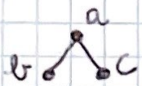
E1 INTERPOLATIVE CODE

DO ONE LEVEL OF RECURSION OF INTERPOLATIVE CODE ON THE SEQUENCE:

$$S = (11, 14, 16, 19, 20, 21, 22).$$



(a) $l = 1 \quad r = 7 \quad low = \overbrace{11}^{S[l]} \quad hi = \overbrace{22}^{S[r]}$

$m = \lfloor \frac{7+1}{2} \rfloor = 4 \Rightarrow S[m] = 19$

$S[m]$ LIES IN $[low + m - l, hi - r + m]$

$[11 + 4 - 1, 22 - 7 + 4]$

$[14, 19]$

SO WE CODE $S[m] - 14$ IN $\lceil \log(19 - 14 + 1) \rceil = 3$ BITS

OUTPUT: "101"

(b) $l = 1 \quad r = 3 \quad low = S[l] = 11 \quad hi = \overbrace{S[4]}^{S[m] \text{ FROM STEP (a)}} - 1 = 18$

$m = \lfloor \frac{1+3}{2} \rfloor = 2 \Rightarrow S[m] = 14$

$S[m]$ LIES IN $[low + m - l, hi - r + m]$

$[11 + 2 - 1, 18 - 3 + 2]$

$[12, 17]$

SO WE CODE $S[m] - 12$ IN $\lceil \log(17 - 12 + 1) \rceil = \lceil \log 6 \rceil = 3$ BITS

OUTPUT: "010"

(c) $l = 5 \quad r = 7 \quad low = \overbrace{S[4]}^{S[m] \text{ FROM STEP (a)}} + 1 = 20 \quad hi = \overbrace{22}^{S[r]}$

$m = \lfloor \frac{5+7}{2} \rfloor = 6 \Rightarrow S[m] = 21$

$S[m]$ LIES IN $[low + m - l, hi - r + m]$

$[20 + 6 - 5, 22 - 7 + 6]$

$[21, 21]$

SO WE CODE $S[m] - 21$ IN $\lceil \log(21 - 21 + 1) \rceil = 0$ BITS

OUTPUT: " "

E2 DELTA- AND RICE-CODE

ENCODE THE SEQUENCE $S = (1, 6, 15, 18, 21, 24, 30)$ USING δ-CODE AND RICE CODE WITH $k=3$

FIRST, WE REPLACE S WITH THE GAP SEQUENCE:

$$S' = (1, \underbrace{5}_{6-1}, \underbrace{9}_{15-6}, \underbrace{3}_{18-15}, 3, 3, 6) \quad \ldots$$

RECALL $\delta(x) = \gamma(\underbrace{|BIN(x)|}_{\text{LENGTH OF BINARY REPRESENTATION OF } x})\, BIN(x)$, WHERE $\gamma(y) = \overbrace{0 \cdots 0}^{|BIN(y)|-1} BIN(y)$

$\delta(1) = \gamma(1)\ 1 = 11$

$\delta(5) = \gamma(3)\ 101 = 011\ 101$

$\delta(9) = \gamma(4)\ 1001 = 00100\ 1001$

$\delta(3) = \gamma(2)\ 11 = 010\ 11$

$\delta(6) = \gamma(3)\ 110 = 011\ 110$

RECALL $R_k(x) = \underbrace{\overbrace{0 \cdots 0 1}^{\text{UNARY OF } q}}_{q+1 \text{ BITS}}\ \underbrace{BIN(r)}_{k \text{ BITS}}$, WHERE $q = \left\lfloor \frac{x-1}{2^k} \right\rfloor$, $r = x - 1 - 2^k q$

$R_3(1) = 1\ 000$ SINCE $q = \left\lfloor \frac{1-1}{8} \right\rfloor = 0$ $r = 0$

$R_3(5) = 1\ 100$ SINCE $q = \left\lfloor \frac{5-1}{8} \right\rfloor = 0$ $r = 4$

$R_3(9) = 01\ 000$ SINCE $q = \left\lfloor \frac{9-1}{8} \right\rfloor = 1$ $r = 0$

$R_3(3) = 1\ 010$ SINCE $q = \left\lfloor \frac{3-1}{8} \right\rfloor = 0$ $r = 2$

$R_3(6) = 1\ 101$ SINCE $q = \left\lfloor \frac{6-1}{8} \right\rfloor = 0$ $r = 5$

E3 (γ, c)-DENSE CODES

SHOW THE (1,3)-CODEWORD OF THE INTEGER 8 OVER 2-BIT "WORDS". HOW MANY BITS FOR THE 36TH CODEWORD?

