Linguaggi di Programmazione

Prontuario

Algoritmo di unificazione

delete

$\mathcal{G} \cup \{t \stackrel{?}{=} t\}$

becomes

$\mathcal{G}$

eliminate

$\mathcal{G} \cup \{x \stackrel{?}{=} t\}$

becomes if $x \in vars(\mathcal{G}) \setminus vars(t)$

$\mathcal{G}[x = t] \cup \{x \stackrel{?}{=} t\}$

swap

$\mathcal{G} \cup \{f(t_1, \ldots, t_m) \stackrel{?}{=} x\}$

becomes

$\mathcal{G} \cup \{x \stackrel{?}{=} f(t_1, \ldots, t_m)\}$

decompose

$\mathcal{G} \cup \{f(t_1, \ldots, t_m) \stackrel{?}{=} f(u_1, \ldots, u_m)\}$

becomes

$\mathcal{G} \cup \{t_1 \stackrel{?}{=} u_1, \ldots, t_m \stackrel{?}{=} u_m\}$

occur-check

$\mathcal{G} \cup \{x \stackrel{?}{=} f(t_1, \ldots, t_m)\}$

fails if $x \in vars(f(t_1, \ldots, t_m))$

conflict

$\mathcal{G} \cup \{f(t_1, \ldots, t_m) \stackrel{?}{=} g(u_1, \ldots, u_h)\}$

fails if $f \neq g$ or $m \neq h$

Sintassi IMP

Definition 3.1 (IMP: syntax). The following productions define the syntax of IMP:

$$
\begin{array}{lcl}
a \in Aexp & ::= & n \mid x \mid a_0 + a_1 \mid a_0 - a_1 \mid a_0 \times a_1 \\
b \in Bexp & ::= & v \mid a_0 = a_1 \mid a_0 \leq a_1 \mid \neg b \mid b_0 \vee b_1 \mid b_0 \wedge b_1 \\
c \in Com & ::= & \textbf{skip} \mid x := a \mid c_0 ; c_1 \mid \textbf{if } b \textbf{ then } c_0 \textbf{ else } c_1 \mid \textbf{while } b \textbf{ do } c
\end{array}
$$

SOS rules

$$\frac{}{\langle x, \sigma \rangle \to \sigma(x)} \text{(ide)}$$

$$\frac{\langle a_0, \sigma \rangle \to n_0 \quad \langle a_1, \sigma \rangle \to n_1}{\langle a_0 + a_1, \sigma \rangle \to n_0 + n_1} \text{(sum)}$$

$$\frac{\langle a_0, \sigma \rangle \to n_0 \quad \langle a_1, \sigma \rangle \to n_1}{\langle a_0 \times a_1, \sigma \rangle \to n_0 \times n_1} \text{(prod)}$$

SOS Rules

$$\frac{}{\langle v, \sigma \rangle \to v} \text{(bool)}$$

$$\frac{\langle b, \sigma \rangle \to v}{\langle \neg b, \sigma \rangle \to \neg v} \text{(not)}$$

$$\frac{\langle a_0, \sigma \rangle \to n_0 \quad \langle a_1, \sigma \rangle \to n_1}{\langle a_0 = a_1, \sigma \rangle \to (n_0 = n_1)} \text{(equ)}$$

$$\frac{\langle b_0, \sigma \rangle \to v_0 \quad \langle b_1, \sigma \rangle \to v_1}{\langle b_0 \vee b_1, \sigma \rangle \to (v_0 \vee v_1)} \text{(or)}$$

$$\frac{\langle a_0, \sigma \rangle \to n_0 \quad \langle a_1, \sigma \rangle \to n_1}{\langle a_0 \leq a_1, \sigma \rangle \to (n_0 \leq n_1)} \text{(leq)}$$

$$\frac{\langle b_0, \sigma \rangle \to v_0 \quad \langle b_1, \sigma \rangle \to v_1}{\langle b_0 \wedge b_1, \sigma \rangle \to (v_0 \wedge v_1)} \text{(and)}$$

