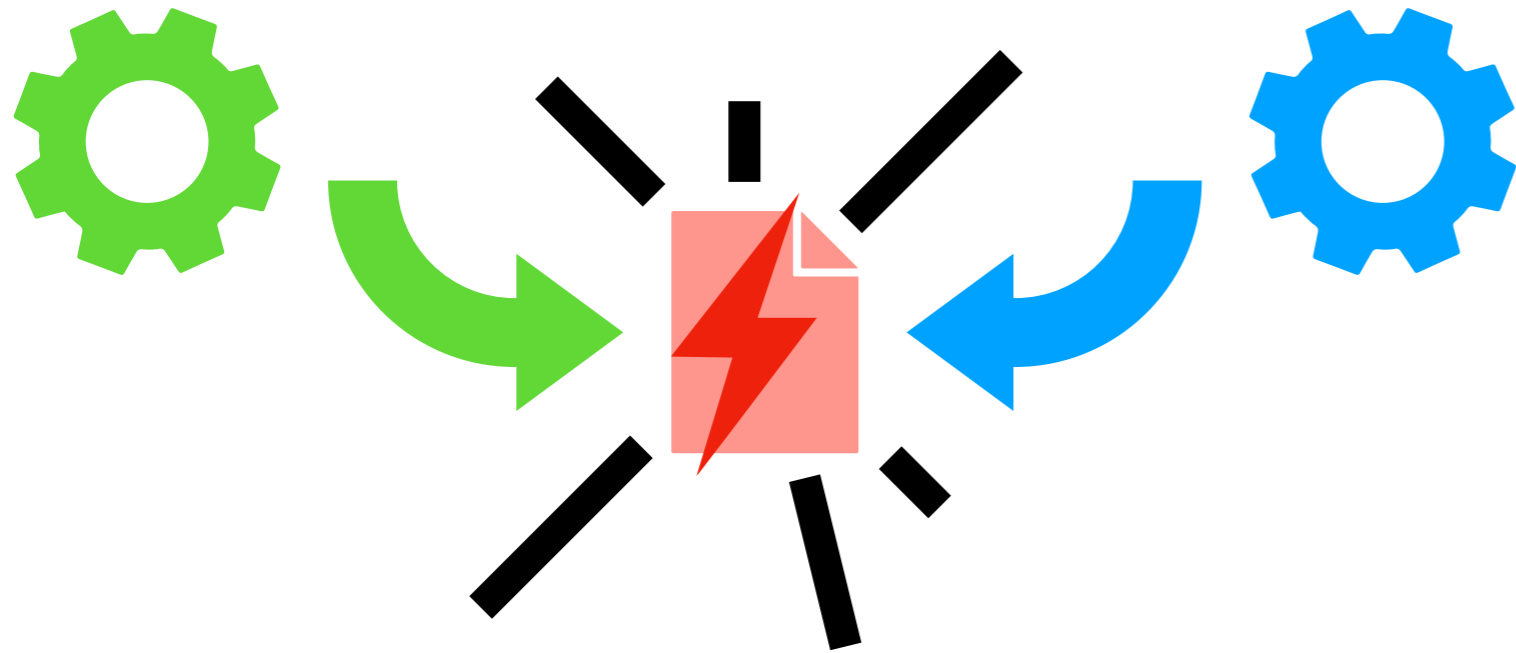


# Linguaggi di Programmazione



Roberta Gori

Esercitazione #1

# Prolog, unificazione, derivazioni

**Ex.1** Sia  $\Sigma_0 = \{0\}$  e  $\Sigma_1 = \{s\}$ . Estendete il programma logico che definisce il predicato  $sum \in \Pi_3$  (visto a lezione) per definire:

1. un predicato  $prod \in \Pi_3$  per calcolare il prodotto di 2 numeri;
2. un predicato  $pow \in \Pi_3$  per calcolare la potenze;
3. un predicato  $div \in \Pi_3$  per verificare che il primo argomento divida il secondo.

# Ex. 1 (1,2,3)

sum(0,y,y) .

sum(s(x),y,s(z)) :- sum(x,y,z) .

