Fano in the two arrays:
L = 011100010100111100 1100 and H= 110 110 10 0 10 10 10 110 0 0 10 0 0 0 0 0 where the original encoding of the integers is in 6 bits. (*hint*: derive first the number of keys, and then the length of the low and high part)

Question #6 [rank 4]. Show the binary succinct encoding of the tree T = { a→b (left child); b→c (left child); b→e (right child); c→d (right child) } of root "a".

Question #7 [rank 5]. Given the set of strings (aa, ba, bb, bc, ca) and you wish to construct a minimal ordered perfect hash function where rank(a,b,c) = (2, 3, 4) and

h1(c' c'') = 2 * rank(c') * rank(c'') mod 11, and
h2(c' c'') = 5 * rank(c') + rank(c'') + 1 mod 11.

Construct the final h(t).