SOS Rules

$$\frac{}{\langle \mathbf{skip}, \sigma \rangle \to \sigma} \text{(skip)}$$

$$\frac{\langle a, \sigma \rangle \to m}{\langle x := a, \sigma \rangle \to \sigma[^m/_x]} \text{(assign)}$$

$$\frac{\langle c_0, \sigma \rangle \to \sigma'' \quad \langle c_1, \sigma'' \rangle \to \sigma'}{\langle c_0; c_1, \sigma \rangle \to \sigma'} \text{(seq)}$$

$$\frac{\langle b, \sigma \rangle \to \mathbf{false} \quad \langle c_1, \sigma \rangle \to \sigma'}{\langle \mathbf{if}\ b\ \mathbf{then}\ c_0\ \mathbf{else}\ c_1, \sigma \rangle \to \sigma'} \text{(ifff)}$$

$$\frac{\langle b, \sigma \rangle \to \mathbf{true} \quad \langle c_0, \sigma \rangle \to \sigma'}{\langle \mathbf{if}\ b\ \mathbf{then}\ c_0\ \mathbf{else}\ c_1, \sigma \rangle \to \sigma'} \text{(iftt)}$$

$$\frac{\langle b, \sigma \rangle \to \mathbf{true} \quad \langle c, \sigma \rangle \to \sigma'' \quad \langle \mathbf{while}\ b\ \mathbf{do}\ c, \sigma'' \rangle \to \sigma'}{\langle \mathbf{while}\ b\ \mathbf{do}\ c, \sigma \rangle \to \sigma'} \text{(whtt)}$$

$$\frac{\langle b, \sigma \rangle \to \mathbf{false}}{\langle \mathbf{while}\ b\ \mathbf{do}\ c, \sigma \rangle \to \sigma} \text{(whff)}$$

Semantica denotazione di IMP

$$\mathscr{A}[\![n]\!]\sigma \stackrel{\text{def}}{=} n$$

$$\mathscr{A}[\![x]\!]\sigma \stackrel{\text{def}}{=} \sigma x$$

$$\mathscr{A}[\![a_0 + a_1]\!]\sigma \stackrel{\text{def}}{=} (\mathscr{A}[\![a_0]\!]\sigma) + (\mathscr{A}[\![a_1]\!]\sigma)$$

$$\mathscr{A}[\![a_0 - a_1]\!]\sigma \stackrel{\text{def}}{=} (\mathscr{A}[\![a_0]\!]\sigma) - (\mathscr{A}[\![a_1]\!]\sigma)$$

$$\mathscr{A}[\![a_0 \times a_1]\!]\sigma \stackrel{\text{def}}{=} (\mathscr{A}[\![a_0]\!]\sigma) \times (\mathscr{A}[\![a_1]\!]\sigma)$$

$$\mathscr{B}[\![v]\!]\sigma \stackrel{\text{def}}{=} v$$

$$\mathscr{B}[\![a_0 = a_1]\!]\sigma \stackrel{\text{def}}{=} (\mathscr{A}[\![a_0]\!]\sigma) = (\mathscr{A}[\![a_1]\!]\sigma)$$

$$\mathscr{B}[\![a_0 \leq a_1]\!]\sigma \stackrel{\text{def}}{=} (\mathscr{A}[\![a_0]\!]\sigma) \leq (\mathscr{A}[\![a_1]\!]\sigma)$$

$$\mathscr{B}[\![\neg b]\!]\sigma \stackrel{\text{def}}{=} \neg(\mathscr{B}[\![b]\!]\sigma)$$

$$\mathscr{B}[\![b_0 \vee b_1]\!]\sigma \stackrel{\text{def}}{=} (\mathscr{B}[\![b_0]\!]\sigma) \vee (\mathscr{B}[\![b_1]\!]\sigma)$$

$$\mathscr{B}[\![b_0 \wedge b_1]\!]\sigma \stackrel{\text{def}}{=} (\mathscr{B}[\![b_0]\!]\sigma) \wedge (\mathscr{B}[\![b_1]\!]\sigma)$$

$$\mathscr{C}[\![\mathbf{skip}]\!]\sigma \stackrel{\text{def}}{=} \sigma$$

$$\mathscr{C}[\![x := a]\!]\sigma \stackrel{\text{def}}{=} \sigma[^{\mathscr{A}[\![a]\!]\sigma}/_x]$$

$$\mathscr{C}[\![c_0; c_1]\!]\sigma \stackrel{\text{def}}{=} \mathscr{C}[\![c_1]\!]^*(\mathscr{C}[\![c_0]\!]\sigma)$$