1 prod(0,y,0) .  
prod(s(x),y,z) :- prod(x,y,w) , sum(w,y,z) .

2 pow(s(x),0,s(0)) .  
pow(x,s(y),z) :- pow(x,y,w) , prod(w,x,z) .

3 div(s(y),z) :- prod(x,s(y),z) .

[Ex. 2] Data la sintassi in Ex.1, risolvere il seguente problema di unificazione

$$1. G_1 \stackrel{\text{def}}{=} \{ \text{prod}(s(x), y, s(z)) \stackrel{?}{=} \text{prod}(y, z, x) \}$$

$$2. G_2 \stackrel{\text{def}}{=} \{ \text{pow}(x, s(y), x) \stackrel{?}{=} \text{pow}(s(y), z, z) \}$$

$$3. G_3 \stackrel{\text{def}}{=} \{ \text{div}(x, s(y)) \stackrel{?}{=} \text{div}(z, x) , \text{div}(y, s(z)) \stackrel{?}{=} \text{div}(u, s(u)) \}$$

# Unificazione

delete

$\mathcal{G} \cup \{t \stackrel{?}{=} t\}$   
becomes  
 $\mathcal{G}$

eliminate

$\mathcal{G} \cup \{x \stackrel{?}{=} t\}$   
becomes if  $x \in vars(\mathcal{G}) \setminus vars(t)$   
 $\mathcal{G}[x = t] \cup \{x \stackrel{?}{=} t\}$

swap

$\mathcal{G} \cup \{f(t_1, \dots, t_m) \stackrel{?}{=} x\}$   
becomes  
 $\mathcal{G} \cup \{x \stackrel{?}{=} f(t_1, \dots, t_m)\}$

decompose

$\mathcal{G} \cup \{f(t_1, \dots, t_m) \stackrel{?}{=} f(u_1, \dots, u_m)\}$   
becomes  
 $\mathcal{G} \cup \{t_1 \stackrel{?}{=} u_1, \dots, t_m \stackrel{?}{=} u_m\}$

occur-check

$\mathcal{G} \cup \{x \stackrel{?}{=} f(t_1, \dots, t_m)\}$   
fails if  $x \in vars(f(t_1, \dots, t_m))$

conflict

$\mathcal{G} \cup \{f(t_1, \dots, t_m) \stackrel{?}{=} g(u_1, \dots, u_h)\}$   
fails if  $f \neq g$  or  $m \neq h$

# Ex. 2 (1)

$$\{\text{prod}(s(x), y, s(z)) \stackrel{?}{=} \text{prod}(y, z, x)\}$$

decompose

$$\{s(x) \stackrel{?}{=} y, y \stackrel{?}{=} z, s(z) \stackrel{?}{=} x\}$$

swap

$$\{y \stackrel{?}{=} s(x), y \stackrel{?}{=} z, s(z) \stackrel{?}{=} x\}$$

eliminate

$$\{y \stackrel{?}{=} s(x), s(x) \stackrel{?}{=} z, s(z) \stackrel{?}{=} x\}$$

swap

$$\{y \stackrel{?}{=} s(x), z \stackrel{?}{=} s(x), s(z) \stackrel{?}{=} x\}$$

eliminate

$$\{y \stackrel{?}{=} s(x), z \stackrel{?}{=} s(x), s(s(x)) \stackrel{?}{=} x\}$$

swap

$$\{y \stackrel{?}{=} s(x), z \stackrel{?}{=} s(x), x \stackrel{?}{=} s(s(x))\}$$

occur-check

# Ex. 2 (2)

$$\{\text{pow}(x, s(y), x) \stackrel{?}{=} \text{pow}(s(y), z, z)\}$$

decompose

$$\{x \stackrel{?}{=} s(y), s(y) \stackrel{?}{=} z, x \stackrel{?}{=} z\}$$

eliminate

$$\{x \stackrel{?}{=} s(y), s(y) \stackrel{?}{=} z, s(y) \stackrel{?}{=} z\}$$

swap

$$\{x \stackrel{?}{=} s(y), z \stackrel{?}{=} s(y), s(y) \stackrel{?}{=} z\}$$

eliminate

$$\{x \stackrel{?}{=} s(y), z \stackrel{?}{=} s(y), s(y) \stackrel{?}{=} s(y)\}$$