FIRST WE OBSERVE THAT $\gamma + c = 2^{\text{BIT LENGTH}}$

$1 + 3 = 2^2$ HOLDS

$\gamma = 1$ STOPPERS $\Rightarrow \{\overbrace{00}^{2\text{ BITS}}\}$

$c = 3$ CONTINUERS $\Rightarrow \{01, 10, \overbrace{11}^{2\text{ BITS}}\}$

VALUES	CODEWORDS
0	00
1	01 00
2	10 00
3	11 00
4	01 01 00
5	01 10 00
6	01 11 00
7	10 01 00
8	10 10 00

ABOUT THE 36TH CODEWORD WE OBSERVE THAT

$\gamma = 1$ INTS CAN BE ENCODED WITH ONE WORD

$\gamma c = 3$ INTS " " " " TWO WORDS, FROM 1 TO $1 + \gamma c - 1 = 3$

$\gamma c^2 = 9$ INTS " " " " THREE ", FROM 4 TO $4 + \gamma c^2 - 1 = 12$

$\gamma c^3 = 27$ INTS " " " " FOUR ", FROM 13 TO $13 + \gamma c^3 - 1 = 39$

HENCE, THE 36TH CODEWORD OF THE (1,3)-DENSE CODE REQUIRES FOUR 2-BIT WORDS, I.E. A TOTAL OF 8 BITS

E4 ELIAS-FANO

ENCODE $S = (1, 3, 4, 5, 9, 16, 23, 27, 28, 31, 40)$ WITH ELIAS-FANO AND SHOW HOW TO ANSWER ACCESS(4), NEXT GEQ(8), NEXT GEQ(32)

$u = S[-1]+1 = 41 \quad n = 11 \quad b = \lceil \log u \rceil = 6 \quad l = \lceil \log \frac{u}{n} \rceil = 2 \quad h = b - l = 4$

	$h = 4$ BITS	$l = 2$ BITS
1	0000	01
3	0000	11
4	0001	00
5	0001	01
9	0010	01
16	0100	00
23	0101	11
27	0110	11
28	0111	00
31	0111	11
40	1010	00

$L = 01\ 11\ 00\ 01\ 01\ 00\ 11\ 11\ 00\ 11\ 00$

$H = \underbrace{110}_{0}\ \underbrace{110}_{1}\ \underbrace{10}_{2}\ \underbrace{0}_{3}\ \underbrace{10}_{4}\ \underbrace{10}_{5}\ \underbrace{10}_{6}\ \underbrace{110}_{7}\ \underbrace{0}_{8}\ \underbrace{0}_{9}\ \underbrace{10}_{10}\ \underbrace{0}_{11}\ \underbrace{0}_{12}\ \underbrace{0}_{13}\ \underbrace{0}_{14}\ \underbrace{0}_{15}$

OBSERVE THAT WE HAVE $2^h = 16$ BUCKETS

① ACCESS(4) → LOW PART ≡ 4^{TH} PAIR OF BITS IN $L \equiv (01)_2$

↘ HIGH PART = $\underbrace{\underbrace{SELECT_1(4)}_{\text{POS OF THE } 4^{TH} \text{ 1-BIT IN } H} - \underbrace{4}_{\text{\# 1-BITS UP TO THAT POS}}}_{\text{\#. 0S BEFORE THE } 4^{TH} \text{ 1-BIT} \equiv \text{BUCKET ID CONTAINING } S[4]} = 5 - 4 = 1 \xrightarrow{\text{USING } h \text{ BITS}} (0001)_2$

② NEXT GEQ(8) = NEXT GEQ$((\underbrace{0010}_{\text{HIGH PART} \Rightarrow \text{BUCKET } 2}\ 00)_2)$

STARTING POS OF BUCKET 0010 IN H

$p = SELECT_0(2) + 1 = 6 + 1 = 7$

$H[p] = 1$, SO THE BUCKET IS NON-EMPTY AND WE SCAN ITS ELEMENTS STARTING FROM $p - (0010)_2 = 7 - 2 = 5$. WE RETURN $9 = (001001)_2$

③ NEXT GEQ(32) = NEXT GEQ$((\underbrace{1000}_{\text{BUCKET } 8}\ 00)_2)$

$p = SELECT_0(8) + 1 = 18 + 1 = 19$

$H[p] = 0$, SO THE BUCKET IS EMPTY AND WE RETURN ACCESS$(p - (1000)_2)$

$= ACCESS(19 - 8)$

$= ACCESS(11)$

$= 40$