$$\mathscr{C}[\![\mathbf{while}\ b\ \mathbf{do}\ c]\!] = \Gamma_{b,c}\,\mathscr{C}[\![\mathbf{while}\ b\ \mathbf{do}\ c]\!]$$

$$\mathscr{C}[\![\mathbf{while}\ b\ \mathbf{do}\ c]\!] \stackrel{\text{def}}{=} \text{fix}\ \Gamma_{b,c} = \bigsqcup_{n\in\mathbb{N}} \Gamma^n_{b,c}(\bot_{\Sigma\to\Sigma_\bot})$$

$$\Gamma_{b,c} \stackrel{\text{def}}{=} \lambda\varphi.\ \lambda\sigma.\ \mathscr{B}[\![b]\!]\sigma \to \varphi^*(\mathscr{C}[\![c]\!]\sigma), \sigma$$

Sintassi HOFL

$$
\begin{array}{rcl}
t & ::= & x \mid n \mid t_0 \ \mathsf{op}\ t_1 \mid \mathbf{if}\ t\ \mathbf{then}\ t_0\ \mathbf{else}\ t_1 \\
 & \mid & (t_0, t_1) \mid \mathbf{fst}(t) \mid \mathbf{snd}(t) \\
 & \mid & \lambda x.\ t \mid t_0\ t_1 \\
 & \mid & \mathbf{rec}\ x.\ t
\end{array}
$$

Sistema di tipi

$$\frac{}{x : \widehat{x}} \qquad \frac{}{n : int} \qquad \frac{t_0 : int \quad t_1 : int}{t_0 \ \mathsf{op}\ t_1 : int} \qquad \frac{t : int \quad t_0 : \tau \quad t_1 : \tau}{\mathbf{if}\ t\ \mathbf{then}\ t_0\ \mathbf{else}\ t_1 : \tau}$$

$$\frac{t_0 : \tau_0 \quad t_1 : \tau_1}{(t_0, t_1) : \tau_0 * \tau_1} \qquad \frac{t : \tau_0 * \tau_1}{\mathbf{fst}(t) : \tau_0} \qquad \frac{t : \tau_0 * \tau_1}{\mathbf{snd}(t) : \tau_1}$$

$$\frac{x : \tau_0 \quad t : \tau_1}{\lambda x.\ t : \tau_0 \to \tau_1} \qquad \frac{t_1 : \tau_0 \to \tau_1 \quad t_0 : \tau_0}{t_1\ t_0 : \tau_1}$$

$$\frac{x : \tau \quad t : \tau}{\mathbf{rec}\ x.\ t : \tau}$$

Semantica op. Lazy

$$\frac{}{n \rightarrow n} \qquad \frac{t_0 : \tau_0 \quad t_1 : \tau_1 \quad t_0, t_1 \text{ closed}}{(t_0, t_1) \rightarrow (t_0, t_1)} \qquad \frac{\lambda x.\, t : \tau_0 \rightarrow \tau_1 \quad \lambda x.t \text{ closed}}{\lambda x.\, t \rightarrow \lambda x.\, t}$$

$$\frac{t_0 \rightarrow n_0 \quad t_1 \rightarrow n_1}{t_0 \text{ op } t_1 \rightarrow n_0 \underline{\text{op}}\, n_1} \qquad \frac{t \rightarrow 0 \quad t_0 \rightarrow c_0}{\textbf{if } t \textbf{ then } t_0 \textbf{ else } t_1 \rightarrow c_0} \qquad \frac{t \rightarrow (t_0, t_1) \quad t_0 \rightarrow c_0}{\textbf{fst}(t) \rightarrow c_0}$$

$$\frac{t[^{\textbf{rec } x.\, t}/_x] \rightarrow c}{\textbf{rec } x.\, t \rightarrow c} \qquad \frac{t \rightarrow n \quad n \neq 0 \quad t_1 \rightarrow c_1}{\textbf{if } t \textbf{ then } t_0 \textbf{ else } t_1 \rightarrow c_1} \qquad \frac{t \rightarrow (t_0, t_1) \quad t_1 \rightarrow c_1}{\textbf{snd}(t) \rightarrow c_1}$$

$$\frac{t_1 \rightarrow \lambda x.\, t_1' \quad t_1'[^{t_0}/_x] \rightarrow c}{(t_1\ t_0) \rightarrow c} \quad \text{(lazy)}$$