delete

$$\{x \stackrel{?}{=} s(y), z \stackrel{?}{=} s(y)\}$$

$$[x = s(y), z = s(y)]$$



# Ex. 2 (3)

$$\{\text{div}(x, s(y)) \stackrel{?}{=} \text{div}(z, x), \text{div}(y, s(z)) \stackrel{?}{=} \text{div}(u, s(u))\}$$

decompose

decompose

$$\{x \stackrel{?}{=} z, s(y) \stackrel{?}{=} x, y \stackrel{?}{=} u, s(z) \stackrel{?}{=} s(u)\}$$

eliminate

$$\{x \stackrel{?}{=} z, s(y) \stackrel{?}{=} z, y \stackrel{?}{=} u, s(z) \stackrel{?}{=} s(u)\}$$

swap

$$\{x \stackrel{?}{=} z, z \stackrel{?}{=} s(y), y \stackrel{?}{=} u, s(z) \stackrel{?}{=} s(u)\}$$

eliminate

$$\{x \stackrel{?}{=} s(y), z \stackrel{?}{=} s(y), y \stackrel{?}{=} u, s(s(y)) \stackrel{?}{=} s(u)\}$$

eliminate

$$\{x \stackrel{?}{=} s(u), z \stackrel{?}{=} s(u), y \stackrel{?}{=} u, s(s(u)) \stackrel{?}{=} s(u)\}$$

decompose

$$\{x \stackrel{?}{=} s(u), z \stackrel{?}{=} s(u), y \stackrel{?}{=} u, s(u) \stackrel{?}{=} u\}$$

swap

$$\{x \stackrel{?}{=} s(u), z \stackrel{?}{=} s(u), y \stackrel{?}{=} u, u \stackrel{?}{=} s(u)\}$$

occur-check

[Ex. 3] Data il programma logico in Ex.1, scrivere delle derivazioni per i goals seguenti:

1.  $\text{sum}(x, s(0), s(s(0)))$
2.  $\text{prod}(s(s(0)), y, s(s(0)))$
3.  $\text{div}(z, s(s(0)))$

# Ex. 3 (1)

$\text{sum}(0, y, y)$  .  
 $\text{sum}(s(x), y, s(z)) :- \text{sum}(x, y, z)$  .

$$\text{sum}(x, s(0), s(s(0)))$$

$$\swarrow_{\hat{\sigma}_1} \text{sum}(x_1, s(0), s(0))$$

$$\swarrow_{\hat{\sigma}_2} \square$$

Alternativamente:

$$\swarrow_{\hat{\sigma}_2} \text{sum}(x_2, s(0), 0)$$

$$\sigma_1 = [x = s(x_1), y_1 = s(0), z_1 = s(0)]$$

$$\sigma_2 = [x_1 = 0, y_2 = s(0)]$$

$$x = s(0)$$

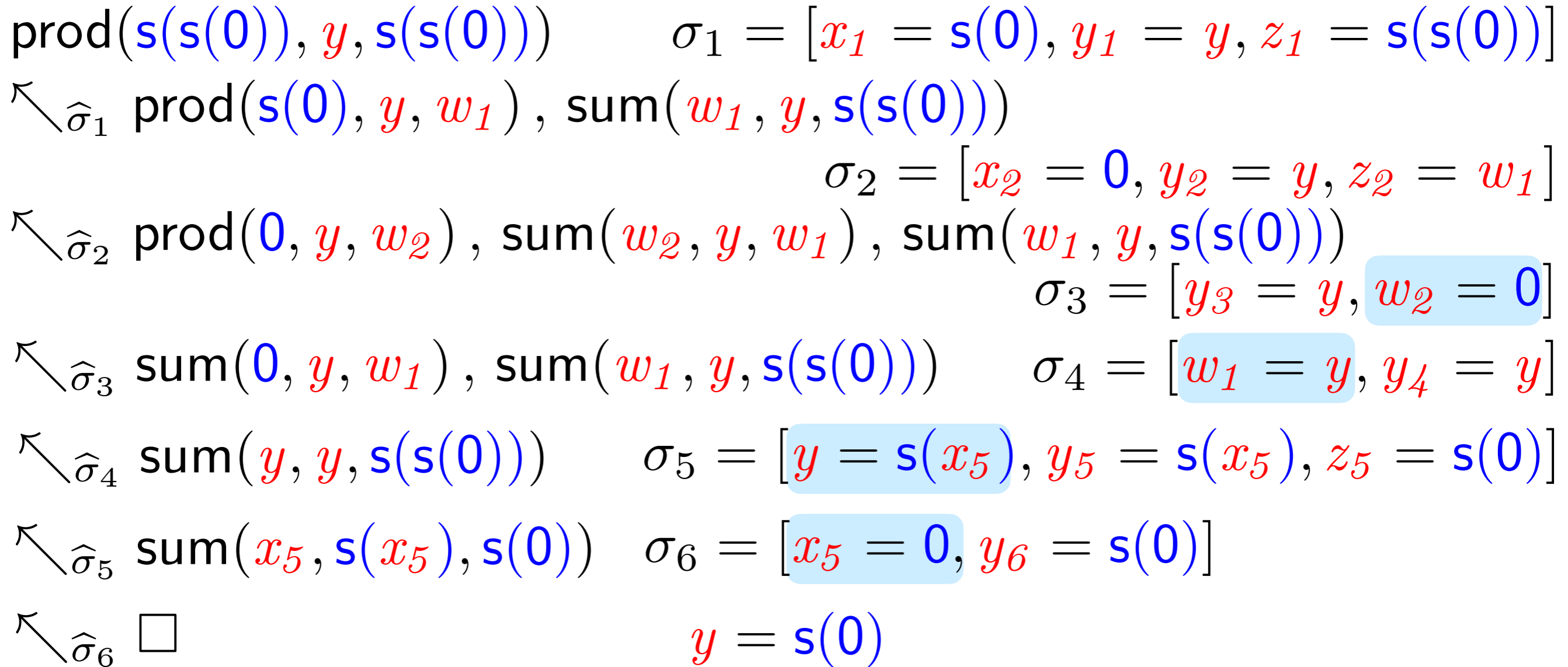
$$\sigma_2 = [x_1 = s(x_2), y_2 = s(0), z_2 = 0]$$

**Fallimento!**

# Ex. 3 (2)

sum(0,y,y) .  
 sum(s(x),y,s(z)) :- sum(x,y,z) .

prod(0,y,0) .  
 prod(s(x),y,z) :- prod(x,y,w) , sum(w,y,z) .