Semantica denotazionale HOFL

$$[\![n]\!]\rho \triangleq \lfloor n \rfloor$$

$$[\![x]\!]\rho \triangleq \rho(x)$$

$$[\![t_1 \ \mathsf{op} \ t_2]\!]\rho \triangleq [\![t_1]\!]\rho \ \underline{\mathsf{op}}_\perp \ [\![t_2]\!]\rho$$

$$[\![\mathbf{if} \ t \ \mathbf{then} \ t_1 \ \mathbf{else} \ t_2]\!]\rho \triangleq \mathsf{Cond}_\tau(\ [\![t]\!]\rho \ , \ [\![t_1]\!]\rho \ , \ [\![t_2]\!]\rho \ )$$

$$[\![(t_1 \ , \ t_2)]\!]\rho \triangleq \lfloor \ (\ [\![t_1]\!]\rho \ , \ [\![t_2]\!]\rho \ ) \ \rfloor$$

$$[\![\mathbf{fst}(\ t \ )]\!]\rho \triangleq \pi_1^* \ (\ [\![t]\!]\rho \ )$$

$$[\![\mathbf{snd}(\ t \ )]\!]\rho \triangleq \pi_2^* \ (\ [\![t]\!]\rho \ )$$

$$[\![\lambda x. \ t]\!]\rho \triangleq \lfloor \ \lambda d. \ [\![t]\!]\rho[^d/_x] \ \rfloor$$

$$[\![\ t \ t_0 \ ]\!]\rho \triangleq \mathbf{let} \ \varphi \Leftarrow [\![t]\!]\rho. \ \varphi([\![t_0]\!]\rho)$$

$$[\![\mathbf{rec} \ x. \ t]\!]\rho \triangleq \mathit{fix} \ \lambda d. \ [\![t]\!]\rho[^d/_x]$$

Sintassi del CCS

$$
\begin{array}{lcl}
p, q & ::= & \mathbf{nil} \\
 & | & x \\
 & | & \mu.p \\
 & | & p\backslash\alpha \\
 & | & p[\phi] \\
 & | & p + q \\
 & | & p|q \\
 & | & \mathbf{rec}\ x.\ p
\end{array}
$$

Semantica operazionale CCS

Act) $$\frac{}{\mu.p \xrightarrow{\mu} p}$$

Res) $$\frac{p \xrightarrow{\mu} q \quad \mu \notin \{\alpha, \overline{\alpha}\}}{p\backslash\alpha \xrightarrow{\mu} q\backslash\alpha}$$

Rel) $$\frac{p \xrightarrow{\mu} q}{p[\phi] \xrightarrow{\phi(\mu)} q[\phi]}$$

SumL) $$\frac{p_1 \xrightarrow{\mu} q}{p_1 + p_2 \xrightarrow{\mu} q}$$

SumR) $$\frac{p_2 \xrightarrow{\mu} q}{p_1 + p_2 \xrightarrow{\mu} q}$$

ParL) $$\frac{p_1 \xrightarrow{\mu} q_1}{p_1|p_2 \xrightarrow{\mu} q_1|p_2}$$

Com) $$\frac{p_1 \xrightarrow{\lambda} q_1 \quad p_2 \xrightarrow{\overline{\lambda}} q_2}{p_1|p_2 \xrightarrow{\tau} q_1|q_2}$$

ParR) $$\frac{p_2 \xrightarrow{\mu} q_2}{p_1|p_2 \xrightarrow{\mu} p_1|q_2}$$

Rec) $$\frac{p[{}^{\mathbf{rec}\ x.\ p}/_{x}] \xrightarrow{\mu} q}{\mathbf{rec}\ x.\ p \xrightarrow{\mu} q}$$

HML: sintassi

$$
\begin{array}{lcl}
F, G & ::= & \mathbf{tt} \\
 & \mid & \mathbf{ff} \\
 & \mid & \bigwedge_{i \in I} F_i \\
 & \mid & \bigvee_{i \in I} F_i \\
 & \mid & \Diamond_{\mu} F \\
 & \mid & \Box_{\mu} F
\end{array}
$$