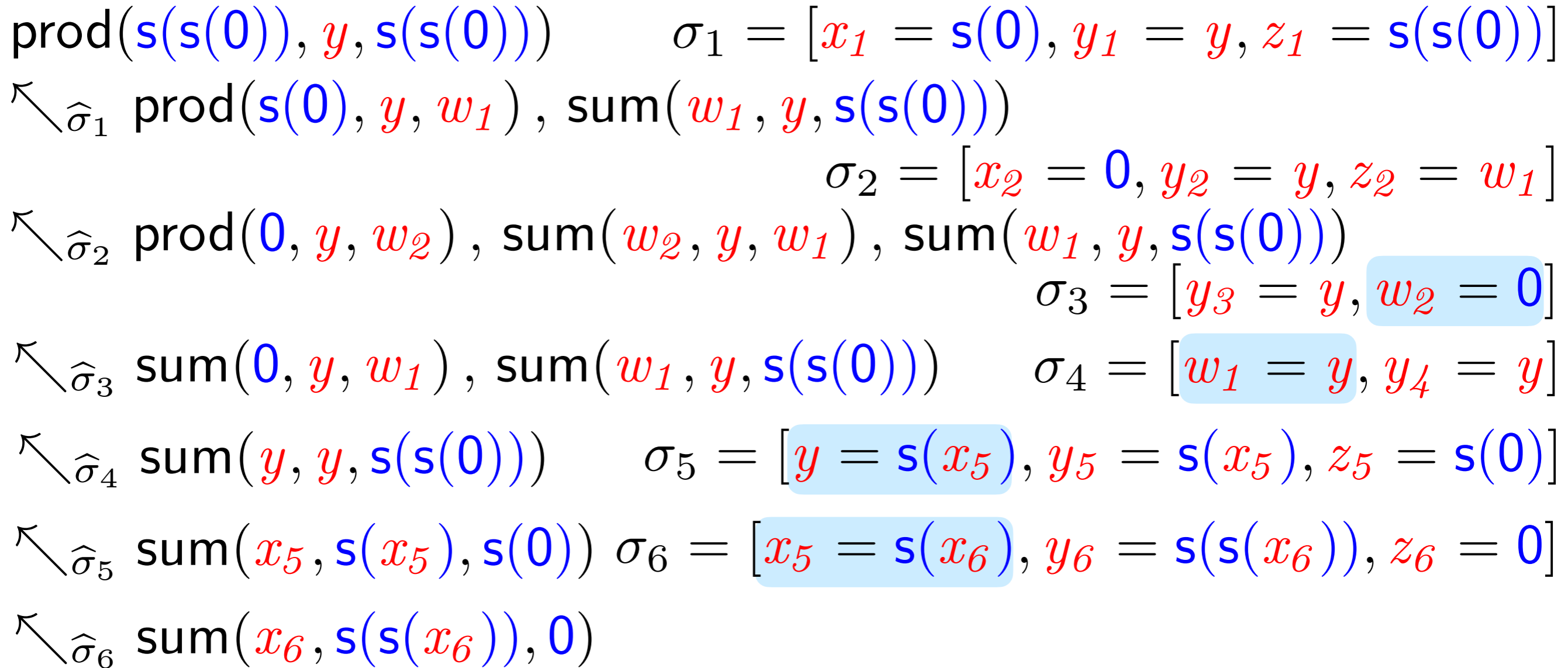


Alternativamente:

# Ex. 3 (2)

sum(0,y,y) .  
 sum(s(x),y,s(z)) :- sum(x,y,z) .

prod(0,y,0) .  
 prod(s(x),y,z) :- prod(x,y,w) , sum(w,y,z) .



**Fallimento!**

# Ex. 3 (3)

sum(0,y,y) .  
sum(s(x),y,s(z)) :- sum(x,y,z) .

prod(0,y,0) .  
prod(s(x),y,z) :- prod(x,y,w) , sum(w,y,z) .

div(s(y),z) :- prod(x,s(y),z) .

$$\text{div}(z, s(s(0))) \quad \sigma_1 = [z = s(y_1), z_1 = s(s(0))]$$

$$\swarrow_{\hat{\sigma}_1} \text{prod}(x_1, s(y_1), s(s(0))) \quad \sigma_2 = [x_1 = s(x_2), y_2 = s(y_1), z_2 = s(s(0))]$$

$$\swarrow_{\hat{\sigma}_2} \text{prod}(x_2, s(y_1), w_2), \text{sum}(w_2, s(y_1), s(s(0))) \quad \sigma_3 = [x_2 = 0, y_3 = s(y_1), w_2 = 0]$$

$$\swarrow_{\hat{\sigma}_3} \text{sum}(0, s(y_1), s(s(0))) \quad \sigma_4 = [y_4 = s(s(0)), y_1 = s(0)]$$

$$\swarrow_{\hat{\sigma}_4} \square \quad z = s(s(0))$$

Alternatively:

# Ex. 3 (3)

sum(0,y,y) .  
 sum(s(x),y,s(z)) :- sum(x,y,z) .

prod(0,y,0) .  
 prod(s(x),y,z) :- prod(x,y,w) , sum(w,y,z) .

div(s(y),z) :- prod(x,s(y),z) .

div(z, s(s(0)))

$\sigma_1 = [z = s(y_1), z_1 = s(s(0))]$

$\swarrow_{\hat{\sigma}_1}$  prod(x<sub>1</sub>, s(y<sub>1</sub>), s(s(0)))

$\sigma_2 = [x_1 = s(x_2), y_2 = s(y_1), z_2 = s(s(0))]$

$\swarrow_{\hat{\sigma}_2}$  prod(x<sub>2</sub>, s(y<sub>1</sub>), w<sub>2</sub>), sum(w<sub>2</sub>, s(y<sub>1</sub>), s(s(0)))

$\sigma_3 = [x_2 = s(x_3), y_3 = s(y_1), z_3 = w_2]$

$\swarrow_{\hat{\sigma}_3}$  prod(x<sub>3</sub>, s(y<sub>1</sub>), w<sub>3</sub>), sum(w<sub>3</sub>, s(y<sub>1</sub>), w<sub>2</sub>), sum(w<sub>2</sub>, s(y<sub>1</sub>), s(s(0)))

$\sigma_4 = [x_3 = 0, y_4 = s(y_1), w_3 = 0]$

$\swarrow_{\hat{\sigma}_4}$  sum(0, s(y<sub>1</sub>), w<sub>2</sub>), sum(w<sub>2</sub>, s(y<sub>1</sub>), s(s(0)))

$\sigma_5 = [y_5 = s(y_1), w_2 = s(y_1)]$

$\swarrow_{\hat{\sigma}_5}$  sum(s(y<sub>1</sub>), s(y<sub>1</sub>), s(s(0)))  $\sigma_6 = [x_6 = y_1, y_6 = s(y_1), z_6 = s(0)]$

$\swarrow_{\hat{\sigma}_6}$  sum(y<sub>1</sub>, s(y<sub>1</sub>), s(0))

$\sigma_7 = [y_1 = 0, y_7 = s(0)]$

$\swarrow_{\hat{\sigma}_7}$  □

$z = s(0)$

Alternativamente:

# Ex. 3 (3)

sum(0,y,y) .  
 sum(s(x),y,s(z)) :- sum(x,y,z) .

prod(0,y,0) .  
 prod(s(x),y,z) :- prod(x,y,w) , sum(w,y,z) .

div(s(y),z) :- prod(x,s(y),z) .

$$\text{div}(z, s(s(0))) \quad \sigma_1 = [z = s(y_1), z_1 = s(s(0))]$$

$$\swarrow_{\hat{\sigma}_1} \text{prod}(x_1, s(y_1), s(s(0))) \quad \sigma_2 = [x_1 = s(x_2), y_2 = s(y_1), z_2 = s(s(0))]$$

$$\swarrow_{\hat{\sigma}_2} \text{prod}(x_2, s(y_1), w_2), \text{sum}(w_2, s(y_1), s(s(0))) \quad \sigma_3 = [x_2 = s(x_3), y_3 = s(y_1), z_3 = w_2]$$

$$\swarrow_{\hat{\sigma}_3} \text{prod}(x_3, s(y_1), w_3), \text{sum}(w_3, s(y_1), w_2), \text{sum}(w_2, s(y_1), s(s(0))) \quad \sigma_4 = [x_3 = 0, y_4 = s(y_1), w_3 = 0]$$

$$\swarrow_{\hat{\sigma}_4} \text{sum}(0, s(y_1), w_2), \text{sum}(w_2, s(y_1), s(s(0))) \quad \sigma_5 = [y_5 = s(y_1), w_2 = s(y_1)]$$

$$\swarrow_{\hat{\sigma}_5} \text{sum}(s(y_1), s(y_1), s(s(0))) \quad \sigma_6 = [x_6 = y_1, y_6 = s(y_1), z_6 = s(0)]$$

$$\swarrow_{\hat{\sigma}_6} \text{sum}(y_1, s(y_1), s(0)) \quad \sigma_7 = [y_1 = s(x_7), y_7 = s(s(x_7)), z_7 = 0]$$

$$\swarrow_{\hat{\sigma}_7} \text{sum}(x_7, s(s(x_7)), 0) \quad \text{Fallimento!}$$



# Induzione Matematica

[Ex. 4] Provare per induzione matematica che

$$\forall n > 0. n^n \geq n!$$

# Ex. 4

$$\forall n > 0. P(n)$$

$$P(n) \triangleq n^n \geq n!$$

$$P(1)$$

$$1^1 = 1 \geq 1 = 1!$$

$$\forall n. P(n) \Rightarrow P(n+1)$$

Prendiamo un generico  $n$

Assumiamo  $P(n) \triangleq n^n \geq n!$

Vogliamo provare  $P(n+1) \triangleq (n+1)^{n+1} \geq (n+1)!$

$$(n+1)^{n+1} = (n+1) \cdot (n+1)^n \geq (n+1) \cdot n^n \geq (n+1) \cdot n! = (n+1)!$$

**[Ex. 5]**

$$a_0 \stackrel{\text{def}}{=} 0 \quad a_{n+1} \stackrel{\text{def}}{=} 2a_n + n$$

Provare per induzione matematica che

$$\forall n \in \mathbb{N}. a_n = 2^n - n - 1$$

# Ex. 5

$$a_0 \triangleq 0 \quad a_{n+1} \triangleq 2a_n + n$$

$$\forall n \in \mathbb{N}. P(n) \quad P(n) \triangleq a_n = 2^n - n - 1$$

$P(0)$

$$a_0 = 0 = 1 - 0 - 1 = 2^0 - 0 - 1$$

$\forall n. P(n) \Rightarrow P(n+1)$

Prendiamo un generico  $n$

Assumiamo  $P(n) \triangleq a_n = 2^n - n - 1$

Vogliamo provare  $P(n+1) \triangleq a_{n+1} = 2^{n+1} - (n+1) - 1$

$$\begin{aligned} a_{n+1} &= 2a_n + n = 2(2^n - n - 1) + n = 2^{n+1} - 2n - 2 + n \\ &= 2^{n+1} - n - 2 = 2^{n+1} - (n+1) - 1 \end{aligned}$$

[Ex. 6] Definiamo i numeri di Fibonacci

$$F_1 \stackrel{\text{def}}{=} 1 \quad F_2 \stackrel{\text{def}}{=} 1 \quad F_{n+2} \stackrel{\text{def}}{=} F_n + F_{n+1}$$

Provare per induzione matematica che

$$\forall n > 0. \sum_{i=1}^n F_i = F_{n+2} - 1$$

# Ex. 6

$$F_1 \triangleq 1 \quad F_2 \triangleq 1 \quad F_{n+2} \triangleq F_n + F_{n+1}$$

$$\forall n > 0. P(n) \quad P(n) \triangleq \sum_{i=1}^n F_i = F_{n+2} - 1$$

$P(1)$

$$\sum_{i=1}^1 F_i = F_1 = 1$$

$$F_{1+2} - 1 = F_3 - 1 = F_1 + F_2 - 1 = 1 + 1 - 1 = 1$$

$P(2)$

$$\sum_{i=1}^2 F_i = F_1 + F_2 = F_3$$

$$F_{2+2} - 1 = F_4 - 1 = F_2 + F_3 - 1 = 1 + F_3 - 1 = F_3$$

# Ex. 6

$$F_1 \triangleq 1 \quad F_2 \triangleq 1 \quad F_{n+2} \triangleq F_n + F_{n+1}$$

$$\forall n > 0. P(n)$$

$$P(n) \triangleq \sum_{i=1}^n F_i = F_{n+2} - 1$$

$$\forall n. P(n) \Rightarrow P(n+1)$$

Prendiamo un generico  $n$

$$\text{Assumiamo } P(n) \triangleq \sum_{i=1}^n F_i = F_{n+2} - 1$$

$$\text{Vogliamo provare } P(n+1) \triangleq \sum_{i=1}^{n+1} F_i = F_{(n+1)+2} - 1$$

$$\sum_{i=1}^{n+1} F_i = F_{n+1} + \sum_{i=1}^n F_i = F_{n+1} + F_{n+2} - 1 = F_{n+3} - 1 = F_{(n+1)+2} - 1$$